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Analysis and Modeling of Interharmonics from Grid-Connected Photovoltaic Systems

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Abstract—The industry of solar Photovoltaic (PV) energy and its application are still booming to enhance the sustainability of the society. When PV systems are connected to the grid, challenging issues should be addressed. One of the challenges is related to interharmonics in PV systems, especially with a large-scale adoption of PV systems. However, the origins of interharmonics remain unclear, although the impact of interharmonics has been reported in literature. Thus, this paper explores the generation mechanisms of interharmonics in PV systems and its characteristics. The exploration reveals that the perturbation from the Maximum Power Point Tracking (MPPT) algorithm is one of the origins of interharmonics appearing in the grid current. Accordingly, the MPPT controller parameters such as the perturbation step-size and the sampling rate have an inevitable impact on the interharmonic characteristics. Furthermore, an approach to characterize the interharmonics in the grid current is proposed. With the proposed model, interharmonics can be predicted according to the designed controller parameters in terms of frequencies and amplitudes. Experimental tests are performed on a single-phase grid-connected PV system. The results are in a close agreement with the analysis, and thus validate the effectiveness of the proposed model.

Index Terms—Photovoltaic (PV) systems, inverters, maximum power point tracking (MPPT), interharmonics, power quality, modeling.

I. INTRODUCTION

In recent years, more and more PV systems have been installed and connected to the grid due to the increasing demand of greener and more sustainable energy systems. However, at the same time, massive connections of PV systems to the grid bring several challenges, e.g., power quality and grid stability issues [1]–[6]. It has been reported that the grid-connected PV systems might be one source of harmonics including interharmonics delivered to the grid, which leads to a poor power quality [7]–[12]. In the latest research, many attempts have been made to identify the impact of harmonics from grid-connected PV inverters. In general, harmonics are generated by PV inverters due to the non-ideal switching behaviors of the power devices (e.g., the dead-time effect) and also the interaction among different controllers (e.g., the dc-link voltage and the current controllers) [13]–[15]. Solutions to mitigate harmonics can also be found in the literature [11]–[15]. On the other hand, the interharmonic issues in PV systems are rarely discussed. Nevertheless, it has been revealed in a few recent publications that a large-scale adoption of grid-connected PV inverters may contribute to interharmonics in the grid currents, causing overloading and flickering [16]–[22]. In the worst scenario, interharmonics may trigger the protection circuit, and thus the PV systems are unintentionally disconnected from the grid [18]. Consequences of such events include considerable energy losses and challenges in the system stability in the case of large-scale PV systems. As the penetration level of PV systems is still increasing, the impact of interharmonics cannot be overlooked. Therefore, the interharmonic issues of PV inverters should be explored and analyzed in order to develop proper mitigation approaches.

By definition, interharmonics are frequency components with non-integer times of the fundamental frequency (e.g., 50 Hz in Europe). In other words, interharmonics can be discrete frequency components (i.e., a single frequency component) or distortion signals with a wide-band spectrum in some in-between integer harmonics [23]. In this regard, the detection of interharmonics is challenging in practice [24]. Concerning the generation of interharmonics, there are several possibilities in power electronic systems [23]: two asynchronous conversion systems (e.g., motor drives) [25]–[27], time-varying loads (e.g., arc furnaces) [28], and mechanical vibrations (e.g., in wind turbines) [29]. Among those, PV systems are usually not considered as a source of interharmonics, since it is typically viewed as a single dc-ac conversion system. However, as the Maximum Power Point Tracking (MPPT) control is mandatory in PV systems and it imposes variations in the dc power during the Maximum Power Point (MPP) searching, interharmonics may also be generated from PV systems [19]–[21]. This has been observed experimentally with some commercial PV inverters [19]–[22], where a considerable amount of interharmonics from PV inverters have been measured during the steady-state MPPT operation. Specifically, it is suggested in [19]–[21] that the interharmonic frequency spectrum is correlated with the MPPT frequency.

Similar characteristics have also been observed in a commercial PV inverter tested at Aalborg University. The test results are shown in Fig. 1 and the frequency spectrum presented in Fig. 2 demonstrates that there are interharmonics in the injected grid current. This can be clearly seen when the PV inverter operates at low-power conditions. Furthermore, as shown in Fig. 2, the interharmonics appear as a series of low-frequency components and to some extent, in

This is the reference copy of the accepted version. When it is published, color versions of one or more of the figures in this paper will be available online at http://ieeexplore.ieee.org.
agreement with the observations seen in a previous study [20]. Nevertheless, the designed controller parameters of the commercial PV inverter under test are not available. Thus, mapping the interharmonic frequency to the MPPT or other control frequencies from the experimental data is a difficult task. That is, the mechanism of the interharmonic emission is not yet fully understood, and the above analysis is done still from observations of measurements. Although a brief discussion on the interharmonic emission mechanisms through simulations has been presented in [30], an in-depth analysis of the mechanisms is still absent and has not been validated experimentally either. Moreover, the influence of controller parameters (e.g., the MPPT controller parameters) on the interharmonic characteristics has not been explored. In all, it calls for a more detailed modeling and a thorough analysis of the interharmonics in grid-connected PV systems.

Accordingly, this paper explores the mechanisms of interharmonic emissions from grid-connected PV systems in § II. It has been observed on a 3-kW single-phase PV system in § II that the perturbation of the MPPT operation is one of the main root-causes of interharmonics. The influence of the controller parameters, such as the perturbation step-size and the MPPT algorithm sampling rate, on the interharmonics has also been investigated in § III. Furthermore, an analytical model is proposed to identify interharmonics according to the pre-designed controller parameters. The analysis results are shown in § IV, which agree well with the experimental tests in § III. This validates the feasibility of the proposed model for interharmonic analysis and identification. Finally, concluding remarks are given in § V.

II. CONTROL AND OPERATION OF SINGLE-PHASE GRID-CONNECTED PV INVERTERS

A. System Configuration and Control Structure

In order to investigate the interharmonics from grid-connected PV systems, experimental tests have been conducted on a single-phase single-stage grid-connected PV system. The system configuration is shown in Fig. 3. The experimental setup shown in Fig. 4 is employed and the system parameters are given in Table I. A PV simulator (Chroma 62000H-S) is employed to emulate the PV array characteristic during the test [31]. With the simulator, it is possible to program the PV array characteristic such as the maximum available power and the voltage at the MPP in order to make the test condition repeatable to compare different cases (e.g., different control parameters). The PV inverter is connected to the grid through an LC-filter and an isolation transformer in order to provide a galvanic isolation between the PV array and the grid.

With this single-stage configuration, the PV inverter (e.g., the full-bridge inverter) plays a major role in controlling the power delivery from the PV arrays to the ac grid [32]. In order to ensure the maximum power extraction from the PV arrays, an MPPT algorithm (e.g., Perturb and Observe - P&O) is employed to determine the reference dc-link voltage $v_{dc}^{*}$ (i.e., the PV voltage) during operation. Then, the dc-link voltage controller, e.g., a Proportional Integral (PI) controller, regulates the dc-link voltage $v_{dc}$ accordingly through the control of the amplitude of the grid current $|i_g|$. The reference grid current $i_{g}^{*}$ is generated considering the phase of the grid voltage $\sin(\theta_g)$, which is obtained from a Phase-Locked Loop (PLL). For single-phase systems, the dc-link voltage $v_{dc}$ contains
double-line frequency components (e.g., 100 Hz) [33]. Thus, a notch filter is used to improve the dc-link voltage control performance [33], [34].

The control performance of the PV system is demonstrated by introducing a step change in the reference dc-link voltage $v^\text{dc}_{\text{ref}}$ at $t = 2$ s. It can be seen from the experimental results in Fig. 5 that the dc-link voltage $v_{\text{dc}}$ can quickly follow a change of the set-point $v^\text{dc}_{\text{ref}}$. The injected grid current also react to the change, as the dc-link voltage error (see Fig. 5(c)) is used to generate the amplitude of the grid current reference. The frequency spectrum of the grid current during the steady-state operation is shown in Fig. 6, where a constant dc-link voltage reference is applied (i.e., the MPPT control is disabled). It can be seen in Fig. 6 that the grid current $i_g$ contains only the fundamental component (i.e., 50 Hz). In other words, the interharmonics in the grid current are negligible under this operating condition. The experiment implies that the interharmonics are potentially related to the dc-link control, more specifically, the dc-link voltage reference.

### B. Maximum Power Point Tracking (MPPT) Operation

According to [19]–[21] and [30], the MPPT operation is considered as a source of interharmonics in PV systems. Therefore, the performance of PV systems under the MPPT operation is investigated in this section. Here, the P&O MPPT algorithm is employed due to its simplicity. In fact, the P&O MPPT algorithm is one of the most widely-used MPPT algorithms in industry [35], and its performance has also been observed previously from the test on the commercial PV inverter in Fig. 1.

![Fig. 4. Experimental setup of a single-phase single-stage grid-connected PV system to explore interharmonics.](image)

### Table I

**PARAMETERS OF THE SINGLE-PHASE GRID-CONNECTED PV SYSTEM.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV rated power</td>
<td>3 kW</td>
</tr>
<tr>
<td>DC-link capacitor</td>
<td>$C_{\text{dc}} = 1100 \ \mu\text{F}$</td>
</tr>
<tr>
<td>LC-filter</td>
<td>$L_{\text{inv}} = 4.8 \ \text{mH}$, $C_f = 4.3 \ \mu\text{F}$</td>
</tr>
<tr>
<td>Leakage inductance of the transformer</td>
<td>$L_{\text{g}} = 2 \ \text{mH}$</td>
</tr>
<tr>
<td>Switching frequency</td>
<td>$f_s = 8 \ \text{kHz}$</td>
</tr>
<tr>
<td>Controller sampling frequency</td>
<td>$f_{\text{inv}} = 20 \ \text{kHz}$</td>
</tr>
<tr>
<td>DC-link voltage</td>
<td>$v^\text{dc}_{\text{ref}} = 450 \ \text{V}$</td>
</tr>
<tr>
<td>Grid nominal voltage (RMS)</td>
<td>$V_g = 230 \ \text{V}$</td>
</tr>
<tr>
<td>Grid nominal frequency</td>
<td>$f_g = 50 \ \text{Hz}$</td>
</tr>
</tbody>
</table>

One important characteristic of the P&O MPPT algorithm (and also other hill-climbing type MPPT algorithms) is the power oscillation during the steady-state operation (e.g., even under a constant solar irradiance condition) [35]. This can be clearly seen from the experimental results in Figs. 7 and 8, where the PV system operates in the steady-state condition with a constant solar irradiance and ambient temperature condition at 100 % and 10 % of the rated power, respectively. Notably, the controller parameters (e.g., PI controller parameters, MPPT perturbation step-size $v_{\text{step}}$, and MPPT algorithm sampling rate $f_{\text{MPPT}}$) are identical for both operating conditions.

![Fig. 5. Experimental results of the PV inverter during a step change in the reference dc-link voltage from $v^\text{dc}_{\text{ref}} = 450 \ \text{V}$ to $v^\text{dc}_{\text{ref}} = 500 \ \text{V}$ (i.e., the PV power is reduced) at $t = 2$ s: (a) dc-link voltage ($v_{\text{dc}}$ is the measured dc-link voltage, $v^\text{dc}_{\text{ref}}$ is the dc component of $v^\text{dc}_{\text{ref}}$), (b) grid current $i_g$, and (c) error in the dc-link voltage $\varepsilon_{\text{dc}}$.](image)

![Fig. 6. Frequency spectrum of the grid current $i_g$ during the constant dc-link voltage operation at 100 % of the rated power (i.e., 3 kW) with the frequency resolution of 0.125 Hz.](image)
conditions. In those cases, the MPPT operation is stable and the dc-link voltage $v_{dc}$ (i.e., the PV voltage) oscillates around three optimum operating points with the MPPT algorithm sampling frequency $f_{MPPT}$ of 5 Hz. This is considered as the optimal MPPT operation, as it is discussed in [36], and similar behaviors have been observed in the commercial PV inverter as shown in Fig. 1. However, it can be noticed that the injected grid current $i_g$ is distorted due to the perturbation, which can be clearly seen at the low-power operation in Fig. 8(b). In that case, the injected grid current $i_g$ presents a large overshoot during the perturbation of the dc-link voltage $v_{dc}$. Since this perturbation will be repeated periodically (during the steady-state operation), interharmonics are generated in the grid current, which will be analyzed in detail in the following.

C. Interharmonic Characteristics

The interharmonics characteristic can be characterized considering the frequency components in the grid current $i_g$. Fig. 9 shows the frequency spectrum of the grid current from the PV system operated at different power levels (i.e., 100 % and 10 % of the rated power). The measurements have been carried out for 8 s during the test (only half of the time-domain waveforms are shown in Figs. 7 and 8). Therefore, the frequency spectrum can be analyzed with a frequency resolution of $1/8 = 0.125$ Hz. It can be seen from the results in Fig. 9 that the interharmonics in the grid current appear as a series of frequency components with a constant distance between each consecutive component. The envelope of the frequency component is almost symmetrical around the fundamental component of the grid current. This resembles to the test results from the commercial PV inverter in Fig. 2 and also in the literature [20]. In this case, the distance between each consecutive interharmonic frequency is 2.5 Hz (e.g., 56.25 Hz, 58.75 Hz, 61.25 Hz, ...). This is applied for the
operation at 100 % and 10 % of the rated power. In both cases, the interharmonics appear at the same frequencies. Moreover, the amplitudes of the interharmonics are also similar (in absolute values), although the amplitude of the fundamental-frequency component at 100% of the rated power is around 10 times higher than that at 10 % of the rated power.

D. Inter harmonic Emission Mechanism

It is necessary to understand the root-causes of interharmonics in order to map the interharmonic characteristics with the control parameters. From the results in Fig. 8, it can be seen that the interharmonics in the grid current are induced by the MPPT perturbation, where the reference dc-link voltage \( v_{dc}^* \) experiences step changes with an amplitude corresponding to the MPPT perturbation step-size \( \Delta v_{dc} \). This will result in an error \( \epsilon_{dc} \) at the input of the dc-link voltage controller as shown in Fig. 8(c). Due to the periodic MPPT perturbation, the error in the dc-link voltage \( \epsilon_{dc} \) is also a periodic waveform, which contains a set of frequency components \( f_n \). Since the dc-link voltage controller is the outer control loop (see Fig. 3), the frequency components of the dc-link voltage error \( \epsilon_{dc} \) can propagate to the grid current amplitude reference \( |i_g^*| \). Then, the multiplication between the grid current amplitude reference \( |i_g^*| \) and the phase of the grid voltage \( \sin(\theta_g) \) will induce an amplitude modulation between the two signals. As a consequence, the reference grid current \( i_g^* \) will contain the frequency components of \( f_g \pm f_n \), where \( f_g \) is the fundamental grid frequency. Assuming an ideal current controller, the injected grid current \( i_g \) will follow its reference \( i_g^* \), inducing the interharmonic components with the frequencies being \( f_g \pm f_n \) in the grid current. Fig. 10 summarizes the frequency mapping of each part in the control structure. Notably, the response of the MPPT perturbation is almost independent on the operating power of the PV system, as it can be seen in Figs. 7(c) and 8(c). Consequently, the interharmonic characteristics in the grid current under both operating conditions (i.e., 100 % and 10 % of the rated power) are similar in terms of frequency and amplitude.

To verify the above analysis, the frequency spectrum of the error in the dc-link voltage is shown in Fig. 11. It can be observed in Fig. 11 that the frequency of the interharmonic components is \( f_n = (2n - 1)f_{MPPT}/4 \), where \( f_{MPPT} \) is the MPPT algorithm sampling frequency and \( n \) is an integer number (e.g., \( f_n = 1.25 \ Hz, 3.75 \ Hz, 6.25 \ Hz \ldots \) for \( f_{MPPT} = 5 \ Hz \)). The dominant frequency components with high amplitudes are around 6.25 Hz, 8.75 Hz, and 11.25 Hz. If the grid voltage only contains the fundamental component (e.g., \( f_g = 50 \ Hz \)), which is the case in this experiment, the dominant frequencies after the amplitude modulation should be 38.75 Hz, 41.25 Hz, and 43.75 Hz (i.e., \( f_g - f_n \)) and 56.25 Hz, 58.75 Hz and 61.25 Hz (i.e., \( f_g + f_n \)). This is in a close agreement with the frequency spectrum of the grid current shown in Fig. 9. Thus, the above analysis in terms of frequency mapping is valid. In order to further analyze the amplitude of each interharmonic component, a control system model is required for determining the system response due to the MPPT perturbation. This will be explained in § IV.

III. Controller Parameter Impact on Interharmonics

In the above discussion, the interharmonics have been observed in the grid current during the MPPT operation. Although the experimental results suggest that the interharmonic characteristics of the grid current are almost not affected by the operating power of the PV system, the controller parameters may have certain impacts. More specifically, the parameters of the MPPT algorithm may change the interharmonic characteristics according to the frequency mapping in Fig. 10. The perturbation step-size \( \Delta v_{step} \) and the sampling rate \( f_{MPPT} \) are two typical parameters of the MPPT algorithms, whose influence on the interharmonics will be discussed in the following.

A. Impact of the MPPT Perturbation Step-Size

The perturbation step-size \( \Delta v_{step} \) strongly affects the tracking performance of the MPPT algorithms. A large step-size can improve the tracking speed due to the reduced number of iterations (e.g., during the solar irradiance change) and also increase the noise immunity (e.g., due to ripples in the dc-link voltage and measurement noise) [36]. However, it will result in large power oscillations during steady-state operation, increasing the power losses and thus lowering the overall efficiency.

This phenomenon is demonstrated in Fig. 12, where three different perturbation step-sizes of \( \Delta v_{step} = 6 \ V, 12 \ V, \) and \( 18 \ V \) are employed in P&O MPPT algorithm. Observations in Fig. 12 indicate that the step-size clearly affects the interharmonics. Notably, the operating power of the PV system is kept at 10
% of the rated power (i.e., 300 W) for all cases, where the impact of the perturbation step-size can be clearly seen in the time-domain waveforms of the grid current \( i_g \) (the amplitude of the interharmonic component becomes comparable with the fundamental frequency component). The other control parameters (e.g., parameters for the PI and current controllers) are identical. It can be concluded from the above tests that large step-sizes will result in higher overshoots in the grid current during the perturbation. The corresponding frequency spectrum of the grid current is analyzed and also shown in the same figure. It further confirms that the perturbation step-size has a strong influence on the amplitude of the interharmonic components. However, it is worth mentioning that the frequencies of the interharmonics are the same regardless of the perturbation step-size amplitude, implying that the frequency is not dependent on the perturbation step-size.

### B. Impact of the MPPT Sampling Rate

Another important parameter of MPPT algorithms is the sampling rate \( f_{\text{MPPT}} \). Increasing the MPPT sampling rate can improve the tracking performance in the case of fast solar irradiance changes. However, the maximum sampling rate of an MPPT algorithm is limited by the dynamic of the dc-link voltage controller, as the dc-link voltage (PV voltage) should reach the steady-state operation before applying the next perturbation. Typically, the MPPT sampling rate is about 1-20 Hz \([37]-[39]\).

In this paper, three MPPT sampling rates (i.e., \( f_{\text{MPPT}} = 2 \) Hz, 5 Hz, and 10 Hz) are chosen. The grid currents with different MPPT sampling rates are shown in Fig. 13. Clearly, the perturbation in the grid current occurs more frequently as the MPPT sampling rate increases. On the other hand, the overshoot in the grid current is almost the same with the three MPPT sampling rates. Furthermore, when seeing from the frequency spectrum of the grid current in Fig. 13, the frequency distance between the consecutive interharmonic component increases along with the MPPT sampling rate. The same observations go for the amplitude of the interharmonic components, although the perturbation step-size \( v_{\text{step}} \) is the same for all cases. This brings more insights into the design of MPPT algorithms – increasing the MPPT sampling rate may challenge the power quality of the injected grid current in terms of interharmonics.

### IV. MODELING OF INTERHARMONICS

As discussed in the last section, the controller parameters (e.g., the perturbation step-size \( v_{\text{step}} \) and the MPPT sampling rate \( f_{\text{MPPT}} \)) strongly affect the interharmonics in the grid current. In order to map the interharmonic characteristic with the designed controller parameters, modeling of interharmonics is presented in this section. The flow diagram of the interharmonic modeling is illustrated in Fig. 14, where three modeling parts are involved: 1) Periodic MPPT oscillation, 2) Response of the dc-link voltage perturbation, and 3) Amplitude modulation.

#### A. Periodic MPPT Oscillation

Since the main cause of interharmonics in the grid current is the MPPT perturbation, the representation of the dc-link voltage during the MPPT perturbation is required as an input to the analysis. In that case, the Fourier analysis should be applied to the variation in the reference dc-link voltage during

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**Fig. 12.** Experimental results of the PV system operating at 10% of the rated power (i.e., 300 W) with the MPPT perturbation step-size of: (a) \( v_{\text{step}} = 6 \) V, (b) \( v_{\text{step}} = 12 \) V, and (c) \( v_{\text{step}} = 18 \) V, where \( v_{\text{dc}} \) is the dc-link voltage (PV voltage), and \( i_g \) is the grid current.
Fig. 13. Experimental results of the PV system operating at 10 % of the rated power (i.e., 300 W) with the MPPT algorithm sampling rate of: (a) $f_{\text{MPPT}} = 2 \text{ Hz}$, (b) $f_{\text{MPPT}} = 5 \text{ Hz}$, and (c) $f_{\text{MPPT}} = 10 \text{ Hz}$, where $v_{\text{dc}}$ is the dc-link voltage (PV voltage), and $i_g$ is the grid current.

![Grid current waveforms](image)

Fig. 14. Proposed modeling approach to identify interharmonics in the grid current due to the MPPT perturbation.

the MPPT operation. Due to the power oscillation characteristic of the P&O MPPT algorithm, the time-domain waveform of the reference dc-link voltage $v_{\text{dc}}^*(t)$ has a fundamental period of $T_0 = 4T_{\text{MPPT}}$ (in the steady-state operation), as it is shown in Fig. 15. Thus, the variation in the reference dc-link voltage $v_{\text{dc}}^*(t)$ can be expressed as

$$v_{\text{dc}}^*(t) = \begin{cases} V_{\text{dc}} & 0 \leq t < T_{\text{MPPT}} \\ V_{\text{dc}} + v_{\text{step}} & T_{\text{MPPT}} \leq t < 2T_{\text{MPPT}} \\ 2V_{\text{dc}} & 2T_{\text{MPPT}} \leq t < 3T_{\text{MPPT}} \\ V_{\text{dc}} - v_{\text{step}} & 3T_{\text{MPPT}} \leq t < 4T_{\text{MPPT}} \end{cases}$$

(1)

where $T_{\text{MPPT}} = 1/f_{\text{MPPT}}$ is the MPPT sampling period and $V_{\text{dc}}$ is the average dc-link voltage. It can be noticed that the time-domain expression of the reference dc-link voltage in (1) is a function of the perturbation step-size $v_{\text{step}}$ and the MPPT sampling rate $f_{\text{MPPT}}$, which is also observed in § III. In this way, the influence of the MPPT algorithm parameters on the interharmonics in the grid current can be analyzed.

Applying the Fourier analysis to the time-domain function of the reference dc-link voltage in (1) gives

$$v_{\text{dc}}^*(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left( a_n \cos(\omega_n t) + b_n \sin(\omega_n t) \right)$$

(2)

with

$$a_0 = 2V_{\text{dc}}, \quad a_n = \frac{2v_{\text{step}}}{\pi n}, \quad b_n = \frac{v_{\text{step}}}{\pi n} \cos(\pi n - 1)$$

in which $n$ denotes the harmonic order with the fundamental period of $T_0 = 4T_{\text{MPPT}}$. $a_0$ is the Fourier coefficient of the dc component, and $a_n$ and $b_n$ are the Fourier coefficients of the $n$-th order harmonic. In fact, the dc component $a_0$ can be neglected in a small-signal analysis (which is used for modeling the system response). According to (2), the reference dc-link voltage is decomposed into a set of periodic signals with the frequencies corresponding to integer times of the fundamental frequency $f_0 = 1/T_0$.

The Fourier analysis can also be represented with a single sinusoid, where the reference dc-link voltage at the $n$-th order harmonic $v_{\text{dcn}}^*(t)$ can be expressed as

$$v_{\text{dcn}}^*(t) = A_n \sin(\omega_n t + \phi_n)$$

(3)
B. Response of the DC-link Voltage Perturbation

Once the reference dc-link voltage \( v_{dc}^*(t) \) is decomposed into a summation of \( n \) frequency components, it can be used as an input to determine the controller response and also the output response of each frequency component. Firstly, the transfer functions of different controllers in the dc-link voltage control loop are needed. The dc-link voltage control loop can be modeled as shown in Fig. 16(a) [40], [41], where \( G_{pi}(s) \) is the transfer function of the dc-link voltage controller (i.e., a PI controller), \( G_{cc}(s) \) is the transfer function of the current controller, \( G_{plant}(s) \) is the transfer function of the plant model (assuming a no-loss condition), and \( G_{notch}(s) \) is the transfer function of the notch filter employed in the feedback loop. Notably, this model is based on an assumption that the current control (inner loop) dynamics are much faster than the dc-link voltage control (outer loop) dynamics, and thereby they are decoupled from each other [41]. This is also practical in many applications. The details of each transfer function are given as

\[
G_{pi}(s) = k_p + \frac{k_i}{s}, \quad G_{cc}(s) = \frac{1}{1 + 3T_s s^2 + \omega_h^2}, \quad G_{plant}(s) = \frac{V_g}{\sqrt{2} V_d C_d s}, \quad G_{notch}(s) = \frac{k_p + k_i}{s + k_h s + \omega_h^2}
\]

where \( k_p \) and \( k_i \) are the proportional and integral gain of the dc-link voltage controller, respectively, \( T_s \) is the digital controller sampling period, \( V_g \) is the Root-Mean-Square (RMS) value of the grid voltage, \( V_d \) is the average dc-link voltage (which is the condition where the linearization takes place), \( \omega_h \) is the notch frequency (e.g., \( \omega_h = 2\pi 100 \text{ rad/s} \)), and \( k_h \) represents the quality factor of the notch filter.

The dc-link voltage control loop can then be re-arranged in order to determine the response of the grid current amplitude \( |i_g| \) due to the dc-link voltage perturbation, as it is shown in Fig. 16(b). The block diagram can be further simplified in order to determine the closed-loop transfer function \( G_{cl}(s) \) between the reference dc-link voltage \( v_{dc}^* \) and the grid current amplitude \( |i_g| \) as in Fig. 16(c), which is given as

\[
G_{cl}(s) = \frac{G_{pi}(s) \cdot G_{cc}(s)}{1 + G_{notch}(s) \cdot G_{plant}(s) \cdot G_{pi}(s) \cdot G_{cc}(s)}
\]

\[
= \left( k_p + \frac{k_i}{s} \right) \left( \frac{1}{\sqrt{2} V_d C_d s} \right) \left( k_p + k_i \right) \left( \frac{1}{1 + 3T_s s^2} \right)
\]

During the steady-state, a frequency response of the closed-loop transfer function can be represented as \( G_{cl}(j\omega) \). At each particular frequency \( \omega_n \), the frequency response of the closed-loop transfer function \( G_{cl}(j\omega_n) \) becomes \( |G_{cl}(j\omega_n)| \cdot \angle G_{cl}(j\omega_n) \). Thus, the amplitude of the grid current at the \( n \)-th order harmonic \( |i_{g,n}|(t) \) can be determined as

\[
|i_{g,n}|(t) = A_n |G_{cl}(j\omega_n)| \sin(\omega_n t + \phi_n + \angle G_{cl}(j\omega_n))
\]

Then, according to the superposition principle, the amplitude of the grid current \( |i_g|(t) \) can be obtained by summing up all the frequency components as

\[
|i_g|(t) = \sum_{n=1}^{\infty} |i_{g,n}|(t)
\]

\[
= \sum_{n=1}^{\infty} A_n |G_{cl}(j\omega_n)| \sin(\omega_n t + \phi_n + \angle G_{cl}(j\omega_n))
\]

\[
= \sum_{n=1}^{\infty} A_n \sin(\omega_n t + \phi_n)
\]

with

\[
A_n' = A_n |G_{cl}(j\omega_n)|, \quad \phi_n' = \phi_n + \angle G_{cl}(j\omega_n)
\]
where it can be seen that the frequency response of the closed-loop transfer function has modified the amplitude (and also the phase angle) of the input signal. However, it has no contribution to the change in the interharmonic frequencies. This is observed in an example of the frequency spectrum of the reference dc-link voltage and the amplitude of the grid current as shown in Fig. 17(a) and (b), respectively.

C. Amplitude Modulation

Following the control structure in Fig. 10, the grid current $i_g(t)$ is determined by multiplying the amplitude of the grid current $|i_g(t)|$ with the phase of the grid voltage $\sin(\theta_g)$, where $\theta_g = 2\pi f_g t$. Since the amplitude of the grid current contains a wide-band spectrum as $|i_g(t)| = \sum_{n=1}^{\infty} |i_g|_n(t)$, the multiplication between the amplitude of the grid current $|i_g(t)|$ and the phase $\sin(\theta_g)$ will result in an amplitude modulation. This can be mathematically derived as

$$|i_g|_n(t) \sin(\theta_g) = |i_g|_n(t) \sin(2\pi f_g t)$$

$$= A'_n \sin(\omega_n t + \phi'_n) \sin(2\pi f_g t)$$

$$= \frac{A'_n}{2} \left[ \cos(2\pi t(f_g - f_n) + \phi'_n) - \cos(2\pi t(f_g + f_n) + \phi'_n) \right]$$

which indicates that the $n$-th order harmonic of the amplitude grid current $|i_g|_n$ will contribute to two frequency components: $f_g \pm f_n$. The amplitude of each resultant frequency is equal to the half of the original signal amplitude, and the phase is shifted by $\pi/2$. For instance, the 8.75-Hz component of the grid current amplitude $|i_g|$ in Fig. 17(b) will contribute to the components of 41.25 Hz and 58.75 Hz appearing in the grid current $i_g$, as it is demonstrated in Fig. 17(c).

It should be pointed out that the frequency mapping between the input and the output signal is not a simple one-to-one mapping after the amplitude modulation, as illustrated in Fig. 18. Therefore, each interharmonic component of the grid current is the sum of two modulated signals which have equal frequency difference. By taking all frequency components into consideration, the grid current can be calculated as

$$i_g(t) = |i_g|_n(t) \sin(\theta_g) = \sum_{n=1}^{\infty} |i_g|_n(t) \sin(2\pi f_g t)$$

$$= \sum_{n=1}^{\infty} A'_n \sin(\omega_n t + \phi'_n) \sin(2\pi f_g t)$$

$$= \sum_{n=1}^{\infty} \frac{A'_n}{2} \left[ \cos(2\pi t(f_g - f_n) + \phi'_n) - \cos(2\pi t(f_g + f_n) + \phi'_n) \right]$$

D. Model Validation and Discussion

In order to validate the model and also the previous observations, the interharmonic model is applied to the PV system with different controller parameters: the MPPT perturbation step-size $v_{\text{step}}$ and the sampling rate of the MPPT algorithm $f_{\text{MPPT}}$. Then, the obtained results from the proposed model
are compared with the experimental results under the same operating condition, as shown in § III.

The influence of the perturbation step-size on the interharmonic in the grid current is demonstrated in Fig. 19. In this case, three different perturbation step-sizes of $v_{\text{step}}$ = 6 V, 12 V, and 18 V are employed (same as the case in Fig. 12). It can be seen in Fig. 19 that the amplitude of the interharmonic increases as the amplitude of the perturbation step-size increases, while the interharmonic frequency remains unchanged. The results obtained from the proposed model are in a close agreement with the experimental results in Fig. 12 both in terms of the interharmonic frequency and the amplitude of each individual component.

Furthermore, the operating condition with different MPPT sampling rates is also demonstrated in Fig. 20, where the MPPT sampling rates of $f_{\text{MPPT}}$ = 2 Hz, 5 Hz, and 10 Hz (same as the case in Fig. 13) are applied to the proposed model. It can be observed in Fig. 20 that the distance between the two consecutive interharmonic frequencies increases as the MPPT sampling frequency increases. The interharmonic frequency also matches well with the observed experimental results in Fig. 13. In fact, the amplitudes of the interharmonic components from the proposed model shown in Fig. 20 are also in a close agreement with the experiments in Fig. 13.

From the above results, it can be seen that the interharmonics in the grid current can be predicted and analyzed with the proposed model, where the results agree well with the experiments. Therefore, the effectiveness of the proposed model is validated. Moreover, it can be used as a tool to design the controller parameters according to the specific interharmonic requirements, if they are required by standards. In addition, possible mitigation solutions have been discussed through simulations in [30]. However, more comprehensive study including the experimental validation is still required in this field.

It is worth mentioning that this paper only focuses on the interharmonic emission mechanism related to the MPPT operation, which is unique for the PV systems. In reality, there could be other sources of interharmonics. For instance, they can be either generated by the PV inverter itself or induced through the interaction between PV inverters and other power electronic units. This requires further investigation, where variations in the grid condition (e.g., stiff and non-stiff grid) should be one consideration. Moreover, only the P&O MPPT algorithm is considered in this study, where the power oscillation occurs during the steady-state operation. This behavior also applies to other hill-climbing type MPPT algorithms such as the incremental conductance MPPT algorithm [42], where the above analysis can be applied with minor modifications. In order to further apply the analysis discussed in this paper to the other MPPT algorithms, it is necessary to model the steady-state behavior of the corresponding algorithm (e.g., if it causes power oscillation or not) in the frequency domain by analyzing the frequency spectrum of the dc-link voltage. Afterwards, the same approach as discussed in Fig. 14 can be followed.
V. CONCLUSION

In this paper, the interharmonics were observed experimentally in PV systems, and thus the mechanisms were explored. First, the observations from experiments showed that the perturbation from the MPPT algorithm is one of the main causes of interharmonics. A further exploration has demonstrated that the interharmonics appear in the grid current due to the amplitude modulation between the response of the dc-link voltage (i.e., amplitude of the grid current) and the phase of the grid voltage. The interharmonic characteristics were also investigated under different controller parameters. It turned out that the perturbation step-size of the MPPT algorithm has a strong influence on the amplitudes of the interharmonic components. On the other hand, the MPPT sampling rate affects both the amplitude and the frequency of the interharmonics. Based on the experimental observations, a model of interharmonics was proposed in this paper. The interharmonic model was then used to identify the characteristics of interharmonics in PV systems under the same conditions of experiments. Comparisons with the experiments validated the effectiveness of the proposed model. Hence, this model can be used to predict and analyze interharmonics in PV systems. It can also be a guiding tool to design the controller.

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