Harmonic Instability Source Identification in Large Wind Farms

Esmaeil Ebrahimzadeh, Frede Blaabjerg, Xiongfei Wang, and Claus Leth Bak

Department of Energy Technology
Aalborg University
Aalborg, Denmark
ebb@et.aau.dk, fbl@et.aau.dk, xwa@et.aau.dk, and clb@et.aau.dk

Abstract—A large-scale power electronics based power system like a wind farm introduces the passive and active impedances. The interactions between the active and passive impedances can lead to harmonic-frequency oscillations above the fundamental frequency, which can be called harmonic instability. This paper presents an approach to identify which wind turbine and which bus has more contribution to the harmonic instability problems. In the approach, a wind farm is modeled as a Multi-Input Multi-Output (MIMO) dynamic system. The poles of the MIMO transfer matrix are used to predict the system instability and the eigenvalues sensitivity analysis in respect to the elements of the MIMO matrix locates the most influencing buses of the wind farm. Time-domain simulations in PSCAD software environment for a 400-MW wind farm validate that the presented approach is an effective tool to determine the main source of the instability problems.

Keywords—harmonics; power quality; wind farm; stability; multi-input multi-output dynamic system

I. INTRODUCTION

High penetration of power electronic converters into the electric power system has initiated technical challenges in respect to the stability and power quality of the system [1]. In a large power electronics based power system like a wind farm, the cables, transformers, capacitor banks, shunt reactors, etc., present the passive impedances, while power electronic converters present the active impedances [2]. The mutual interactions between the passive and active impedances may lead to instability problems [3]. The interactions of the active impedances resulting from the fast inner control loops of the power converters may lead to high frequency oscillations above the fundamental frequency, which can be called harmonic instability [4]. Many of researches about harmonic instability discuss how to predict the instability conditions but pay less much attention to identify the main source of instability [4]-[14]. For stability analysis, a general approach is based on the state-space model, where the contribution of each component to the system stability can be identified by the participation factor analysis [4]-[9]. However, since the detailed models of power converters, loads, cables, transformers, etc, are required, the formulation of the state matrices for systems with a high integration of power converters may become complicated [10]. Apart from the state-space analysis, the impedance-based analysis approach is effective tool to predict the harmonic instability by calculating the ratio of the converter active impedance to the grid equivalent impedance at Point of Common Coupling (PCC) [11]-[14]. However, it cannot identify which bus and which converter in a large power electronics based power systems contribute more to the harmonic instability.

Another powerful tool for predicting instability problems is presented in [15], [16], where a wind farm is introduced as a Multi-Input Multi-Output (MIMO) dynamic system by using a transfer function matrix including passive elements and converter controller parameters. However, the contributions of different components and buses on harmonic-frequency oscillations have not been identified.

In order to fill in this gap, this paper presents an analytical approach to calculate participation factors of the different buses and converters in a wind farm. The participation factors of the different components and buses are calculated by means of modal analysis of the introduced MIMO system. A power converter with a larger participation factor has more contribution to the harmonic instability, and consequently, a power converter with the largest participation factor is identified as the main source of harmonic instability. The proposed methodology has been verified by time-domain simulations of a 400-MW wind farm in PSCAD software environment.
II. A LARGE WIND FARM AS A MULTI-INPUT MULTI-OUTPUT DYNAMIC SYSTEM

Fig. 1(a) shows a Wind Turbine (WT) with full-scale converters in a wind farm and Fig. 1(b) depicts the equivalent circuit of the Grid-Side Converter (GSC) of the WT for harmonic stability analysis. In Fig. 1(b), \( G_{\text{cont,k}} \) is the current controller, and \( G_{\text{delay,k}} \) is the approximated delay of the digital control. In this paper, \( G_{\text{cont,k}} \) and \( G_{\text{delay,k}} \) are assumed to be as

\[
G_{\text{cont,k}} = K_{p,k} + \frac{K_{i,k} s}{s^2 + \alpha f}\tag{1}
\]

\[
G_{\text{delay,k}} = e^{-1.5T_{sk,s}} \approx \frac{1 - 1.5T_{sk,s}}{2} + \frac{(1.5T_{sk,s})^2}{10} - \frac{(1.5T_{sk,s})^3}{120} s^3
\]

where \( \omega f \) is the fundamental frequency of the grid and \( T_{sk,s} \) is the sampling period. The objective of this paper is to identify the main source of harmonic instability (harmonic-frequency oscillations above the fundamental frequency). Therefore, outer power control and dc-link oscillations, which may cause low frequency oscillations, are neglected [11]. Since the dc-link is considered constant, the Turbine-Side Converters (TSCs) can also be neglected. The relationship between the current references and the bus voltages in the wind farm can be written as a Multi-Input Multi-Output (MIMO) dynamic system by

\[
V(s) = G_f(s)U(s)
\]

\[
V(s), G_f(s), \text{and } U(s)
\]

are

\[
V(s) = \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix}, G_f(s) = \begin{bmatrix} G_f(s) & G_f(s) \\ G_f(s) & G_f(s) \end{bmatrix}, U(s) = \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix}
\]

where

\[
\begin{align*}
U_{w1}(s) &= \begin{bmatrix} I_{w1}(s) \\ I_{w2}(s) \end{bmatrix} \\
V_{w1}(s) &= \begin{bmatrix} V_{w1}(s) \\ V_{w2}(s) \end{bmatrix} \\
U_{sw1}(s) &= \begin{bmatrix} I_{sw1}(s) \\ I_{sw2}(s) \end{bmatrix} \\
V_{sw1}(s) &= \begin{bmatrix} V_{sw1}(s) \\ V_{sw2}(s) \end{bmatrix}
\end{align*}
\]

\[
V(s) = \begin{bmatrix} V_{w1}(s) \\ V_{w2}(s) \end{bmatrix}, U(s) = \begin{bmatrix} U_{w1}(s) \\ U_{w2}(s) \end{bmatrix}
\]

\[
G_f(s), G_f(s), G_f(s), \text{and } G_f(s) \text{ are given in [16], } V(s) \text{ is the voltage of } n^\text{th} \text{ bus}, I_{w1}(s) \text{ is the current reference of the } k^\text{th} \text{ GSC, and the } V(s) \text{ is the main grid voltage.}
\]

A. Poles of the system

Poles of the MIMO dynamic system can be obtained by solving the following equation:

\[
\text{det}[G_f(s)] = 0
\]

\[
\Rightarrow P_i = \alpha_i + j\beta_i, P_j = \alpha_j + j\beta_j, \ldots, P_q = \alpha_q + j\beta_q
\]

where the oscillation frequency (\( f_i \)) and the damping ratio (\( \zeta_i \)) of oscillations can also be obtained as

\[
f_i = \frac{\beta_i}{2\pi}, \quad \zeta_i = \frac{-\alpha_i}{\sqrt{\alpha_i^2 + \beta_i^2}}
\]

The system is stable if and only if all the poles have negative real parts. The pole with the largest real part is called the critical pole (\( s_c \)), i.e.,

\[
s_c = \alpha_c + j\beta_c, \quad \alpha_c = \text{Max}(\alpha_1, \alpha_2, \ldots, \alpha_q)
\]

B. Harmonic Instability source identification

By substituting the critical pole (\( s_c \)) for \( s \) in the function matrix of \( G_f(s) \), \( \tilde{G}_f(s_c) \) can be numerically obtained. Based on the idea of the eigenvalue decomposition [17]-[21], the matrix \( \tilde{G}_f(s_c) \) can be decomposed into three matrices as

\[
\tilde{G}_f(s_c) = R_f \Lambda_f L_f = R_f
\]

\[
\begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_m
\end{bmatrix}
\]

where \( \Lambda_f \) is a diagonal matrix whose diagonal elements are the eigenvalues of \( \tilde{G}_f(s_c) (\lambda_1, \lambda_2, \ldots, \lambda_m) \). \( R_f \) is a matrix whose columns are the corresponding right eigenvectors, i.e.,

\[
G_f(s_c)R_f = R_f \Lambda_f
\]

\[
L_f \text{ is a matrix whose rows are transposed left eigenvectors, i.e.,}
\]

\[
L_f G_f(s_c) = \Lambda_f L_f
\]

The following equation can be derived from (10) and (11):

\[
L_f = R_f^T
\]

Using (9) and (12), the inverse of \( \tilde{G}_f(s_c) \) can be calculated from
Fig. 2. The configuration of the 400-MW wind farm with the aggregated strings which is studied in this paper.

Table I. Frequency, damping, the largest participation factor, and the most influencing bus for the oscillatory modes of the wind farm

<table>
<thead>
<tr>
<th>Pole</th>
<th>Real part $a_i$</th>
<th>Frequency $f_i$</th>
<th>Damping $\zeta_i$</th>
<th>The largest Participation Factor (PF)</th>
<th>The most influencing bus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-107.438</td>
<td>51.40616</td>
<td>0.315628</td>
<td>0.535925</td>
<td>Bus-2</td>
</tr>
<tr>
<td>2</td>
<td>-65.7248</td>
<td>52.29672</td>
<td>0.196136</td>
<td>0.36575</td>
<td>Bus-3</td>
</tr>
<tr>
<td>3</td>
<td>-76.9366</td>
<td>56.52478</td>
<td>0.211717</td>
<td>0.185176</td>
<td>Bus-4</td>
</tr>
<tr>
<td>4</td>
<td>-361</td>
<td>62.88671</td>
<td>0.674504</td>
<td>0.234466</td>
<td>Bus-5</td>
</tr>
<tr>
<td>6</td>
<td>-884.23</td>
<td>508.8612</td>
<td>0.266552</td>
<td>1.233141</td>
<td>Bus-2</td>
</tr>
<tr>
<td>7</td>
<td>-558.831</td>
<td>834.3687</td>
<td>0.105996</td>
<td>0.598959</td>
<td>Bus-2</td>
</tr>
<tr>
<td>8</td>
<td>62.19831</td>
<td>840.1478</td>
<td>-0.01178</td>
<td>0.431535</td>
<td>Bus-2</td>
</tr>
<tr>
<td>9</td>
<td>-36.3585</td>
<td>889.9904</td>
<td>0.006502</td>
<td>0.253933</td>
<td>Bus-2</td>
</tr>
<tr>
<td>10</td>
<td>-778.346</td>
<td>900.1397</td>
<td>0.136335</td>
<td>0.791803</td>
<td>Bus-5</td>
</tr>
<tr>
<td>11</td>
<td>-1466.69</td>
<td>908.4157</td>
<td>0.248888</td>
<td>0.633163</td>
<td>Bus-2</td>
</tr>
<tr>
<td>12</td>
<td>-966.408</td>
<td>1010.935</td>
<td>0.150414</td>
<td>0.963276</td>
<td>Bus-4</td>
</tr>
<tr>
<td>13</td>
<td>-146.546</td>
<td>1262.467</td>
<td>0.018471</td>
<td>0.259429</td>
<td>Bus-16</td>
</tr>
<tr>
<td>15</td>
<td>-13453.3</td>
<td>1520.601</td>
<td>0.815316</td>
<td>0.989698</td>
<td>Bus-2</td>
</tr>
<tr>
<td>17</td>
<td>-13060.2</td>
<td>1571.691</td>
<td>0.797646</td>
<td>0.990838</td>
<td>Bus-3</td>
</tr>
<tr>
<td>18</td>
<td>-12976.1</td>
<td>1595.823</td>
<td>0.791289</td>
<td>0.993135</td>
<td>Bus-5</td>
</tr>
<tr>
<td>20</td>
<td>-12659.8</td>
<td>1664.792</td>
<td>0.770898</td>
<td>0.996</td>
<td>Bus-4</td>
</tr>
<tr>
<td>21</td>
<td>-172.072</td>
<td>2396.805</td>
<td>0.011425</td>
<td>0.288203</td>
<td>Bus-8</td>
</tr>
<tr>
<td>22</td>
<td>-193.822</td>
<td>2400.508</td>
<td>0.012849</td>
<td>0.28709</td>
<td>Bus-7</td>
</tr>
<tr>
<td>23</td>
<td>-90.7824</td>
<td>2646.583</td>
<td>0.005459</td>
<td>0.273503</td>
<td>Bus-9</td>
</tr>
<tr>
<td>24</td>
<td>-95.4275</td>
<td>2649.73</td>
<td>0.005372</td>
<td>0.272747</td>
<td>Bus-6</td>
</tr>
<tr>
<td>25</td>
<td>-57.638</td>
<td>6464.016</td>
<td>0.001419</td>
<td>0.288726</td>
<td>Bus-10</td>
</tr>
<tr>
<td>26</td>
<td>-57.9728</td>
<td>6464.04</td>
<td>0.001427</td>
<td>0.288816</td>
<td>Bus-13</td>
</tr>
<tr>
<td>27</td>
<td>-56.9193</td>
<td>6480.975</td>
<td>0.001398</td>
<td>0.291282</td>
<td>Bus-11</td>
</tr>
</tbody>
</table>

Table II. Participation factors of the buses 2 to 5 for the critical pole

<table>
<thead>
<tr>
<th>Bus number</th>
<th>PF for the critical pole $(P_f=62.2=52791)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus-2 (GSC-1)</td>
<td>0.43</td>
</tr>
<tr>
<td>Bus-3 (GSC-2)</td>
<td>0.09</td>
</tr>
<tr>
<td>Bus-4 (GSC-3)</td>
<td>0.16</td>
</tr>
<tr>
<td>Bus-5 (GSC-4)</td>
<td>0.14</td>
</tr>
</tbody>
</table>

\[
G_i^T(s_e) = R_i L_i = R_i \begin{bmatrix} 1/\lambda_i & 0 & 0 & 0 \\ 0 & 1/\lambda_2 & 0 & 0 \\ 0 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 1/\lambda_m \end{bmatrix} \tag{13}
\]

Since $s_e$ is a pole of the $G_i^T(s)$, one of the eigenvalues of $G_i (s_e)$ ($\lambda_1, \lambda_2, \ldots, \lambda_m$) should ideally be equal to zero. However, it is close to, but not exactly zero, because the decomposition is performed using floating-point computations, which can suffer from round-off errors. The mention eigenvalue, i.e, the smallest eigenvalue is called the critical eigenvalue ($\lambda_c$) and its right and left eigenvectors are called the critical right and left eigenvectors ($r_i$ and $l_i$). If the $i^{th}$ eigenvalue is $\lambda_i$, the $i^{th}$ column of the matrix $R_i$ is $r_i$ and the $i^{th}$ row of the matrix $L_i$ is $l_i$. The sensitivity of the critical eigenvalue with respect to the $G_i(s_e)$ entries can then be obtained by

\[
S_{i} = r_i l_i \tag{14}
\]

$S_{i}$ is the sensitivity matrix and its $k^{th}$ diagonal element is Participation Factor (PF) of the $k^{th}$ bus. The bus with the largest PF is the most influencing bus on the critical eigenvalue, where, in fact, is the main source of the instability and can be called the critical bus.
A. Theoretical analysis

Fig. 2 shows the configuration of a 400-MW wind farm [10], which is used in this paper to identify the harmonic instability source. The network consists of four branches and each branch is an aggregated 100-MW wind turbine. A simple Thévenin’s equivalent voltage source is used to represent the grid. The transformers are modeled as a short-circuit impedance and the cables are modeled as a nominal π-model. In this case study, the dimension of the matrix $G_f(s)$ is 16x16, which shows the relations between Bus-1 to Bus-16 (see Fig. 2). The parameters of the wind farm are given in Appendix A. The GSC controllers are designed with acceptable bandwidths. More detailed information about the model can be found in [16], [22], [23]. Table I shows the oscillation frequency, the damping ratio, the largest participation factor, and the most influencing bus for the oscillatory modes of the wind farm. It can be seen that most poles with low and medium frequencies are related to power electronic converter (Bus-2 to Bus-5) and high-frequency poles are related to the buses where power cables and transformers are connected. The wind farm has one unstable pole, $P_8$, (critical pole) with the frequency of 840 Hz. Therefore, harmonic-frequency oscillations around 840 Hz propagate into the wind farm because of instability problems. Bus-2 has the largest PF for the critical pole, which shows that the GCS-2 is the main source of the harmonic instability. Table II shows the participation factors of the buses 2 to 5 for the critical pole. As it can be seen from Table II, Bus-2 (GSC-1) is the most influencing bus on instability, while Bus-3 (GSC-2) is less critical compared to the other buses.

III. A 400-MW WIND FARM AS A CASE STUDY

Fig. 3. Disconnecting the power converter (GSC-2) with the small Participation Factor (PF) under unstable conditions at $t = 0.3$ s.

Fig. 4. Disconnecting the power converter (GSC-1) with the large Participation Factor (PF) under unstable conditions at $t = 0.3$ s.
B. Time-domain simulation results

In order to confirm the predicted theoretical results, time-domain simulations have been provided in the PSCAD software environment. In Fig. 3, the GSC-2 is disconnected from the wind farm at t = 0.3s. Since the GCS-2 (Bus-3) has a very small PF (PF = 0.09), the wind farm will be remained unstable even after disconnecting the GSC-2. In Fig. 4, the GSC-1, the converter with the largest PF (PF = 0.43) is disconnected. Fig. 4 shows the wind farm becomes stable after disconnecting the GSC-1, which confirms the GSC-1 is the main source of the instability (as predicted in Table II).

IV. CONCLUSION

This paper has attempted to identify the contribution of each power converter to harmonic instability and to locate the main source of harmonic instability in large wind farms. A large wind farm is introduced as a Multi-Input Multi-Output (MIMO) dynamic system and the critical converter are identified by the modal analysis of the introduced MIMO system. Under unstable conditions, the theoretical analysis for a 400-MW wind farm shows that some power converters can have larger Participation Factor (PF) than the other converters. Time-domain simulations in PSCAD software confirm that disconnecting the power converters with larger PFs can transform the wind farm from an unstable condition to a stable condition.

V. APPENDIX A

TABLE A. 400-MW WIND FARM PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value (P.U.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer T₁ Leakage inductance</td>
<td>3.18x10⁻⁴</td>
</tr>
<tr>
<td>Transformer T₁ Shunt capacitance</td>
<td>7.84x10⁻⁵</td>
</tr>
<tr>
<td>Transformer T₁ Series inductance</td>
<td>1.80x10⁻⁴</td>
</tr>
<tr>
<td>Transformer T₁ Series resistance</td>
<td>0.022</td>
</tr>
<tr>
<td>33 kV cable (Cable 15A) Leakage inductance</td>
<td>3.8x10⁻⁴</td>
</tr>
<tr>
<td>33 kV cable (Cable 15A) Shunt capacitance</td>
<td>7.5x10⁻⁴</td>
</tr>
<tr>
<td>33 kV cable (Cable 15A) Series inductance</td>
<td>5.8x10⁻⁴</td>
</tr>
<tr>
<td>33 kV cable (Cable 15A) Series resistance</td>
<td>0.018</td>
</tr>
<tr>
<td>Transformer T₃ Leakage inductance</td>
<td>4.46x10⁻⁴</td>
</tr>
<tr>
<td>Transformer T₃ Resistance ( R₁ )</td>
<td>2</td>
</tr>
<tr>
<td>Transformer T₃ Inductance (L₁)</td>
<td>10.61x10⁻⁸</td>
</tr>
<tr>
<td>Transformer T₃ Capacitance (C₁)</td>
<td>9.55x10⁻⁸</td>
</tr>
<tr>
<td>SCR</td>
<td>5</td>
</tr>
<tr>
<td>Grid</td>
<td></td>
</tr>
<tr>
<td>Controller Bandwidth</td>
<td></td>
</tr>
<tr>
<td>GSC-1</td>
<td>710 Hz</td>
</tr>
<tr>
<td>GSC-2</td>
<td>450 Hz</td>
</tr>
<tr>
<td>GSC-3</td>
<td>625 Hz</td>
</tr>
<tr>
<td>GSC-4</td>
<td>780 Hz</td>
</tr>
</tbody>
</table>

$S_{max} = 450$ MVA, $f_b = 50$ Hz

REFERENCES


