Novel Statistical Approaches for Radio Channel Modelling:

Path Arrival Rate For In-room Channels With Directive Antennas.

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Stochastic multipath models

For the transmitted (complex baseband) signal $s(\tau)$, the received signal reads:

$$y(\tau) = \sum_k \alpha_k s(\tau - \tau_k),$$

The delay and gain pairs form a marked point process $\{(\tau_k, \alpha_k)\}$ with intensity function intensity function, or *path arrival rate*, $\lambda(\tau)$ and *conditional gain distribution* $p(\alpha|\tau)$.

Numerous such multipath models have been proposed, with delays generated from various point processes and gain distributions.

Example: In Turin's model [Turin et al., 1972], $\{(\tau_k, \alpha_k)\}$ is a marked Poisson point process fully specified by $\lambda(\tau)$ and $p(\alpha|\tau)$.

Factorization of the power delay spectrum

The arrival rate and mark distribution determines second moment of the received signal. For zero-mean and conditionally uncorrelated gains:

$$\mathbb{E}[|y(\tau)|^2] = \int_{-\infty}^{\infty} P(\tau - t)|s(t)|^2 dt,$$

with a *power-delay spectrum* $P(\tau)$ that factorizes as

$$P(\tau) = \underbrace{\sigma_\alpha^2(\tau)}_{\substack{\text{Variance of gain} \\ \text{at delay } \tau \\ \text{(variance of } p(\alpha|\tau))}} \times \underbrace{\lambda(\tau)}_{\substack{\textit{Path arrival rate}, \\ \text{(intensity function of} \\ \text{the point process } \{\tau_k\})}} .$$

Thus, two of these three entities should be defined to specify the second moment of the model.

How to obtain the arrival rate?

Measurement of arrival rate can be challenging:

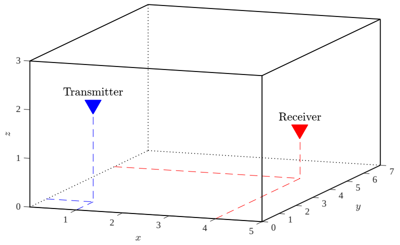
- Requires estimators for arrival rate based on received signal.
- Results are affected by imperfections of the estimators as well as noise limitations.
- Often, the (within cluster) arrival rate is set to a constant for convenience. This choice does not replicate the specular-diffuse transition observed in measurements.

Here, we attempt to *analyze* the propagation environment:

- Analysis of realistic environments is intractable.
- The method is feasible for simplistic propagation environments.
- We focus here on the arrival rate for a room-electromagnetic setting which can be analyzed using mirror theory.

Rectangular room channel

Rectangular room with directional transmit at receive antennas.



The antenna gain in direction Ω is denoted $G(\Omega)$.

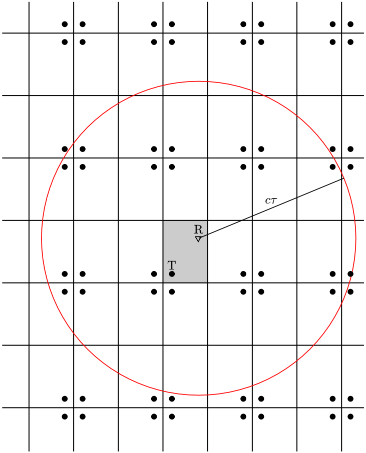
The *beam coverage fraction*,defined as the fraction of the sphere through which the antenna radiates power, i.e.

$$\omega = \frac{1}{4\pi} \int_{\mathcal{O}} d\Omega, \qquad \mathcal{O} = \{\Omega : G(\Omega) \geq \epsilon \cdot G_{max}\},$$

where $G_{\max}$ is the max. antenna gain and ϵ is a small constant set according to the application.

Mirror sources for a rectangular room

Iteratively mirroring the transmitter in the boundaries of the room give a set of mirror sources with corresponding mirror rooms index by a triplet k.



For each mirror source k we can compute

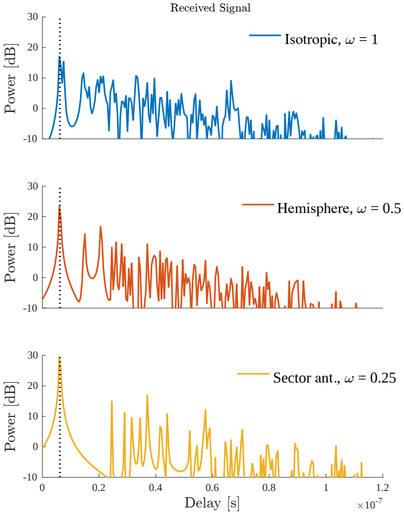
- propagation delay τ_k
- direction of departure Ω_{Tk}
- direction of arrival Ω_{Rk}
- gain α_k

$$|\alpha_k|^2 = g^{|k|} \cdot \frac{G_T(\Omega_{Tk})G_R(\Omega_{Rk})}{(4\pi c\tau_k/l_c)^2}$$

where g is the wall reflection gain, $|k|$ is the reflection order for source k, l_c is the carrier wavelength and c is the speed of light.

Simulation example: Received signal power

We plot the received signal power for identical sector antennas with spherical cap gain patterns specified by the beam coverage fraction ω.



Simulation Settings

Room dim.,	$5 \times 5 \times 3\,\mathrm{m}^3$
Reflection gain, g	0.6
Center Frequency	$60\,\mathrm{GHz}$
Bandwidth, B	$2\,\mathrm{GHz}$

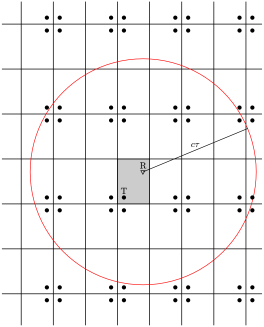
Antennas point in line-of-sight direction.

Observations:

- Specular-diffuse transition.
- More directive antennas lead to sparse received signal with higher per component power.

Approximate arrival count and rate

The *arrival count* $N(\tau)$ is the number of signal components received up until delay τ.



By adaptation of the room acoustical reasoning in [Eyring, 1930] we obtain

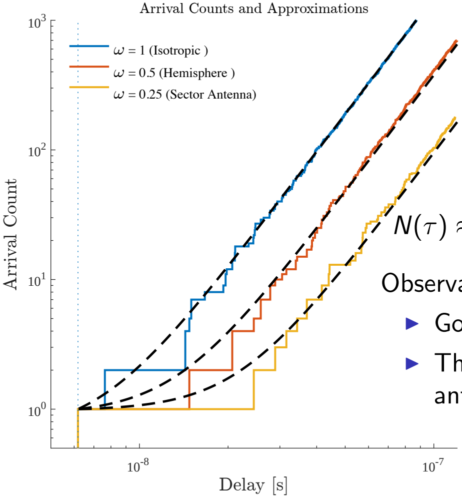
$$N(\tau) \approx \mathbb{1}(\tau \geq \tau_0) \left[1 + \frac{4\pi c^3(\tau^3 - \tau_0^3)}{3V} \right] \omega_T \omega_R$$

where τ_0 is the line-of-sight delay and V is the room volume.

Differentiation gives the arrival rate

$$\lambda(\tau) \approx \delta(\tau - \tau_0)\omega_T\omega_R + \mathbb{1}(\tau > \tau_0)\frac{4\pi c^3 \tau^2}{V}\omega_T\omega_R$$

Simulation example (contd.): Arrival count



$$N(\tau) \approx \mathbb{1}(\tau \geq \tau_0)\left[1 + \frac{4\pi c^3(\tau^3 - \tau_0^3)}{3V}\right]\omega_T\omega_R$$

Observations:

- Good approximation at large delays.
- The approximation account well for the antenna directivity.

Random transmitter position and orientation

By randomizing transmitter position and orientation, we can derive exact result for the arrival rate by use of stochastic geometry:

For uniformly distributed transmitter position and orientation the mean arrival count reads exactly

$$\mathbb{E}[N(\tau)] = \frac{4\pi c^3 \tau^3}{3V} \omega_T \omega_R \mathbb{1}(\tau > 0)$$

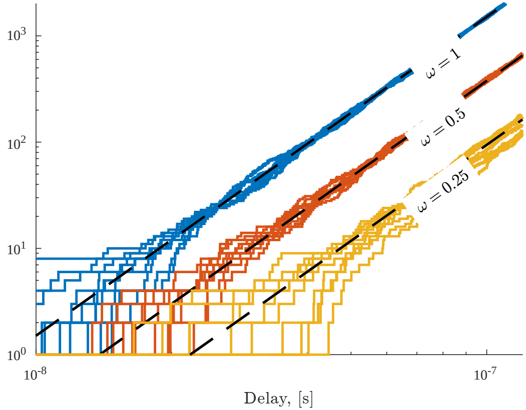
with corresponding arrival rate

$$\lambda(\tau) = \frac{4\pi c^3 \tau^2}{V} \omega_T \omega_R \mathbb{1}(\tau > 0).$$

Proof: Observe that the set of mirror source positions forms a homogeneous point process and apply Campbell's theorem to the mean arrival count. [Pedersen, 2018]

Simulation example (contd.): Mean arrival count

(a) Arrival Count, 10 realizations. Sector antenna



Observations:

- The mean count is affected by the antenna directivity.
- Individual realizations of the count fluctuate about the mean.
- The fluctuations are largest (compared to the mean) at low delays.

Power delay spectrum

The power-delay spectrum can be approximated as [Pedersen, 2018]

$$P(\tau) = \underbrace{\sigma_\alpha^2(\tau)}_{\approx \frac{e^{-\tau/T}}{(4\pi c\tau/l_c)^2}\cdot\frac{1}{\omega_T\omega_R}} \times \underbrace{\lambda(\tau)}_{=\frac{4\pi c^3\tau^2}{V}\omega_T\omega_R \mathbb{1}(\tau>0).} \approx \mathbb{1}(\tau>0)\frac{e^{-\tau/T}}{4\pi V/l_c^2 c}.$$

with the (Eyring-Kuttruff) reverberation time defined as

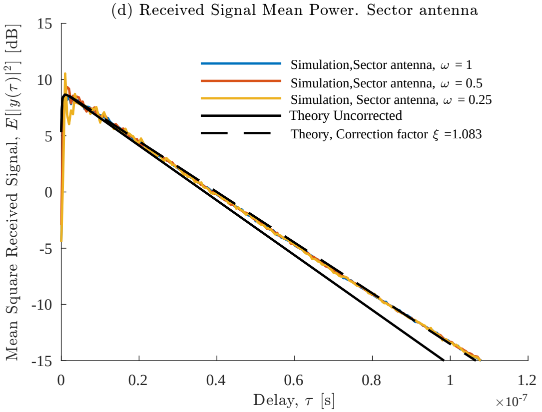
$$T = -\frac{4V}{cS\ln(g)}\cdot\xi, \qquad \text{where} \qquad \xi = \frac{1}{1+\gamma^2\ln(g)/2}.$$

The constant γ^2, depends on the aspect ratio of the room and is typically in the range 0.3 to 0.4 [Kuttruff, 2000].

The delay power spectrum does not depend on the antenna directivity!

Simulation example (contd.): Power delay spectrum

Average received power (10000 Monte Carlo runs)

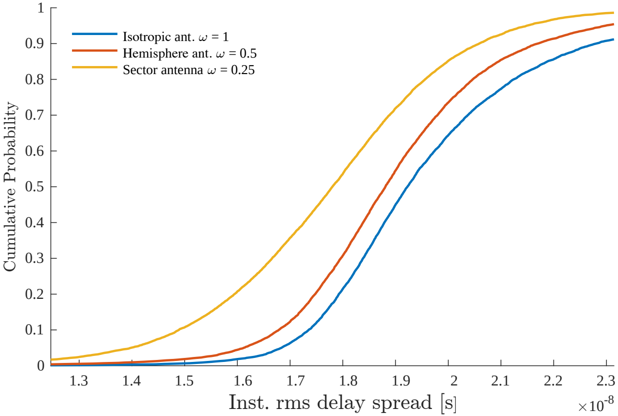


Observations:

- The approximation gives an excellent fit when applying the correction factor. In this case, $\gamma^2 = 0.35$ and $\xi \approx 1.08$.
- The power-delay spectrum is unaffected by the antenna directivity.

Simulation example (contd.): RMS Delay Spread

CDF, Mean delay:



- Higher antenna directivity gives lower rms delay spread.
- This occurs even in models with identical power delay spectrum.

Conclusion

- For simplistic scenarios, we can derive the path arrival rate by randomizing the antenna positions and orientations.
- Averaging over the uniformly distributed transmit antenna position and orientation, we see that the path arrival rate

$$\lambda(\tau) = \frac{4\pi c^3 \tau^2}{V} \omega_T \omega_R \mathbb{1}(\tau > 0)$$

  - quadratic in delay (gives a specular-diffuse transition)
  - inversely proportional to room volume (larger rooms lead to a slower transition)
  - proportional to the product of antenna beam coverage fractions (more directive antennas yield lower arrival rate).
- Even though the power delay spectrum is *not* affected by antenna directivity, the distribution of rms delay spread is.
- To accurately model system related entities such as rms delay spread, the model should account for the arrival rate.

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