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A Formal Semantics for Concept Understanding Relying on Description Logics

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Keywords: Concept Understanding, Conceptualisation, Terminology, Interpretation, Formal Semantics, Description Logics, Ontologies.

Abstract: In this research, Description Logics (DLs) will be employed for logical description, logical characterisation, logical modelling and ontological description of concept understanding in terminological systems. It's strongly believed that using a formal descriptive logic could support us in revealing logical assumptions whose discovery may lead us to a better understanding of 'concept understanding'. The Structure of Observed Learning Outcomes (SOLO) model as an appropriate model of increasing complexity of humans' understanding has supported the formal analysis.

1 INTRODUCTION

The central focus of this research is on *concepts*. My point of departure is the special focus on the fact that there is a general problem concerning the notion of 'concept', in linguistics, psychology, philosophy and computer science. This research aims at providing a logical description (and analysis) of the use of 'concepts' in terminological knowledge representation systems, and, thus, I need to assume concepts' applications in order to be comprehensible in the context and in my logical formalism. Taking into consideration (Baader et al., 2010) and (Rudolph, 2011), a concept might be correlated with a distinct 'entity' or to/with its essential features, characteristics and properties. Note that an entity's properties express its relationships with itself and with other entities. Through the lens of Predicate Logic, a concept might be considered to be equivalent to a [unary] predicate. It shall be emphasised that this remark is not about language, but this is how concepts are perceived by logicians. Accordingly, it could be claimed that predicates could—logically—express the characteristics of concepts in terminological systems. More specifically, predicates assign characteristics, features and properties of concepts into some subjects. It's believed that predicates may determine the applications of logical descriptions. As all logicians know, predicates play fundamental roles in reasoning processes and in giving satisfying conditions for definitions of [logical] truth. By taking into consideration that 'a predicate expresses a condition that the entities referred to may satisfy, in which case the resulting sentence will be true (see (Blackburn, 2016))', predicates can be applied in expressing meanings within formal semantics. Subsequently, the formal semantics could focus on multiple conditions through definitions of truth (and falsity). The central objective of formal semantics can be said to be formalising and manipulating the relationships between the signifiers of a description and what the signifiers do [or have been designed to do], see (Jackendoff, 1990; Gray et al., 1992; Barsalou, 1999; Resnik, 1999).

As mentioned, the central focus of this research is on concepts (and through the lens of Predicate Logic). Concepts and their interrelationships will be used to establish the basic terminology adopted in the modelled domain regarding the hierarchical structures. My logical descriptions will have a special focus on my methodological assumption that expresses that 'human beings can find out that an individual thing/phenomenon is an instance of a formed concept, and, thus, their individual grasp of that concept (in the form of their conceptions) provide foundations for producing their own conceptualisations'. This article will focus on describing and characterising humans' concept understandings and will deal with a formal-semantic model for figuring out the underlying logical

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assumptions of 'concept understanding'. The term 'understanding' will be observed from multiple perspectives, and, subsequently, the expressiveness of the semantic model's descriptions will be improved. The Structure of Observed Learning Outcomes (SOLO) taxonomy is an appropriate model of increasing complexity of humans' understanding. SOLO as a descriptive model of knowing and understanding can support my formalism. Additionally, its taxonomical structure could be expressed in the form of some logical inclusions. In this research, the formal semantic analysis of [concept] understanding is based on Description Logics (DLs). I believe that DLs can support me in proposing an understandable logical description for clarifying concept understanding. DLs as the profound formalism are used for representing predicates and for formal reasoning over them. They mainly focus on terminological knowledge. It is of a terminological system's particular importance in providing a logical formalism for knowledge representation systems, and, also, for semantic representations and ontologies (as formal and explicit specification of a shared conceptualisation on the domains of interest), see (Davies et al., 2003; Staab and Studer, 2009).

The main contributions of this research are: (i) providing a formal semantics (relying on DLs) for conceptual analysis of concept understanding, and analysing a knowledge representation formalism for expressing concept understanding, and (ii) designing and formalising an ontology that provides a structural representation of concept understanding within the analysed semantic model.

2 DESCRIPTION LOGICS

First, I shall mention that (Baader et al., 2010) is my main reference to Description Logics. Description Logics (DLs) represent knowledge in terms of *individuals* (objects, things), *concepts* (classes of things), and *roles* (relationships between things). Individuals correspond to constant symbols, concepts to unary predicates, and roles to binary (or any other *n*-ary) predicates and relations in Predicate Logic. Reconsidering the predicate P in Predicate Logic, we have [possibly specified] concept C in DLs. There are two kinds of atomic symbols, which are called *atomic concepts* and *atomic roles*. These symbols are the elementary descriptions from which we can inductively (by employing concept constructors and role constructors) build the specified descriptions. Considering N_C, N_R and N_O as the sets of atomic concepts, atomic roles and individuals respectively, the ordered triple $\langle N_C, N_R, N_O\rangle$ represents a signature. The set of main logical symbols in ALC (Attributive Language with Complements: the Prototypical DL, see (Schmidt-Schauss and Smolka, 1991)) is: { Conjunction ($\sqcap$: And), Disjunction ($\sqcup$: Or), Negation ($\neg$: Not), Existential Restriction ($\exists$: There exists ...), Universal Restriction ($\forall$: For all ...) }. We also have Atomic Concepts (A), Top Concept ($\top$: Everything) and Bottom Concept ($\bot$: Nothing) in ALC.

In order to define a formal semantics, we need to apply terminological interpretations over our signatures. More particularly, any [terminological] interpretation consists of (i) a non-empty set Δ (that is the interpretation domain and consists of any variable that occurs in any of the concept descriptions), and (ii) an interpretation function $.^I$ (let me call it 'interpreter'). The interpreter assigns to every individual (like a) a '$a^I \in \Delta^I$'. Also, it assigns to every atomic concept A (or every atomic unary predicate) a set $A^I \subseteq \Delta^I$, and to every atomic role P (or every atomic binary predicate) a binary relation $P^I \subseteq \Delta^I \times \Delta^I$. Table 1 reports the syntax and the semantics of ALC.

Table 1: The Prototypical Description Logic.

Syntax	Semantics
A	$A^I \subseteq \Delta^I$
P	$P^I \subseteq \Delta^I \times \Delta^I$
$\top$	Δ^I
$\bot$	$\varnothing$
$C \sqcap D$	$(C \sqcap D)^I = C^I \cap D^I$
$C \sqcup D$	$(C \sqcup D)^I = C^I \cup D^I$
$\neg C$	$(\neg C)^I = \Delta^I \setminus C^I$
$\exists R.\ C$	$\{ a \mid \exists b.(a,b) \in R^I \wedge b \in C^I \}$
$\forall R.\ C$	$\{ a \mid \forall b.(a,b) \in R^I \supset b \in C^I \}$

A knowledge base in DLs usually consists of a number of terminological axioms and world descriptions (so-called: 'assertions'), see Table 2. The terminological interpretation I is called a *model* of an axiom [or a model of a basic world description], if, and only if, it can semantically satisfy it, see Tables 2 and 3. In the following Tables P is an atomic role, R and S are role descriptions, A is an atomic concept, and C and D are concept descriptions.

Table 2: Axioms and World Descriptions in DLs.

Name	Syntax	Semantics
Concept Inclusion Axiom	$C \sqsubseteq D$	$C^I \subseteq D^I$
Role Inclusion Axiom	$R \sqsubseteq S$	$R^I \subseteq S^I$
Concept Equality Axiom	$C \equiv D$	$C^I = D^I$
Role Equality Axiom	$R \equiv S$	$R^I = S^I$
Concept Assertion	$C(a)$	$a^I \in C^I$
Role Assertion	$R(a, b)$	$(a^I, b^I) \in R^I$

Table 3: Inductive Concept Descriptions.

Over Concept	Over Role
$A^I \subseteq \Delta^I$	$P^I \subseteq \Delta^I \times \Delta^I$
$\bot^I = \varnothing$	$\bot^I = \varnothing$
$(\neg C)^I = \Delta^I \setminus C^I$	$(\neg R)^I = (\Delta^I \times \Delta^I) \setminus R^I$
$(C \sqcap D)^I = C^I \cap D^I$	$(R \sqcap S)^I = R^I \cap S^I$

Let me start the logical analysis with two examples:

Example 1. Mary has verified that 'there is a young student' and 'there is a non-old student' are expressing the same matter. Her verification between these two propositions is expressible in DLs by: $\exists$hasStudent.Young $\equiv$ $\exists$hasStudent.¬Old. It's realisable that Mary has assumed the axiom stating that Young and Old are two disjoint concepts, and, in fact, the logical term 'Young $\sqcap$ Old $\sqsubseteq \bot$' has formed a terminological axiom for Mary. It's obvious that Mary's interpretation over (i) Young $\sqcap$ Old $\sqsubseteq \bot$ (meaning that Young and Old are disjoint concepts) and (ii) Person $\sqsubseteq$ Young $\sqcup$ Old (meaning that every person is either young or old) has played crucial roles here. In fact, Mary has interpreted, and, respectively, understood that these two sentences ('there is a young student' and 'there is a non-old student') have the same meanings. More specifically, Mary's terminological interpretation (over i and ii) has produced her understanding of an equivalence between the concept descriptions $\exists$hasStudent.Young and $\exists$hasStudent.¬Old. We can see that Mary's interpretation has been restricted (limited) to her understanding of disjointness of the concept descriptions $\exists$hasStudent.Young and $\exists$hasStudent.¬Old. At this point I shall claim that the concepts (concept descriptions) C and D are logically and semantically equivalent, when 'for all' possible terminological interpretations like I, we have: $C^I = D^I$. In this example, if one person, say John, does not assume the axioms stating that 'Young and Old are two disjoint concepts' and 'every person is either young or old', then there will not be an equivalence relation between $\exists$hasStudent.Young and $\exists$hasStudent.¬Old. Let me conclude that Mary's and John's understandings are dissimilar, because they have had different terminological interpretations in their minds (and it is because of their different conceptions and concept formations). For example, regarding John's terminological interpretation, the proposition 'there is a middle-aged student' could be added beside 'there is a young student' and 'there is a non-old student'. In fact, John could have the axiom 'Person $\sqsubseteq$ Young $\sqcup$ MiddleAged $\sqcup$ Old (meaning that every person is young or middle aged or old)' in his mind. Consequently, John by taking this axiom (based on his own conception) into consideration doesn't understand '$\exists$hasStudent.Young' and '$\exists$hasStudent.¬Old' as equivalent concept descriptions.

Example 2. Mary has verified that 'Anna has a child who is a philosopher' and 'Anna has a child who is a painter' could be jointly expressed by 'Anna has a child who is a philosopher and painter'. Translated into DLs we have her expression as followings: $\exists$hasChild.Philosopher $\sqcap$ $\exists$hasChild.Painter $\equiv$ $\exists$hasChild.(Philosopher $\sqcap$ Painter). Suppose that Anna has two children and one is a philosopher and the other one is a painter. Then, $\exists$hasChild.(Philosopher $\sqcap$ Painter) is not equivalent to $\exists$hasChild.Philosopher $\sqcap$ $\exists$hasChild.Painter. Actually, Mary has not proposed a correct description, and this is because of her inappropriate terminological interpretation. Accordingly, her understanding has followed her inappropriate interpretation. In fact, she incorrectly (semantically: False) has understood that the proposition 'Anna has a child who is a philosopher and painter' expresses the same matter. Reconsidering the proposed formalism, $\exists$hasChild.Philosopher $\sqcap$ $\exists$hasChild.Painter and $\exists$hasChild.(Philosopher $\sqcap$ Painter) are not—semantically—the same and there should not be an equivalence symbol between them. Thus, Mary's interpretation has not been satisfactory. Subsequently, her understanding is not satisfactory and appropriate.

3 A SEMANTIC MODEL FOR CONCEPT UNDERSTANDING

In this section I clarify my logical conceptions of 'concept understanding'. The term 'understanding' is very complicated and sensitive in psychology, neuroscience, cognitive science, philosophy and epistemology. There has not been any complete model for describing this term, but there are some proper models of (i) understanding of understanding (see (Foerster, 2003)), (ii) understanding representation (see (Peschl and Riegler, 1999; Webb, 2009)), and (iii) specification of the components of understanding (i.e., from the cognition's and from the affects' perspectives), see (Chaitin, 1987; Kintsch et al., 1990; di Pellegrino et al., 1992; MacKay, 2003; Zwaan and Taylor, 2006; Uithol et al., 2011; Uithol and Paulus, 2014). This research—by analysing a formal semantics—focuses on the junctions of 'understanding of concept understanding' and 'concept understanding representation' in terminological systems, and, more specifically, it focuses on logical analysis of concept understanding and its terminological representation.

3.1 Concept Understanding as a Relation (and Function)

I shall claim that *concept understanding*—as a relation—could relate 'the characteristics and attributes of a concept' with 'a description'. More specifically, *understanding* is a function (mapping) from a concept into some propositions (and statements) which could be interpreted as 'concept descriptions'. In fact, the characteristics and properties of a concept by means of the *understanding* function become mapped into concept descriptions. Let me be more specific:

A. A human being—by concept understanding—attempts to map the significant characteristics of concepts into some concept descriptions. For example, 'breathing'—as a biological and psychological process—is a characteristic and trait of all animals, and, thus, $\mathsf{breathing}$ (that is a role) is the characteristic of the concept Animal. Then, (i) knowing the fact that the individual horse is an instance of the concept Animal (Formally: $\mathsf{Animal(horse)}$), and (ii) drawing the [concept subsumption] inference '$\mathsf{Horse} \sqsubseteq \mathsf{Animal}$', collectively lead us to knowing and to understanding that 'horses breathe' (or equivalently: 'horses do breathing'). The role $\mathsf{breathing}$ could be manifested in the concept Breath. Therefore, (i) and (ii) collectively lead us to expressing the concept description '$\mathsf{Animal(horse)} \sqcap \exists \mathsf{hasTrait.Breath}$' for the individual horse (as an instance of the concept Animal), and, respectively, for the concept Horse (as a sub-concept of Animal).

B. A human being—by concept understanding—attempts to map the concepts' properties and their interrelationships with themselves into some concept descriptions. For example, the one who knows that 'male horses breathe', by taking the terminological and assertional axioms $\{\mathsf{Animal(horse)},\ \mathsf{Horse} \sqsubseteq \mathsf{Animal},\ \mathsf{MaleHorse} \sqsubseteq \mathsf{Horse},\ \mathsf{FemaleHorse} \sqsubseteq \mathsf{Horse}\}$ into consideration can know and understand that 'female horses breathe' as well.

C. A human being—by concept understanding—attempts to map the concepts' properties and their relationships with other concepts into some concept descriptions. For example, the one who knows that 'horses breathe' (and as described: $\mathsf{Animal(horse)} \sqcap \exists \mathsf{hasTrait.Breath}$), could—respectively—know and understand that the individual rabbit (that is an animal) breathes as well. So, she/he could express that 'rabbits breathe', and, in fact, $\mathsf{Animal(rabbit)} \sqcap \exists \mathsf{hasTrait.Breath}$.

Conclusion. Relying on Predicate Logic [and on DLs], the phenomenon of 'concept understanding' could be interpreted as a 'binary predicate' [and as a 'role'] of human beings on expressing some concept descriptions. Let me represent this role by '$\mathsf{understanding}$'.

3.2 Concept Understanding as a Conceptualisation

The concept understanding could be interpreted to be the limit/type of conceptualisation. Accordingly, humans need to conceptualise concepts in order to understand them. In (Badie, 2016a) and (Badie, 2016b), I have defined a 'conceptualisation' as "a uniform specification of the separated understandings". In fact, any concept understanding could be interpreted as a local manifestation of a global conceptualisation. Additionally, human beings' grasps of concepts could provide proper foundations for generating their own conceptualisations. I shall claim that 'concept understanding' could be acknowledged as a limited type of humans' concept constructions, when the concept constructions are supported by their own conceptualisations. Therefore, $\mathsf{conceptualising}$ is a role of human beings. This conclusion—relying on DLs—could be represented by the 'role inclusion (or role subsumption)' $\mathsf{understanding} \sqsubseteq \mathsf{conceptualising}$. In other words, 'understanding a concept' has been acknowledged as the sub-role of 'conceptualising that concept'. On the other hand, *it is not the case that all conceptualisations are understandings*. In

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fact, all the conceptualised concepts could not be understood.

3.3 Concept Understanding as an Interpretation-based Model

Generally, an interpretation is the act of elucidation, explication and explanation, see (Simpson and Weiner, 1989). According to (Honderich, 2005) and through the lens of philosophy, "…in existential and hermeneutic philosophy, 'interpretation' becomes the most essential moment of human life: The human being is characterized by having an 'understanding' of itself, the world, and others. This understanding, to be sure, does not consist—as in classical ontology or epistemology—in universal features of universe or mind, but in subjective–relative and historically situated interpretations of the social. …". Regarding (Blackburn, 2016) and through the lens of logic, an 'interpretation' of a logical system assigns meanings (or semantic values) to the formulae and their elements. At this point I shall take into consideration that the phenomenon of 'interpretation' could have a conjunction with the phenomenon of 'terminological interpretation' in formal languages. More specifically, the one who has engaged her/his interpretations to explicate [and justify] what [and why] she/he means by classifying a thing/phenomenon as an instance of a concept, needs to interpret the non-logical signifiers of different concept descriptions within her/his linguistic expressions.

Considering any set of non-logical symbols (that have no logical consequences) in a terminology, a terminological interpretation over humans' languages could be described to be constructed based on the tuple ⟨*Interpretation Domain*, *Interpretation Function*⟩. The interpretation domain (or the universe of interpretation) might be called 'universe of discourse'. As mentioned in previous section, the interpretation domain must be non-empty. This non-empty set forms the range of any variable that occur in any of the concept descriptions within linguistic expressions. It's a fact that the collection of the rules and the processes that manage different terms and descriptions in linguistic expressions, cannot have any meaning until the non-logical signifiers and constructors are given terminological interpretations. The interpretations prepare humans for producing their personal meaningful [and understandable] concept descriptions. Hence, I have recognised all 'concept understandings' as 'concept interpretations'. This conclusion—relying on DLs—could be represented by the 'role inclusion' understanding $\sqsubseteq$ interpreting. Therefore, 'concept understanding' has been expressed as the sub-role of 'concept interpreting'. But, on the other hand, not all interpretations (of concepts) imply understandings (of concepts). Equivalently, *it is not the case that all interpretations are understandings*. In other words, all the interpreted concepts may not be understood. Accordingly, considering any interpretation as a function, 'concept understanding' is recognised as an 'interpretation function'.

From this point I apply the function *UND* (as a limit of the interpretation function *I*) in my formalism. Then, C^{UND} represents 'Concept Understanding', where C stands for Concept. Consequently, considering *UND* as a kind of interpretation, there exists a tuple like $\langle D_U, C_{understood}\rangle$, where (i) D_U represents the understanding domain (that consists of the variables that occur in any of the concept descriptions which are going to be understood), and (ii) $C_{understood}$ is the understood concept. $C_{understood}$ is achievable based on the understanding function $-^{UND}$. Relying on the function $-^{UND}$,

$$C^{UND} \subseteq C^I \subseteq \Delta^I \;\;\&\;\; D_U^{UND} \subseteq \Delta^I.$$

It shall be stressed that D_U^{UND} expresses 'understanding all concepts belonging to the understanding domain'. Note that $-^{UND}$ (that is a function) can provide a model for a terminological [and assertional] axiom. Therefore, the desired model (i) is a restricted form of a terminological-interpretation-based model, and (ii) can satisfy the semantics of the terminological and assertional axioms (read '$UND \vDash$ Axiom': *UND* satisfies the axiom), see Table 4. Consequently:

$$C^{UND} \subseteq C^I \subseteq \Delta^I \quad \& \quad -^{UND} : C \rightarrow C^{UND}$$
$$\text{Where: } C^{UND} \subseteq D_U^{UND} \subseteq \Delta^I .$$

I shall emphasise that we are not able to conclude that $C^I \subseteq D_U^{UND}$. On the other hand, we certainly know that , $C^{UND} \subseteq \Delta^I$ (because $C^{UND} \subseteq C^I$ and $C^I \subseteq \Delta^I$). According to the analysed characteristics, the *UND* understanding model in my terminology is constructed based upon the tuple ⟨*Understanding Domain*, *Understanding Function*⟩ as:

$$UND = \langle D_U^{UND}, -^{UND}\rangle.$$

Table 4: *Understanding* Model: Terminologies, World Descriptions and their Semantics.

Name	Description, Semantics
Understanding a Concept Inclusion	$UND \vDash (C \sqsubseteq D) \Rightarrow C^{UND} \subseteq D^{UND}$
Understanding a Role Inclusion	$UND \vDash (R \sqsubseteq S) \Rightarrow R^{UND} \subseteq S^{UND}$
Understanding a Concept Equality	$UND \vDash (C \equiv D) \Rightarrow C^{UND} = D^{UND}$
Understanding a Role Equality	$UND \vDash (R \equiv S) \Rightarrow R^{UND} = S^{UND}$
Understanding a Concept Assertion	$UND \vDash C(a) \Rightarrow a^{UND} \in C^{UND}$
Understanding a Role Assertion	$UND \vDash R(a, b) \Rightarrow (a^{UND}, b^{UND}) \in R^{UND}$

Table 5 is based on Table 4 and itemises inductive concept descriptions and their semantics as the products of the understanding model.

Table 5: Understanding Inductive Concept Descriptions.

Model Satisfies the Vocabulary	Semantics
$UND \vDash \top$	$\top^{UND} = \top$
$UND \vDash \bot$	$\bot^{UND} = \varnothing$
$UND \vDash \neg R$	$(\neg R)^{UND} = \top \setminus R^{UND}$
$UND \vDash \neg C$	$(\neg C)^{UND} = D_U^{UND} \setminus C^{UND}$
$UND \vDash (R \sqcap S)$	$(R \sqcap S)^{UND} = R^{UND} \cap S^{UND}$
$UND \vDash (C \sqcap D)$	$(C \sqcap D)^{UND} = C^{UND} \cap D^{UND}$

3.4 Concept Understanding as a Product of Functional Roles

How could we employ DLs in order to describe an understanding function as a [functional] role of a human being? Let me interpret functional roles (features) as the roles that are existentially functions, and, thus, they can express *functional* actions, movements, procedures and manners of human beings. Let N_F be a set of functional roles and N_R be the set of role [descriptions]. Obviously: $N_F \subseteq N_R$, and informally, functional roles are some kinds of roles.

Lemma. The *UND* understanding model is–semantically—structured over:

a. the understanding domain (or D_U),
b. the understanding function (or -^{UND}), and
c. the set D_U^{UND} (or equivalently, the effect of the understanding function -^{UND} on the Top concept) that represents understanding all atomic concepts (everything) in the understanding domain.

Analysis. The *UND* understanding model associates with each atomic concept a subset of D_U^{UND}, and with each ordinary atomic role a binary relation over $D_U^{UND} \times D_U^{UND}$. Note that any functional role can be recognised as a partial function. More specifically, considering $F = f_1 \circ \cdots \circ f_n$ (F is a chain of functional roles), the chain $f_1^{UND} \circ \cdots \circ f_n^{UND}$ represents the composition of n partial understanding functions. In fact, by employing *UND*, any f_i^{UND}—semantically—supports the [overall] functional role F^{UND}. Note that for all i in $(1,n)$, f_{i+1} produces the input of f_i. Therefore, the understanding of f_{i+1} (the output of f_{i+1}) provides the input of the understanding of f_i. In particular, any concept description could be understood over the subsets of D_U^{UND}. This characteristic is very useful in making a strong linkage between the terms ‘understanding’ and ‘chain of functional roles’. It supports my semantic model in scheming and describing "*the understanding as the product of a chain of functional roles, where the functional roles are the partial understanding functions*". You will see how it works.

3.5 Humans’ Functional Roles through SOLO’s Levels

According to (Biggs and Collis, 2014), the Structure of Observed Learning Outcomes (SOLO) taxonomy is a proper model that can provide an organised framework for representing different levels of humans’ understandings. This model is concerned with various complexities of understanding on its different layers. According to SOLO and focusing on humans’ levels of knowledge with regard to a concept, we have:

- Pre-structured knowledge. Here humans’ knowledge of a concept is pre-structured (and is the product of their pre-conceptions).
- Uni-structured knowledge. Humans have a limited knowledge about a concept. They may know one or few isolated fact(s) about a concept.
- Multi-structured knowledge. They are getting to know a few facts relevant for a concept, but they are still unable to link and relate them together.
- Related Knowledge. They have started to move towards deeper levels of understanding of a concept. Here they are able to link different facts and to explain several conceptions of a concept.

- Extended Abstracts. This is the most complicated level. Humans are not only able to link lots of related conceptions [of a concept] together, but they can also link them to other specified and complicated conceptions. Now they are able to link multiple facts and explanations in order to produce more complicated extensions relevant for a concept.

Obviously, the extended abstracts are the products of deeper comprehensions of related structures. Related structures are the products of deeper comprehensions of multi-structures. The multi-structures are the products of deeper comprehensions of uni-structures, and the uni-structures are the products of deeper comprehensions of pre-structures. Let me select a process (as a sample of humans' functional roles) from any of the SOLO's levels and formalise it. According to SOLO, *creation* (with regard to an understood concept) is an instance of 'extended abstracts', *justification* (with regard to an understood concept) is an instance of 'related structures', *description* (with regard to an understood concept) is an instance of 'multi-structures' and *identification* (with regard to an understood concept) is an instance of 'uni-structures'. Therefore, *Creation*, *Justification*, *Description* and *Identification* are four processes which could be analysed as functions in the model. Any of these functions can support a functional role as a 'partial understanding function':

i. *Creation* has interrelatedness with creatingOf that is a functional role and extends the humans' mental abstracts.

ii. *Justification* has interrelatedness with the functional role justifyingOf that relates the lower structures.

iii. *Description* has correlation with the functional role describingOf that produces the multi-structures.

iv. *Identification* has correlation with the functional role identifyingOf that generates the uni-structures.

It shall be emphasised that identifyingOf, describingOf, justifyingOf and creatingOf are only four examples of functional roles within SOLO's categories, and, in fact, the SOLO's levels are not limited to these functions. For example, followingOf and namingOf are two other instances of uni-structures, combiningOf and enumeratingOf are two other instances of multi-structures, analysingOf and arguingOf are two other instances of related structures, and formulatingOf and theorisingOf are two other instances of extended abstracts.

As mentioned, the functional roles creatingOf, justifyingOf, describingOf and identifyingOf represent the equivalent roles of the *creation*, *justification*, *description* and *identification* functions respectively. Furthermore, these functions are the partial functions of the *understanding* function. Obviously, the understanding function (that is a process) could also be considered to be equivalent to a functional role like understandingOf. Employing the 'role inclusion' axiom we have: (1) creatingOf $\sqsubseteq$ understandingOf, (2) justifyingOf $\sqsubseteq$ understandingOf, (3) describingOf $\sqsubseteq$ understandingOf, and (4) identifyingOf $\sqsubseteq$ understandingOf. Equivalently: (1) $creation \subseteq understanding$, (2) $justification \subseteq understanding$, (3) $description \subseteq understanding$, and (4) $identification \subseteq understanding$.

It shall be claimed that understandingOf—conceptually and logically—supports 'the *understanding* function based on the analysed understanding model (or *UND*)'. Similarly, we can define *CRN*, *JSN*, *DSN* and *IDN* as sub-models of *UND* for representing *creation*, *justification*, *description* and *identification* respectively. Any of these models can—semantically—satisfy the terminologies and world descriptions in Table 4. Accordingly—relying on inductive rules—they can satisfy concept descriptions in Table 5.

Note that *CRN* (as a model) fulfils the desires of *UND* better (and more satisfying) than *JSN*, *DSN* and *IDN*. Considering D_U as the understanding domain, we have:

$$D_U^{UND} \subseteq D_U^{CRN} \subseteq D_U^{JSN} \subseteq D_U^{DSN} \subseteq D_U^{IDN}.$$

More specifically:

- D_U^{CRN} represents the model of *creation* over the understanding domain. It consists of concepts which are (or could be) 'created' by human beings. Formally: $C^{CRN} \in D_U^{CRN}$.
- D_U^{JSN} represents the model of *justification* over the understanding domain. It consists of concepts which are (or could be) 'justified' by human beings. Formally: $C^{JSN} \in D_U^{JSN}$.
- D_U^{DSN} represents the model of *description* over the understanding domain. It consists of concepts which are (or could be) 'described' by human beings. Formally: $C^{DSN} \in D_U^{DSN}$.
- D_U^{IDN} represents the model of *Identification* over the understanding domain. It consists of

concepts which are (or could be) 'identified' by human beings. Formally: $C^{IDN} \in D_U^{IDN}$.

Proposition. The terminological axioms and the world descriptions (in Table 4) and inductive concept descriptions (in Table 5) are all valid and meaningful for *CRN*, *JSN*, *DSN* and *IDN*. Therefore, inductive concept descriptions are also valid and meaningful over the concatenation of the *creation*, *justification*, *description* and *identification* functions that have supported these terminological models.

Proposition. All satisfactions based on *IDN* are already satisfied by *DSN*, *JSN* and *CRN* over D_U^{DSN}, D_U^{JSN} and D_U^{CRN} respectively. Informally, if a human being is able to describe, justify and create with regard to her/his conception of a concept, so, she/he is already capable of identifying that concept. Furthermore, she/he might be able to identify something else with regard to her/his conception of that concept.

Formal Analysis. The semantics of the composite function '*creation* (*justification* (*description* (*identification* (*C*))))'—that is the product of the chain of functional roles—supports the proposed semantic model on D_U^{UND}, which is the central domain of the understanding (central part of the understanding domain). Considering all the roles relevant for the concept *C*, we have:

1. $(\forall R_1.C)^{CRN} =$
$\{ a \in D_U^{CRN} \mid \forall b.(a,b) \in R_1^{CRN} \rightarrow b \in C^{CRN} \}$.

Therefore:

2. $(\forall R_2.C)^{JSN} =$
$\{ a \in D^{JSN} \mid \forall b.(a,b) \in R_2^{JSN} \rightarrow b \in C^{JSN} \}$.

Therefore:

3. $(\forall R_3.C)^{DSN} =$
$\{ a \in D_U^{DSN} \mid \forall b.(a,b) \in R_3^{DSN} \rightarrow b \in C^{DSN} \}$.

Therefore:

4. $(\forall R_4.C)^{IDN} =$
$\{ a \in D_U^{IDN} \mid \forall b.(a,b) \in R_4^{IDN} \rightarrow b \in C^{IDN} \}$.

In the afore-itemised formalism R_1, R_2, R_3 and R_4 stand for creatingOf, justifyingOf, describingOf and identifyingOf respectively. Consequently, *CRN*, *JSN*, *DSN* and *IDN* have been observed as roles of human beings. Accordingly, it's possible to represent the chain of functional roles in the form of a collection of implications as following:

$$(\forall R_1.C)^{CRN} \Rightarrow (\forall R_2.C)^{JSN} \Rightarrow (\forall R_3.C)^{DSN} \Rightarrow (\forall R_4.C)^{IDN}.$$

It must be concluded that 'any role based on a conception of *C*' to the left of any of arrows makes a logical premise for 'other roles based on conceptions of *C'* to the right of that arrow. It shall be stressed that this is a very important terminological fact. The concluded logical relationship represents a flow of concept understanding from deeper layers to shallower layers.

4 AN ONTOLOGY FOR CONCEPT UNDERSTANDING

According to (Grimm et al., 2007; Staab and Studer, 2009), an ontology—from the philosophical point of view—is described as studying the science of being and existence. Ontologies must be capable of demonstrating the structure of the reality of a thing/phenomenon. They check multiple attributes, particularities and properties that belong to a thing/phenomenon because of its natural and structural existence. An ontology—from another perspective and through the lenses of information and computer sciences—is described as an explicit and formal specification of a shared conceptualisation in a domain of interest. However, in my opinion, there could be very strong relationship between these two descriptions of ontologies. In fact, ontologies in information sciences attempt to mirror the things'/phenomena's structures in virtual and artificial systems. The ontological descriptions in information sciences tackle to provide appropriate logical and formal descriptions of a phenomenon [and of its structure] considering various concepts relevant for that phenomenon. From this perspective, an ontology can be schemed and demonstrated by semantic networks and semantic representations. A semantic network is a graph whose nodes represent concepts (e.g., unary predicates) and whose arcs represent relations (e.g., binary/*n*-ary predicates) between the concepts. Accordingly, semantic networks provide structural representations of a thing/phenomenon. In Figure 1 I have designed a semantic network as an ontology for 'concept understanding'. This hierarchical semantic representation, (1) specifies the conceptual relationships between the most important ingredients of this research, (2) demonstrates the logical representation of concept understanding. It shows how the proposed model attempts to represent concept understanding. This semantic representation can be interpreted as a specification of the shared conceptualisation of concept understanding within terminological systems. The proposed ontology can be reformalised and formalised in ALC in the form of a collection of fundamental terminologies as following:

A Formal Ontology for Concept Understanding.

{ UnaryPredicate $\sqsubseteq$ Predicate, BinaryPredicate $\sqsubseteq$ Predicate, Concept $\sqsubseteq$ UnaryPredicate, Concept $\sqsubseteq$ $\exists$hasInstance.Individual, BinaryPredicate $\sqsubseteq$ ($\exists$hasNode.Individual $\sqcap$ $\exists$hasNode.Individual), Role $\sqsubseteq$ BinaryPredicate, Relation $\sqsubseteq$ BinaryPredicate, Function $\sqsubseteq$ Relation , Interpretation $\sqsubseteq$ Function, Conceptualisation $\sqsubseteq$ Function, ConceptUnderstanding $\sqsubseteq$ Interpretation, ConceptUnderstanding $\sqsubseteq$ Conceptualisation, PartialFunction $\sqsubseteq$ Function, FunctionalRole $\sqsubseteq$ Role, FunctionalRole $\sqsubseteq$ hasEquivalence.PartialFunction, FunctionalRole $\sqsubseteq$ Function, SubModel $\sqsubseteq$ Model, SemanticModel $\sqsubseteq$ Model, InterpretationSemanticModel $\sqsubseteq$ SemanticModel, UnderstandingSemanticModel $\sqsubseteq$ SemanticModel, UnderstandingSemanticSubModel $\sqsubseteq$ SubModel, UnderstandingSemanticSubModel $\sqsubseteq$ SemanticModel, InterpretationSemanticModel $\sqsubseteq$ $\exists$hasSupport.Interpretation, UnderstandingSemanticModel $\sqsubseteq$ $\exists$hasSupport.InterpretationSemanticModel, UnderstandingSemanticModel $\sqsubseteq$ $\exists$hasSupport.UnderstandingSemanticSubModel, UnderstandingSemanticSubModel $\sqsubseteq$ $\exists$hasSupport.FunctionalRole **}**

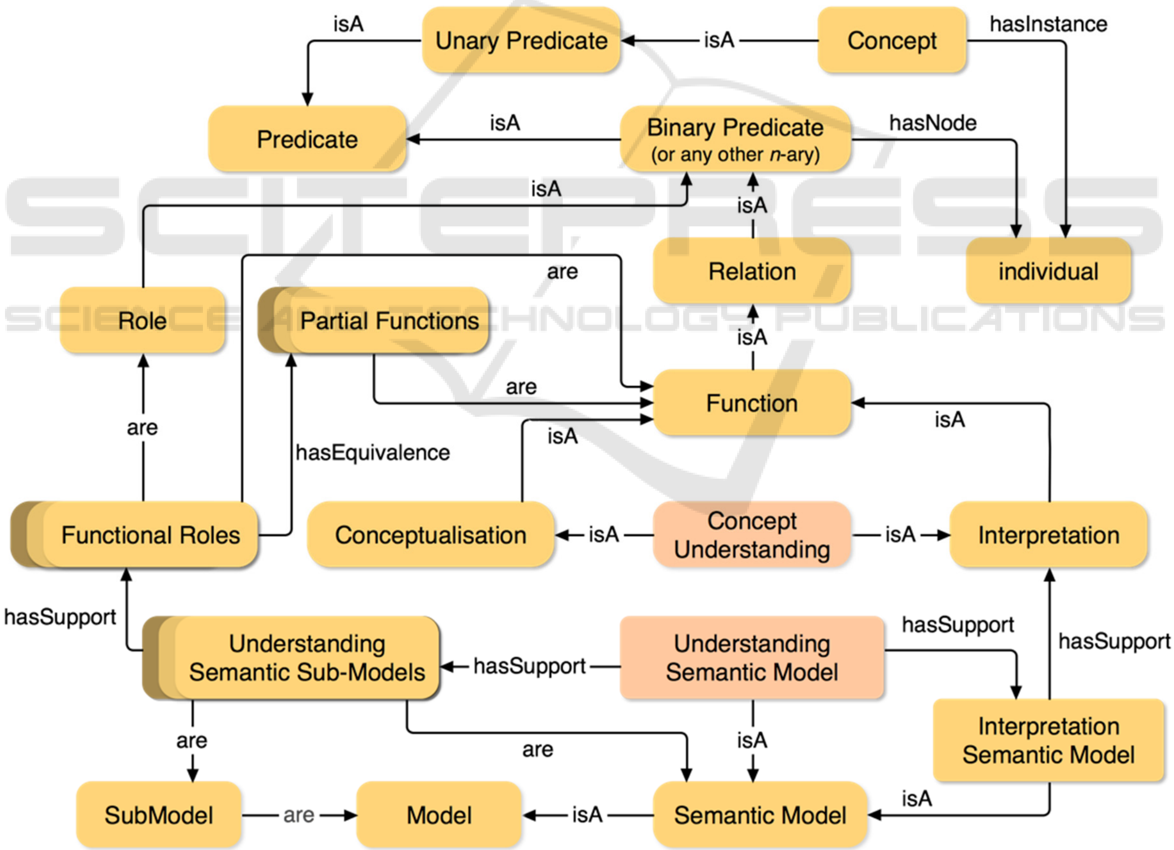

Figure 1: An Ontology for Concept Understanding.

5 CONCLUSIONS

The readers of this article may ask "if the term 'understanding' in this research is related to the real human beings, or if this research's domain is only information and computer sciences?" Actually, that's why I have employed Description Logics. Under a plethora of names (among them terminological systems and Concept Languages), Description Logics (DLs) attempt to provide descriptive knowledge representation formalisms based on formal semantics to establish common [conceptual and logical] grounds and interrelationships between human beings and machines. Description Logics supported me in revealing some hidden conceptual assumptions that could support me in having a better understanding of 'concept understanding'. DLs—by considering concepts as unary predicates and by applying terminological interpretations over them—have proposed a realisable logical description for explaining the humans' concept understanding. The central contribution of the article has been providing a formal semantics for logical analysis of concept understanding. According to the logical analysis, a background for terminological representation of concept understanding has been expressed. Consequently, a semantic representation [as an ontology and a specification of the shared conceptualisation of 'concept understanding'] has been designed and formalised.

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