Dynamic modeling of networks, microgrids, and renewable sources in the dq0 reference frame: A survey

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Abstract—With increasing penetration of distributed and renewable sources into power grids, and with increasing use of power electronics based devices, the dynamic behavior of large-scale power systems is becoming increasingly complex. These recent developments have led to several models attempting to simplify the analysis of dynamic phenomena, among them are models based on the dq0 transformation. Many recent works present dq0-based models of various power system components, ranging from small renewable sources to complete networks. The purpose of this paper is to review and categorize these works, with an objective to promote a straightforward modeling and analysis of complex systems, based on dq0 quantities. The paper opens by recalling basic concepts of the dq0 transformation and dq0-based models. We then review several recent works related to dq0 modeling and analysis, considering models of passive components, complete passive networks, synchronous machines, wind turbine systems, photovoltaic inverters, and others.

I. INTRODUCTION

For many years the dynamic behavior of power grids was dictated by large synchronous machines with high inertia and slow dynamic responses. As a result, dynamic processes in interconnected and large-scale power systems were successfully analyzed based on time-varying phasor models (quasi-static models) [1]. A key assumption of time-varying phasor models is that phasors are slowly changing in comparison to the system frequency [2], [3], and therefore AC quantities are mapped to quasi-constant signals. A key advantage of this approach is that the resulting models are time-invariant, and have a well-defined operating point. Due to these properties, quasi-static models have been used extensively in the analysis of dynamic interactions that occur in time frames of seconds to minutes, and have historically enabled studies of machines stability, inter-area oscillation, and other slow dynamic phenomena [4]–[7].

However, in recent years, the increasing penetration of small distributed generators and fast power electronics based devices creates new challenges, one of which is that the system is in many cases not quasi-static. Due to these emerging technologies, voltage and current signals may contain high harmonic components, and can exhibit fast amplitude and phase variations [2]. These developments have led to several alternative models attempting to bridge this gap, among them are models based on the dq0 transformation. Such models combine two properties of interest: similar to time-varying phasor models, dq0-based models map AC signals to quasi-constant signals, so the resulting model is often time-invariant, with a well-defined operating-point. In addition, dq0 models are inherently transient models, and are derived directly from physical representations. As a result, dq0-based models do not rely on the assumption of a quasi-static system, and remain accurate at high frequencies [8]. The relations between different types of dynamic models are presented in Table I.

<table>
<thead>
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<th>operating point</th>
<th>small-signal</th>
<th>high frequencies</th>
<th>nonsymmetric networks</th>
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</table>

Due to these trends, many recent works present dq0-based models of various power system components, ranging from small renewable sources to complete networks [9]. The purpose of this paper is to review and categorize these works, with an objective to promote a straightforward modeling and analysis of complex systems, based on dq0 quantities. The paper opens by recalling basic concepts of the dq0 transformation and dq0-based models. We then review several recent works related to dq0 modeling and analysis, considering models of passive components, complete passive networks, synchronous machines, wind turbine systems, photovoltaic inverters, and others. We hope that the paper may be a step toward a general dynamic model of large-scale power systems, based on dq0 quantities. This in turn may allow a better understanding of the complex dynamics associated with the integration of distributed and renewable sources.
II. OVERVIEW OF DIFFERENT MODELING TECHNIQUES

Several approaches exist for modeling the dynamic behavior of three-phase power systems. One technique is to model the system in the time domain, using the native abc reference frame. This approach is often the most general, since it applies, for instance, to non-symmetric or unbalanced systems, and is valid over a wide range of frequencies. Another popular approach is to model the power system using time-varying phasors, often by means of the network power flow equations. This approach has many benefits, one of them is that the network is described using purely algebraic equations. However, time-varying phasors are only applicable at low frequencies, under the assumption that the system is quasi-static. A solution that complements these two well-known approaches is to model power systems on the basis of dq0 quantities. This approach is not as general as abc-based models, and is advantageous mainly when the network and units are symmetrically configured, see [8] and brief discussion in Section III-B for more details.

Another popular modeling technique is dynamic phasors. Dynamic phasors generalize the idea of quasi-static phasors, and represent voltage and current signals by Fourier series expansions in which the harmonic components are evaluated over a moving time window [10], [11]. The idea behind the dynamic phasors approach is therefore to approximate the system with nearly periodic quantities, which allow an accurate representation of the system while using a relatively large numerical step size [12], [13]. A comparison of simulation techniques based on dynamic phasors in the abc and dq0 reference frames may be found in [14], [15], a comparison of several dynamic phasors based models of large-scale networks is presented in [16]. In addition, dynamic phasors have the advantage of efficiently simulating harmonic components, which is not possible with dq0-based models. Dynamic phasors have been demonstrated with many system components, including synchronous and induction machines [11], [17], HVDC systems [12], [18], [19], FACTS devices [3], sub-synchronous resonance [20], asymmetric systems [11], [21], and asymmetric faults [17], [22]. In addition, dynamic phasors are utilized in state estimation [23]–[25], in systems with varying frequencies [26], and also in microgrids [27].

III. MODELING AND ANALYSIS OF POWER SYSTEMS BASED ON DQ0 QUANTITIES

We now recall the basic definition of the dq0 transformation. Consider a reference frame rotating with an angle of \( \theta(t) \). For instance, in a synchronous machine, \( \theta(t) \) is typically selected to be the rotor electrical angle. Let \( \hat{x} \) represent the quantity to be transformed (current, voltage, or flux), and use the compact notation \( x_{abc} = [x_a, x_b, x_c]^T \), \( x_{dq0} = [x_d, x_q, x_0]^T \). The dq0 transformation with respect to the reference frame rotating with the angle \( \theta \) can be defined as [28, Appendix C]

\[
\hat{x}_{dq0} = T_\theta x_{abc}
\]

(1)

By definition, symmetrically configured networks produce symmetric three-phase currents, if fed by symmetric three-phase voltages. For instance, a transmission network with identical circuit in each of the three-phases is symmetrically configured.

Fig. 1. An example showing mapping of abc signals to dq0 signals.

with

\[
T_\theta = \frac{2}{3} \begin{bmatrix}
\cos (\theta) & \cos (\theta - \frac{2\pi}{3}) & \cos (\theta + \frac{2\pi}{3}) \\
-\sin (\theta) & -\sin (\theta - \frac{2\pi}{3}) & -\sin (\theta + \frac{2\pi}{3})
\end{bmatrix}.
\]

(2)

Note that there are several variants of the transformation (2) available in the literature; however, all of them are equivalent up to the proper selection of the angle \( \theta \) and order of the rows [6], [9]. The same applies to the balanced (without zero component) version of \( T_\theta \) known as Park’s transformation [29, Appendix A], which was first introduced in [30].

A fundamental property of the dq0 transformation is that it maps symmetric AC signals to constants. For instance, consider the signals

\[
\begin{align*}
v_a &= A \cos (\theta) + B, \\
v_b &= A \cos \left( \theta - \frac{2\pi}{3} \right) + B, \\
v_c &= A \cos \left( \theta + \frac{2\pi}{3} \right) + B.
\end{align*}
\]

(3)

Using the inverse transformation \( T_\theta^{-1} \), it is easy to see that

\[
\begin{bmatrix}
v_a \\
v_b \\
v_c
\end{bmatrix} = \begin{bmatrix}
\cos (\theta) & -\sin (\theta) & 1 \\
\cos (\theta - \frac{2\pi}{3}) & -\sin (\theta - \frac{2\pi}{3}) & 1 \\
\cos (\theta + \frac{2\pi}{3}) & -\sin (\theta + \frac{2\pi}{3}) & 1
\end{bmatrix} \begin{bmatrix}
A \\
0 \\
B
\end{bmatrix},
\]

(4)

and therefore \( v_d = A, v_q = 0, v_0 = B \), see Fig. 1.

When describing complex systems, models based on dq0 signals may be presented as signal-flow diagrams, in which each component is modeled by dq0 quantities. A simple motivating example is presented in Fig. 2. A question that often appears when modeling systems based on dq0 quantities is how to choose the reference frame, or in other words, how to link machines that rotate at different frequencies. One approach presented in [2], [31] is to model the network and its components using a dq0 transformation that is based on a unified (global or common) reference frame, rotating with a fixed frequency \( \omega_s \). The frequency \( \omega_s \) is chosen as follows: if there is an infinite bus in the system, \( \omega_s \) is selected as the frequency of the infinite bus. If no generator is large enough to be modeled as an infinite bus, then \( \omega_s \) is equal to the steady-state system frequency. In this case dq0 signals will be constant at steady-state, and the system will have well-defined equilibrium point [32].
A dq0 transformation based on \( \omega_s \) is similar to (2), and is obtained by substituting \( \theta = \omega_s t = 2 \pi f_s t \):

\[
x_{dq0} = T_{\omega_s} x_{abc}
\]

with

\[
T_{\omega_s} = \frac{2}{3} \begin{bmatrix} \cos(\omega_s t) & \cos(\omega_s t - \frac{2\pi}{3}) & \cos(\omega_s t + \frac{2\pi}{3}) \\ -\sin(\omega_s t) & -\sin(\omega_s t - \frac{2\pi}{3}) & -\sin(\omega_s t + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}
\]

(6)

A formula that allows to convert signals from the \( \theta \) reference frame to the unified frame can be derived following [31] as

\[
\begin{bmatrix} x_d \\ x_q \\ x_0 \end{bmatrix} = \begin{bmatrix} \sin(\delta) & \cos(\delta) & 0 \\ -\cos(\delta) & \sin(\delta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_d \\ \dot{x}_q \\ \dot{x}_0 \end{bmatrix},
\]

(7)

where \( \delta(t) = \theta(t) - \omega_s t + \pi/2 \). The variables \( x_d, x_q \) are defined with respect to \( \omega_s t \), and \( \dot{x}_d, \dot{x}_q \) are defined with respect to \( \theta \).

The total instantaneous three-phase power \( P_{3d} \) flowing from a unit into the network can be computed as

\[
P_{3d} = v_d i_d + v_q i_q + v_c i_c = V_{abc}^T I_{abc}
\]

\[
= (T_{\omega_s}^{-1} V_{dq0})^T T_{\omega_s}^{-1} I_{dq0}
\]

\[
= (V_{dq0})^T (T_{\omega_s}^{-1})^T T_{\omega_s}^{-1} I_{dq0}
\]

\[
= \begin{bmatrix} v_d & v_q & v_0 \end{bmatrix} \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}
\]

\[
= \frac{3}{4} (v_d i_d + v_q i_q + 2v_0 i_0).
\]

(8)

A. Linear passive components

This section overviews dq0 models of linear passive elements [33]–[35]. Such elements form the basis for modeling a large variety of more complex components. We open this discussion by demonstrating the dynamic model of a symmetric three-phase inductor. The inductor model in the native abc reference frame is given by

\[
L \frac{d}{dt} I_{abc,12} = V_{abc,1} - V_{abc,2},
\]

(9)

and can be converted to the \( dq \) frame as follows. Observe that differentiation of (5) results in

\[
\frac{d}{dt} I_{dq0} = \frac{dT_{dq0}}{dt} I_{abc} + T_{\omega_s} \frac{d}{dt} I_{abc},
\]

(10)

which after simple algebraic manipulations yields

\[
\frac{d}{dt} I_{dq0,12} = \mathcal{W} I_{dq0,1} + \frac{1}{L} (V_{dq0,1} - V_{dq0,2}),
\]

(11)

where

\[
\mathcal{W} = \begin{bmatrix} 0 & \omega_s & 0 \\ -\omega_s & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.
\]

(12)

This equation describes a state-space model of a symmetric three-phase inductor. Similarly, the model of a symmetric three-phase capacitor \( C \) is given as

\[
\frac{d}{dt} (V_{dq0,1} - V_{dq0,2}) = \mathcal{W} (V_{dq0,1} - V_{dq0,2}) + \frac{1}{C} I_{dq0,12}.
\]

(13)

And for a symmetric three-phase resistor \( R \) the model is given by the simple static relations

\[
V_{dq0} = I_3 R I_{dq0},
\]

(14)

where \( I_3 \) denotes the \( 3 \times 3 \) identity matrix.

Now consider briefly how energy is expressed in \( dq0 \) coordinates. Assume a symmetrically configured three-phase inductor (9). By definition, the stored energy is

\[
E = \frac{1}{2} L (i_d^2 + i_q^2 + i_c^2),
\]

(15)

which can be rewritten as

\[
E = \frac{1}{2} LI_{abc}^T I_{abc}
\]

\[
= \frac{1}{2} L \left( T_{\omega_s}^{-1} I_{dq0} \right)^T T_{\omega_s}^{-1} I_{dq0}
\]

\[
= \frac{1}{2} L \begin{bmatrix} i_d & i_q & i_0 \end{bmatrix} \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix}
\]

\[
= \frac{3}{4} L (i_d^2 + i_q^2 + 2i_0^2).
\]

(16)

B. Modeling symmetric transmission networks and microgrids

By combining models of elementary passive components, any symmetric transmission network can be modeled based on the \( dq0 \) reference frame. With various degree of details, this problem was addressed in several works. The following cases of transmission/distribution networks have been studied:

- **RL:** the network and loads are modeled in a common \( dq0 \) reference frame [31] under the assumption of balanced signals [2], [36]. In [9] this assumption is replaced by a less restrictive assumption, requiring the transmission network to be symmetrically configured.

- **RL with shunt elements and transformers:** a non-minimal state-space model is developed in [37]. Continuing this research, the minimal (controllable and observable) state-space model for the large-scale transmission networks is presented in [38]. This model extends the above models relying on a more general topology as defined in [39] and illustrated in Fig. 4.
can be described by a constant admittance matrix $Y$ through the linear relation
\[ I(t) = YV(t), \]
which is equivalent to the well-known power flow equations [47]. Equation (19) can alternatively be written in terms of its real and imaginary parts as
\[
\begin{align*}
\text{Re}\{I(t)\} &= \text{Re}\{Y\} \text{Re}\{V(t)\} - \text{Im}\{Y\} \text{Im}\{V(t)\}, \\
\text{Im}\{I(t)\} &= \text{Im}\{Y\} \text{Re}\{V(t)\} + \text{Re}\{Y\} \text{Im}\{V(t)\}.
\end{align*}
\]

Assume that the network is balanced. Then, a dynamic model based on $dq$ signals is given by [16]
\[
\begin{bmatrix}
I_d(s) \\
I_q(s)
\end{bmatrix} =
\begin{bmatrix}
N_1(s) & jN_2(s) \\
-jN_2(s) & N_1(s)
\end{bmatrix}
\begin{bmatrix}
V_d(s) \\
V_q(s)
\end{bmatrix},
\]
where
\[
\begin{align*}
N_1(s) &:= \frac{1}{2} (Y(s + j\omega_s) + Y(s - j\omega_s)), \\
N_2(s) &:= \frac{1}{2} (Y(s + j\omega_s) - Y(s - j\omega_s)),
\end{align*}
\]
and $Y(s)$ is the frequency dependent nodal admittance matrix. According to [8, Lemma 2], for $s = 0$, the dynamic model in (21) reduces to the quasi-static expressions in (20). This fact allows to realize quasi-static models as
\[
\begin{bmatrix}
I_d \\
I_q
\end{bmatrix} =
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix} =
\begin{bmatrix}
D_{dq} - C_{dq}A_{dq}^{-1}B_{dq}
\end{bmatrix}
\begin{bmatrix}
V_d \\
V_q
\end{bmatrix},
\]
where the system matrices are defined as in (18) with respect to the $dq$ part.

Consider now a simple example of the balanced three-phase inductor. Starting from the quasi-static model, the admittance matrix describing the inductor is $Y = Y(j\omega_s) = 1/(j\omega_s L)$. Using time-varying phasors and (19), (20), the quasi-static model is given by
\[ I = \frac{1}{j\omega_s L} V. \]

By taking the real and imaginary parts of this equation, and using the relations between $dq$ signals and phasors $v_d = \sqrt{2} \text{Re}\{V\}$, $v_q = \sqrt{2} \text{Im}\{V\}$, equivalent expressions for the quasi-static model become
\[
\begin{align*}
v_d &= -\omega_s L i_q, \\
v_q &= \omega_s L i_d,
\end{align*}
\]
In addition, recall that the $dq$ model of the inductor (as in (11)) is given by
\[
\begin{align*}
v_d &= -\omega_s L i_q + L \frac{d}{dt}i_d, \\
v_q &= \omega_s L i_d + L \frac{d}{dt}i_q.
\end{align*}
\]

Direct comparison of these equations reveals that both models are similar, except for the additional time derivative terms in the $dq$ model, which describe high-frequency effects. Note that at low-frequencies, where the time derivatives are negligible, both models are equivalent.
C. Synchronous machines

The dq0 transformation has been used over many years for modeling, analysis, and control of synchronous machines. There exist numerous books (e.g., [6], [28], [31], [48]–[52]) with detailed mathematical models of synchronous generators, excitation systems, and governing systems, based on dq0 quantities. To briefly demonstrate how the dq0 transformation is used for analyzing the dynamics of synchronous machines, the following discussion shows a model of a simple generator connected to an infinite bus. While such a model is popular in many textbooks, we recall here how to develop it based on the unified reference frame \( \omega_s t \), instead of more traditional rotor angle \( \theta \). This is necessary for the model to be compatible with the network model, as discussed in Section III-B.

Consider a simple generator (with droop control mechanism) represented as an ideal voltage source behind an inductance \( L_d \). The generator is connected to an infinite bus, as shown in Fig. 5. The generator voltage is \( \vec{v}_{d,g} = 0, \vec{v}_{q,g} = V_e, v_{0,g} = 0 \), with a reference angle of \( \theta \). In this example \( \theta \) is the electric angle of the rotor in respect to the stator. The infinite bus has a constant frequency of \( \omega_s \), so its voltage is given by \( v_d = V, v_q = 0, v_0 = 0 \), with a reference angle of \( \omega_s t \). Now the goal is to construct a dynamic model of the system based on dq0 signals. However, a potential problem is that the two voltage sources are defined with respect to two different reference frames \( (\theta \text{ and } \omega_s t) \). To solve this, we choose \( \omega_s t \) as a unified reference frame for both the infinite bus and synchronous machine. The synchronous machine voltage is now obtained by substituting \( \vec{v}_{d,g} = 0, \vec{v}_{q,g} = V_e, v_{0,g} = 0 \) into (7), which leads to

\[
\begin{align*}
v_{d,g} &= V_e \cos(\delta) \\
v_{q,g} &= V_e \sin(\delta) \\
v_{0,g} &= 0,
\end{align*}
\]

and \( \delta = \theta - \omega_s t + \pi/2 \). Furthermore, the dynamic behavior of the angle \( \delta \) is described by

\[
\frac{d^2 \delta}{dt^2} = \frac{\text{poles}}{2J\omega_s} \left( -P_{\phi} + 3P_{\text{ref}} - \frac{1}{D} \frac{d}{dt} \delta \right),
\]

which is the classic swing equation with the droop control mechanism. The term \( J \) is the rotor moment of inertia, \( \text{poles} \) is the number of machine poles (must be even), \( P_{\text{ref}} \) is the single-phase reference power, and \( D \) represents the droop control sloop parameter. The three-phase power can be computed by (8). Combination of (28) and (8) with \( \delta = \phi_1 \) results in the state equations

\[
\begin{align*}
\frac{d}{dt} \delta &= \omega - \omega_s, \\
\frac{d}{dt} \omega &= \frac{\text{poles}}{2J\omega_s} \left( -\frac{3}{2} V_e (\cos(\delta))i_d + \sin(\delta)i_q \right) + 3P_{\text{ref}} - \frac{1}{D} (\omega - \omega_s),
\end{align*}
\]

and the outputs are defined by (27). Note that the model represents only the machine’s voltage source, and does not include the synchronous inductance, which is represented separately in (11) with \( L = L_d \).

For completeness, we now recall a more sophisticated (physical) model of a synchronous machine [28]. The model presented herein captures the interaction of the direct-axis magnetic field with the quadrature-axis mmf, and the quadrature-axis magnetic field with the direct-axis mmf, as well as the effects of resistances, transformer voltages, field winding dynamics, and salient poles. The parameters are explained in Table II. Following [28], and omitting laborious algebraic manipulations, the resulting state equations of a synchronous machine in the dq0 reference frame (with respect to \( \omega_s t \)) are given by

\[
\begin{align*}
\frac{d}{dt} \phi_1 &= - \frac{2R_a L_{ff}}{L_0^2} \phi_1 + \phi_2 \phi_3 + \frac{2R_a L_{af}}{L_0^2} \phi_4 \\
&\quad + \sin(\phi_5)v_d - \cos(\phi_5)v_q, \\
\frac{d}{dt} \phi_2 &= - \frac{R_a}{L_0} \phi_2 - \phi_3 \phi_5 + \cos(\phi_5)v_d + \sin(\phi_6)v_q, \\
\frac{d}{dt} \phi_3 &= - \frac{R_a}{L_0} \phi_3 + v_0, \\
\frac{d}{dt} \phi_4 &= \frac{3R_f L_{af}}{L_0^2} \phi_1 - \frac{2R_f L_f}{L_0^2} \phi_4 + v_f, \\
\frac{d}{dt} \phi_5 &= \frac{\text{poles}}{2J} \left( T_m + \frac{3L_0^2}{2L_0^2 L_q} \phi_1 \phi_2 + \frac{3L_{af}}{L_0^2} \phi_2 \phi_4 \right), \\
\frac{d}{dt} \phi_6 &= \phi_5 - \omega_s,
\end{align*}
\]

TABLE II

<table>
<thead>
<tr>
<th>Nomenclature: Synchronous Machine</th>
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<tbody>
<tr>
<td>( \lambda_d )</td>
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<td>( \lambda_f )</td>
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<td>( v_{d,g} ), ( v_{q,g} ), ( v_0 )</td>
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<tr>
<td>( J )</td>
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<tr>
<td>( T_m )</td>
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</tbody>
</table>
and the outputs are defined as

\[ \begin{align*}
    i_d &= \frac{2L_{ff}}{L_\beta} \sin(\phi_0)\phi_1 - \frac{1}{L_q} \cos(\phi_0)\phi_2 + \frac{2L_{af}}{L_\beta} \sin(\phi_0)\phi_4, \\
    i_q &= \frac{2L_{ff}}{L_\beta} \cos(\phi_0)\phi_1 - \frac{1}{L_q} \sin(\phi_0)\phi_2 - \frac{2L_{af}}{L_\beta} \cos(\phi_0)\phi_4, \\
    i_0 &= -\frac{1}{L_0} \phi_3, \\
    i_f &= -\frac{3L_{af}}{L_\beta} \phi_1 + \frac{2L_d}{L_\beta} \phi_4, \\
    \omega &= \phi_5, \quad \delta = \phi_6,
\end{align*} \]

where \( L_\beta^2 = 2L_dL_{ff} - 3L_{af}^2 \). In this model, the state variables are selected as \( \phi_1 = \lambda_d, \phi_2 = \lambda_q, \phi_3 = \lambda_0, \phi_4 = \lambda_f, \phi_5 = \omega, \delta = \phi_6 \) and the inputs as \( v_d, v_q, v_0, v_f, T_m \). Unlike the simplified model (29), this model already includes the inductance terms \( L_d, L_q, L_0 \). A convenient property of the model presented above is that its inputs and outputs are defined with respect to the unified reference frame rotating with \( \omega_s t \), and therefore it can be directly connected to the network. For instance, connecting the synchronous machine model to an infinite bus is immediate by direct substitution of voltages \( v_d, v_q, v_0 \) from Fig. 5 into (30).

### D. Wind energy conversion systems

The dynamics of induction machines, which are typically used in wind energy conversion systems, are often studied in the \( dq \) reference frame. Induction machines can be categorized into two types: fixed and adjustable speed generators. Recently more attention is paid toward adjustable generators, particularly, several such advantages are variable speed operations, independent control of their active and reactive output powers, high energy efficiency, and low size. In addition, work [59] presents a general approach for modeling wind turbine systems, which is useful for power system dynamic simulations, and is compatible with existing software implementations. This paper follows the scheme presented in Fig. 6, but additionally discusses protection system and control of rotor and terminal voltage. Work [57] presents a literature review on the different aspects of DFIGs. Specifically, the paper discusses various control methods used in wind turbine systems, including pitch angle control [60], vector and decoupling control [58], [61], [62], and passivity control methods [63], [64]. Finally, works [65]–[67] give an overview of the recent trends in standalone applications, as well as wind parks. These paper discuss specific grid connections, control issues, and perspectives of using particular types of generators.

![Fig. 6. Block scheme of a typical wind turbine.](image)

### E. Photovoltaic inverters

The continuing growth of distributed generation is leading to networks with a mixture of classic rotating machines and inverter interfaced generators [68], among them photovoltaic inverters. Photovoltaic inverters are typically controlled either

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**TABLE III**

**NOMENCLATURE: INDUCTION MACHINE**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( v_{d,s}, v_{q,s}, v_{d,r}, v_{q,r} )</td>
<td>voltages</td>
</tr>
<tr>
<td>( i_{d,s}, i_{q,s}, i_{d,r}, i_{q,r} )</td>
<td>currents</td>
</tr>
<tr>
<td>( \phi_{d,s}, \phi_{d,r}, \phi_{q,s}, \phi_{q,r} )</td>
<td>fluxes</td>
</tr>
<tr>
<td>( R_s, R_r )</td>
<td>winding resistances</td>
</tr>
<tr>
<td>( L_s, L_r )</td>
<td>self-inductances</td>
</tr>
<tr>
<td>( L_{s,s}, L_{i,s} )</td>
<td>leakage inductances</td>
</tr>
<tr>
<td>( L_{s,r} )</td>
<td>stator to rotor mutual inductance</td>
</tr>
<tr>
<td>( L_m )</td>
<td>magnetizing inductance</td>
</tr>
</tbody>
</table>
as \( PQ \) sources, in which case the active and reactive powers of the inverter are directly controlled, or as \( PV \) sources, where the active power and voltage amplitude are directly controlled [9], [69]–[72]. These two control schemes are associated with the two main operation modes for photovoltaic inverters, which are the grid-feeding [73], [74] and grid-forming modes [29], [75], [76]. Both control schemes can be implemented using lower level \( dq0 \)-based controllers, as depicted in Fig. 7. As shown in a recent number of works, the \( PQ \) control scheme is more popular today, mainly since the output current of the inverter is well regulated, which enables robust and economically competitive designs. In many designs the reactive power is chosen to be zero, and the inverter is operated with a power factor of unity (for instance, as proposed in [77]). This approach leads to the lowest output current per active power, but the trade-off is that the inverter does not provide the reactive power that may be required to stabilize the grid.

A typical \( PQ \) control scheme is shown in Fig. 7(a). The design consists of an outer loop that controls the active power, and an inner current-control loop, which regulates the inverter output current. The outer loop regulates the active power in order to match it to the power produced by the photovoltaic source, where the feedback is provided by the bus-capacitor voltage. The active and reactive power set points are fed to the inner current controller, which regulates the output current using a \( dq0 \) reference frame rotating with the inverter output voltage. The reactive power set point may be chosen directly (for example, \( Q^* = 0 \)), or may be manipulated to support a required voltage profile at the inverter output. More details may be found in [68]. It should be noted that this design omits many practical details, and is by no means the only design available.

Assume now that: \( i \) high frequency switching harmonics are ignored; \( ii \) the inverter is connected to the grid with a constant frequency \( \omega_s \); \( iii \) all three-phase voltages and currents are balanced; \( iv \) the PLL loop is ideal and perfectly extracts the \( dq \) components and frequency of the grid voltage with zero distortion; \( v \) the MPPT unit and DC-DC converter are ideal, and represented by a single constant power source \( (P_{dc}) \); \( vi \) all power conversion stages are assumed to be ideal (lossless and no internal energy storage); \( vii \) the current controller uses a common topology, as proposed in [68]. The current controller and active-power compensators are based on simple proportional-integral controllers. Then, the photovoltaic inverter based on \( PQ \) control scheme (Fig. 7(a)) can be described as follows. The main system components include:

✓ Active power control:

\[
P^* = K_{p,p} (v_{dc} - v_{dc}^{set}) + z_1 + \frac{P_{dc}}{3}
\] (34)

such that

\[
\frac{d}{dt} v_{dc} = \frac{1}{v_{dc}^{set} C} (P_{dc} - P_{pv}),
\]

\[
\frac{d}{dt} z_1 = K_{i,p} (v_{dc} - v_{dc}^{set}),
\]

and the output photovoltaic power is computed as

\[
P_{pv} = \frac{3}{2} (\hat{v}_d i_d + \hat{v}_q i_q).
\]

✓ Current calculation:

\[
\hat{i}_d = \frac{2}{v_d^2 + v_q^2} (P^* v_d + Q^* v_q),
\]

\[
\hat{i}_q = \frac{2}{v_d^2 + v_q^2} (P^* v_q - Q^* v_d).
\]

✓ Current control:

\[
\hat{v}_d = v_d - \omega_s L i_q + K_{p,c} (\hat{i}_q - i_d) + z_2,
\]

\[
\hat{v}_q = v_q + \omega_s L i_d + K_{p,c} (\hat{i}_d - i_q) + z_3,
\]

such that

\[
\frac{d}{dt} z_2 = K_{i,c} (\hat{i}_d - i_d),
\]

\[
\frac{d}{dt} z_3 = K_{i,c} (\hat{i}_q - i_q).
\]
PLLs are usually required when there is a need to synchronize a unit to the grid (e.g., inverters) and also for monitoring and control purposes. A comprehensive review of recent PLL schemes is presented in [78]. Recent challenges in this area include:

- exploring ways to realize/implement PLLs of different types [79]–[82];
- seeking methods to improve the dynamic performance of PLLs using, for example, adaptive frequency estimation loop [83] or adaptive loop gain [84];
- improving the filtering capability and disturbance rejection ability of PLLs by including different filters inside their control loop or before their input such as:
  - moving average filter [85];
  - notch filter [86], [87];
  - multiple synchronous reference frame filtering [88], [89];
  - complex-coefficient-filter [90], [91];
  - delayed signal cancellation [92]–[96];
  - second-order generalized integrator [94], [97].

Finally, it was recently noticed that the droop control and PLLs structurally resemble each other [98], [99]. This relation creates additional possibilities for future research, which can improve and further develop both techniques.

IV. DISCUSSION AND CONCLUSION

With increasing penetration of distributed and renewable sources into power grids, and with increasing use of power electronics based devices, the dynamic behavior of large-scale power systems are becoming increasingly complex. Several systematic issues are:

- Active power balancing
- Reactive power balancing
- Reduced system inertia
- Fast transients
- Stability issues, due to interactions between power electronics based components
- Frequency regulation issues due to increasing integration of power electronics generators
- Resonance due to interactions between power electronics based devices and power lines
- Poor damping of local and inter-area oscillations

Stability and dynamic responses of large-scale power systems are often studied by means of time-varying phasors, under the assumption that the system is quasi-static. However, with increasing integration of fast renewable and distributed sources, this assumption is becoming increasingly inaccurate. In this light, the dq0 transformation provides several important advantages that simplify the system dynamics, especially when modeling balanced or symmetrically configured power systems. One advantage is that sinusoidal three phase signals are mapped to constant signals at steady-state, so dq0-based models have a well-defined operating point, and therefore enable small-signal and stability analysis. In addition, dq0 models capture the actual behavior of networks and devices, and are accurate at high frequencies. Models based on the dq0 transformation also allow a formal extension of quasi-static (time-varying phasor) models to high frequencies, without assuming a quasi-static system, and without sacrificing accuracy.
A current challenge is to merge various dq0-based models appearing in the literature to allow modeling of large-scale power systems. In light of this challenge, this paper provides an overview of recent dq0 models of the network and its main components. We explain basic concepts related to the dq0 transformation and dq0-based models, and review several recent models of passive components, networks, synchronous machines, photovoltaic inverters, wind turbines, rectifiers, and others. A central objective of this paper is to promote dq0-based analysis not only of single machines, but also of large-scale power systems. This in turn may allow a better understanding of the dynamics associated with the integration of distributed and renewable energy sources.

### References


### Table IV

**DQ0 Transformation: Summary of Application Areas**

<table>
<thead>
<tr>
<th>Unit</th>
<th>Model type</th>
<th>Refs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission networks</td>
<td>transient</td>
<td>[2], [9], [16], [36]–[38]</td>
</tr>
<tr>
<td>Microrgids</td>
<td>transient</td>
<td>[9], [41], [42], [44]</td>
</tr>
<tr>
<td>Synchronous machines</td>
<td>transient</td>
<td>[6], [28], [31], [48]–[52], [100], [101]</td>
</tr>
<tr>
<td></td>
<td>small-signal</td>
<td>[6], [37]</td>
</tr>
<tr>
<td>Wind turbines</td>
<td>transient</td>
<td>[53]–[64], [66], [102]–[108]</td>
</tr>
<tr>
<td></td>
<td>small-signal</td>
<td>[109]</td>
</tr>
<tr>
<td>Photovoltaic inverters</td>
<td>transient</td>
<td>[9], [29], [68]–[77], [110]–[113]</td>
</tr>
<tr>
<td>PLL</td>
<td>transient</td>
<td>[68], [78]–[94], [96]–[99], [114]</td>
</tr>
<tr>
<td></td>
<td>small-signal</td>
<td>[93], [95]</td>
</tr>
<tr>
<td>Rectifiers</td>
<td>transient</td>
<td>[115]–[119]</td>
</tr>
<tr>
<td></td>
<td>small-signal</td>
<td>[120]–[124]</td>
</tr>
</tbody>
</table>


