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Polarimetric Wireless Indoor Channel Modelling Based on Propagation Graph
Ramoni Adeogun, Troels Pedersen, Carl Gustafson and Fredrik Tufvesson

Abstract—This paper generalizes a propagation graph model to polarized indoor wireless channels. In the original contribution, the channel is modelled as a propagation graph in which vertices represent transmitters, receivers and scatterers while edges represent the propagation conditions between vertices. Each edge is characterized by an edge transfer function accounting for the attenuation, delay spread and phase shift on the edge. In this contribution, we extend this modelling formalism to polarized channels by incorporating depolarization effects into the edge transfer functions and hence, the channel transfer matrix. We derive closed form expressions for the polarimetric power delay spectrum and cross-polarization ratio of the indoor channel. The expressions are derived considering average signal propagation in a graph and relate these statistics to model parameters, thereby providing a useful approach to investigate the averaged effect of these parameters on the channel statistics. Furthermore, we present a procedure for calibrating the model based on method of moments. Simulations were performed to validate the proposed model and the derived approximate expressions using both synthetic data and channel measurements at 15 GHz and 60 GHz. We observed very good agreement between the model, approximate expressions and measured channel.

Index Terms—Directed graph, polarization, MIMO system, stochastic channel model, dual polarized system, millimetre wave, measurements.

I. INTRODUCTION

UTILIZATION of the additional degrees of freedom offered by polarization in wireless propagation to increase channel spectral efficiency has received considerable attention within the last several years. For example, in MIMO systems, antenna elements having dual polarizations offer significant increase in channel capacity and often require less space for deployment than those with single polarization [1], [2]. More recently, collocated dual-polarized antennas have been identified as a cost- and space-effective configuration in MIMO deployments and have been adopted as the antenna configuration of choice in the 3GPP, LTE and LTE-Advanced. It is also expected that polarization will be an integral part of future generation wireless communication techniques such as millimeter wave propagation [3] and massive MIMO systems [4].

As with other wireless systems, exploiting the full benefits of polarized systems requires adequate understanding of the polarized wireless propagation channels. A common practice in design and performance evaluation of wireless communication systems is therefore, to use mathematical models for characterizing the propagation channel. In addition to temporal, frequency and directional properties in classical channel models, models for polarized channels must incorporate polarization and depolarization effects arising from reflection, diffraction and scattering in the propagation medium. Channel modelling for polarized systems has been the focus of active research within the last several years and a number of models have been developed based on the classical spatial channel modelling approaches for unpolarized systems (see e.g., [5], [6] and the references therein). Polarized channel models have also been defined within 3GPP [7], WINNER [8], and COST [9]. These models are predominantly based on the spatial channel modelling approach without account for recursive scattering.

Motivated by the need to study the effects of recursive and non-recursive scattering on wireless channel characterization, an alternative modelling framework based on directed propagation graph have been presented in [10]–[12]. The graph based model describes the propagation channel as a directed graph with the transmitters, receivers and scatterers as vertices and interactions between vertices defined as a time-invariant transfer function. Based on the graph description, closed-form expressions for the channel transfer function is given in [12]. The graph may be generated using deterministic, stochastic or a combination of deterministic and stochastic approach as done in the example model in [12]. Modelling the channel using a propagation graph offers a number of benefits over classical ray-tracing or geometry based spatial channel modelling. Graph based modelling also allows analytical computation of the channel transfer function based on the concept of electromagnetic wave reverberation. Graph based models have relatively low computational complexity when compared to other modelling methods and are straightforward to generalize to multi-user MIMO and different frequency bands. Another important feature of the graph model is the ability to capture via its recursive structure, the avalanche effects and diffuse components with only specular components.

Several other studies have recently presented applications and/or modifications of the graph based models to various propagation environments such as indoor [13], [14], indoor-to-outdoor [15], high speed railway [16]–[19] and millimeter wave systems [20]. Hybrid models combining the propagation graph based model with ray tracing approaches have also been studied in [21], [22]. To the best of our knowledge, there has been no study on propagation graph modelling for polarized channels.
In this paper, we extend the propagation graph model [12] to wireless channels with polarized antenna and derive expressions for the transfer functions. Polarization dependent propagation characteristics including depolarization, polarization power coupling and antenna polarimetric response are incorporated into the model. This generalization has been partly presented in a previous work [25]. We derive approximate expressions for predicting the co- and cross-polar power, cross-polarization ratio (XPR) and kurtosis of the output of a propagation graph. The expressions relate these important statistics of any polarized channel to model parameters for the propagation graph (i.e., number of scatterers, probability of visibility, polarization coupling parameter and reflection gain). The expressions may also be used for evaluating averaged statistics of the channel without performing Monte Carlo simulations using the model. The basis for deriving approximate models for polarimetric power delay profile has been partly presented in [24]. We further developed a method of moment based procedure for calibrating the model using channel measurements. The procedure involve fitting estimates of channel statistics to equivalent approximate expressions. Finally, we perform Monte Carlo simulations to verify the closed form approximations, evaluate the performance of the model calibration procedure, and validate the proposed model using dual polarized channel measurements at 60 GHz and 15 GHz.

II. PROPAGATION GRAPH BASED CHANNEL MODEL

In this section, we introduce the propagation graph based modeling framework in [12] and develop an extension of the model to polarized wireless channels.

A. Propagation Graph

The signal from a transmit antenna propagates through the environment before arriving at the receive antenna. Depending on the environment, the transmitted signal undergoes multiple interactions (reflection, diffraction and/or scattering) with objects in the propagation medium. These propagation mechanisms can be represented as a directed graph with vertices \( V = \{V_1, \cdots, V_N\} \), where \( V_t = \{S_1, \cdots, S_N\} \) and \( V_r = \{R_1, \cdots, R_N\} \) corresponding to the transmit antennas, scatters and receive antennas, respectively. The edges of the propagation graph denote the propagation condition between the originating and terminating vertices. A propagation graph for a MIMO wireless channel is a directed graph with \( N_t \) transmit vertices, \( N_s \) scattering vertices and \( N_r \) receive vertices. \( N_t(N_s) \) are the total number of transmit(receive) ports in the system. In a single user MIMO channel, \( N_t \) corresponds to the number of antenna ports on the single receiver. In a multi-user MIMO case, \( N_t \) is the total number of antenna ports on all receivers. The polarized channel is therefore considered as a linear time invariant system with \( N_t \) input ports and \( N_r \) output ports. For example, a system with three transmitting terminals each having one dual polarized patch antenna with two input port gives \( N_t = 6 \). To be consistent with existing works, we denote the vertex set \( V \) as a union of three disjoint sets: \( V = V_t \cup V_s \cup V_r \) and the edge set \( E \) as a union of four disjunct sets: \( E = E_d \cup E_t \cup E_s \cup E_r \), where \( E_d \) denotes the set of direct edges between transmit and receive antennas, \( E_t \) denotes transmit to scatterer edges set, \( E_s \) denote inter-scatterer edges and \( E_r \) denotes scatterers to receive antennas edge set. Note that transmit and receive vertices in the directed graph are treated as sources (with only outgoing edges) and sinks (with only incoming edges), respectively. If two vertices \( v, w \in V \) are visible (i.e., a propagation walk or path exists), we denote the edge from the originating vertex, \( v \) to the terminating vertex, \( w \) as \( e = (v, w) \). The originating vertex may be a transmit antenna or a scatterer and the terminating vertex is either a scatterer or receive antenna.

B. Polarized Propagation Mechanism

Modelling changes in the polarization state of a propagating wave is an important feature of models for polarized wireless channels. Signal depolarization and cross-polarization coupling in wireless channels are due to three major mechanisms viz: antenna cross-polar isolation (XPI), array mismatch and interaction of electromagnetic waves with scatterers [25], [26]. We assume that the depolarization effect due to antenna XPI is incorporated into the array response. This is reasonable since XPI is an antenna effect. Depolarization due to array mismatch can be represented as a rotation around an appropriately chosen axis [27]. We therefore assume that depolarization due to array mismatch is incorporated into the polarimetric array responses.

We describe the propagation environment with respect to an earth-related, global Cartesian coordinate system where the vertical (z-axis) corresponds to the zenith and the horizontal axes (x- and y-axis) are parallel to the ground. For simplicity, we assume that wave propagation on edges of the propagation can be described as plane waves in the far-field. With this assumption, a signal, \( Z(f) \), propagating on an edge of the graph with direction \( \Omega = (\theta, \phi) \) has no radial component and can therefore be decomposed into its \( \hat{r} \theta \) and \( \hat{r} \phi \) components \( Z_\theta(f) \) and \( Z_\phi(f) \) in the spherical coordinates, respectively. Here, \( \theta \) and \( \phi \) denote the azimuth and elevation direction of propagation, respectively. The polarization state of a signal on
an edge, $e$ can therefore be fully described (with respect to the propagation direction, $\Omega_e$) by its $\vec{i}_\theta$ and $\vec{i}_\phi$ components.

With this definition, signal propagation from the transmitter to the receiver is as follows:

- Each transmit antenna emits a signal with two orthogonal components. The emitted signal components propagate through the outgoing edges of the transmit antenna.
- The orthogonal components arrive at the receive antenna elements via its incoming edges. The received signal is therefore the sum of signals from all incoming edges weighted by the polarimetric antenna response.
- A scattering vertex emits via its outgoing edges the sum of depolarized version of all signals arriving via its incoming edges. The interaction of polarized signals with a scatterer may result in changes in the polarization state of some or all of the orthogonal components. The depolarized signals are then summed up at the scatterer and distributed to its outgoing edges. The propagation of a polarized signal via the edges in a graph and its interaction with scatterers therefore results in attenuation, delay, phase shift and depolarization.

Without loss of generality, we represent polarization coupling due to interaction with the $n$th scatterer as

$$M = \begin{bmatrix} M_{\theta\theta} & M_{\theta\phi} \\ M_{\phi\theta} & M_{\phi\phi} \end{bmatrix},$$

where $M_{ab}$ denotes power transfer between polarization states $a$ and $b$.

C. Graph Description of Polarized Propagation

Consider a polarized systems with $N_t$ transmitters and $N_r$ receivers. Let the vertices in the graph be indexed according to

$$v_n \in \begin{cases} V_t; & n = 1, \ldots, N_t \\ V_r; & n = N_t + 1, \ldots, N_t + N_r \\ V_s; & n = N_t + N_r + 1, \ldots, N_t + N_r + N_s. \end{cases}$$

(2)

In the graph representation of the polarized channel, the signal at every scattering vertex is a two-dimensional vector. The effect of a scattering vertex on the polarization of the signal can therefore be represented by the $2 \times 2$ scattering matrix in (1). Using the vertex indexing in (2), the propagation graph for the polarized channel can be represented by a $(N_t + N_r + 2N_s) \times (N_t + N_r + 2N_s)$ adjacency matrix,

$$A(f) = \begin{bmatrix} 0 & 0 & 0 \\ D(f) & 0 & R(f) \\ T(f) & 0 & B(f) \end{bmatrix},$$

with sub-matrices:

$D(f) \in \mathbb{C}^{N_t \times N_t}$ contains the transfer function of the direct edges between transmit and receive antenna pairs including the antenna response.

$T(f) \in \mathbb{C}^{2N_r \times N_t}$ contains the $2 \times 1$ transfer function vectors for all the $N_tN_r$ transmit antenna to scatterer edges.

$R(f) \in \mathbb{C}^{N_t \times 2N_r}$ contains the $2 \times 1$ transfer function vectors for all the $N_rN_s$ scatterer to receive antenna edges.

$B(f) \in \mathbb{C}^{2N_s \times 2N_r}$ contains the $2 \times 2$ transfer function sub-matrices for all the $N_rN_s$ scatterer to scatterer edges.

$0$ is an all-zero matrix with the appropriate dimension.

As observed in [12], the structure of $A(f)$ is a consequence of the structure of the propagation graph. The first $N_t$ rows are zeros because the transmit antennas are considered pure sources. Similarly, the $N_t + 1$ to $N_t + N_r$-th columns are zeros because receive antennas are treated as pure sinks. It should be noted that although [3] has the same structure as given in [12] for the uni-polarized channel, the dimension and structure of the polarized submatrices differs from those in [12].

D. Models for Gains and Polarimetric Scattering Matrix

Based on the physical propagation mechanisms in a polarized channel, the transfer function of the edge, $e$ can be expressed as

$$A_e(f) = G_e(f) \odot \exp[j(2\pi f_{\tau} + \Psi_e)],$$

(4)

where $\odot$ and $\exp[\cdot]$ denote entry-wise multiplication and entry-wise exponential, respectively. The edge propagation delay can be calculated for edge, $e = (v_n, v_m)$ from the vertex position vectors as $\tau_e = |(r_{vn} - r_{vm})|/c$, where $c$ is the speed of light. The random phase rotations for each of the polarization components are contained in $\Psi_e$, which is a $2 \times 1$ or $2 \times 2$ matrix. $1$ is an all one matrix or vector with the same dimension as $\Psi_e$. The edge gain, $G_e(f)$ can be calculated from

$$G_e(f) = \begin{cases} F_{\theta}(\Omega_e); & e \in E_{\theta} \\ F_{\phi}(\Omega_e); & e \in E_{\phi} \\ \sqrt{4\pi^2 f_{\mu}(\Omega_e)S(\Omega_e)}; & e \in E_{\mu} \end{cases}$$

(5)

where $\cdot$ denotes set cardinality. Here, $\text{odil}(e)$ denotes the number of outgoing edges from the $n$th scatterer,

$$\mu(\Omega_a) = \frac{1}{|\mathcal{E}_a|} \sum_{e \in \mathcal{E}_a} \tau_e, \quad S(\Omega_a) = \sum_{e \in \mathcal{E}_a} \tau_e^{-2}, \quad \mathcal{E}_a \subset \mathcal{E},$$

(6)

and the entity $\mathbf{F}_e$ is a matrix that captures the polarization dependent power coupling. The dimension and models for $\mathbf{F}_e$ depends on the type of edge.

1) Direct Edges: Signals propagating on a direct edge experiences two polarization dependent effects, viz. array response processing at both the transmitter and receiver. Thus, $\mathbf{F}_e$ can be expressed as

$$\mathbf{F}_e = \mathbf{X}_t(\Omega_e)^T \mathbf{X}_r(\Omega_e), \quad e \in E_{\text{d}},$$

(7)

where $\mathbf{X}_t(\Omega_e)$ and $\mathbf{X}_r(\Omega_e)$ are the $2 \times 1$ transmit and receive polarimetric array response vectors, respectively. Here, $\mathbf{F}_e$ is a scalar, despite the bold notation.

2) Transmitter to Scatterer Edges: On a transmitter to scatterer edge, the polarization related effects experienced by a signal is due to the polarimetric array response processing. The polarimetric vector can therefore be obtained from

$$\mathbf{F}_e = \mathbf{X}_t(\Omega_e), \quad e \in E_{\text{t}}.$$  

(8)

Thus, in this case, $\mathbf{F}_e$ has dimension $2 \times 1$. 
3) Scatterer to Scatterer Edges: Assuming that the polarization dependent effect of a scatterer to scatterer edge, \( e = (v_n, v_m) \), is due to the originating vertex, \( F_e \) is given by
\[
F_e = M_n R(-\Omega_e), \quad e \in \mathcal{E}_a,
\]
where \( R(\Omega) \) is a rotation matrix which rotates the polarization states of a signal with direction \( \Omega \) such that its direction is aligned with the z-axis of the global coordinates.

4) Scatterer to Receiver Edges: On a scatterer to receiver edge, the polarization mechanisms include polarization mixing due to the originating vertex and array response processing due to the receive vertex, the polarization dependent vector can therefore be written as
\[
F_e = R(\Omega_e) M_n A_n^T (\Omega_e), \quad e \in \mathcal{E}_r.
\]

E. Transfer Function for Polarized Channel

Since all vertices in the propagation graph are fixed, the MIMO channel can be assumed to be linear and time-invariant. The signal received can therefore be expressed as
\[
Y(f) = H(f) X(f),
\]
where \( H(f) \) is the \( N_s \times N_t \) complex transfer matrix of the propagation graph. \( X(f) \) and \( Y(f) \) are the \( N_s \times N_t \)-dimensional signals at the transmit and receive signal vertices, respectively. Using \( (9) \), the signal observed at the output of scatterer \( v \) can be written as
\[
Z_v(f) = M_v \sum_{v' \in V_s} R(\Omega_{v',v}) Z^\text{in}_{v',v}(f),
\]
where \( Z^\text{in}_{v',v}(f) \in \mathbb{C}^{2 \times 1} \) denote the signal on edge, \( e = (v', v) \). Equation \( (12) \) implies that the signal at the output of a scattering vertex is a depolarized version of the sum of signals on all incoming edges of the vertex. The signals observed at the scatterers can now be collected into the \( 2N_s \times 1 \) vector
\[
Z(f) = [Z^T_1(f), \ldots, Z^T_{N_s}(f)]^T.
\]

Using the vertex indexing in \( (2) \), the signals at all vertices can be collected into the \( N_t + N_s + 2N_s \) dimensional vector
\[
C(f) = [X(f)^T, Y(f)^T, Z(f)^T]^T.
\]

Using \( (14) \) and following a similar procedure as in \( (11) \), \( (12) \), the transfer function for the dual polarized channel can be shown to be
\[
H(f) = D(f) + R(f)[I - B(f)]^{-1} T(f), \quad \rho(B) < 1,
\]
where \( \rho(B) \) denotes the spectral radius of \( B \).

Algorithm 1: Stochastic Generation of Polarized Channel

**Input:** Model parameters: \( N_s, P_{\text{vis}}, g, \gamma, f_{\text{min}}, f_{\text{max}}, \Delta f \) and room dimensions.

1. Specify the coordinates of the transmitter(s) and receiver(s).
2. Draw the positions of \( N \) scatterers according to the specified spatial scatterer distribution.
3. Generate edges according to the edge occurrence probability in \( (16) \).
4. Compute polarimetric gains using \( (9) \) and edge transfer functions using \( (11) \).
5. Compute \( H(f) \); \( f = f_{\text{min}}, f_{\text{min}} + \Delta f, \ldots, f_{\text{max}} \) using \( (13) \).
6. Compute channel impulse response, \( h(\tau) \) via inverse discrete Fourier transform.

**Output:** \( H(f); \ h(\tau) \)
B. Analysis of the Polarimetric Power Delay Spectrum

We analyze the polarimetric power delay spectrum (PDS)\textsuperscript{1} by using an approximation and validate this approximation via simulations. We approximate the full propagation graph by a simpler graph as shown in Fig. 2. On average, each vertex in the is connected to \((N_s - 1)P_{\text{vis}}\) scatterer vertices. To ensure power balance at a vertex, the signal on each outgoing edge of a vertex is scaled by \(\sqrt{(N_s - 1)P_{\text{vis}}} \). The received signal power after \(k\) inter-scatterer interactions can be approximated by

\[
P_t[k] = P_D(\tau) + \frac{1}{(N_s - 1)P_{\text{vis}}} G_t^T (g^2 M)^k M G_r P_t,  
\]

where \(g\) denotes the average reflection gain. \(P_D\) denotes the power of the line-of-sight (LOS) component. \(P_t\) and \(P_t\) are the output and input power vectors, respectively. The vectors, \(G_t/r\) denote the receiver/transmitter polarimetric array response vector averaged over all directions and is defined as

\[
G_{t/r} = [\mu^\theta_{t/r} \mu^\phi_{t/r}]^T,  
\]

where \(\mu^\theta_{t/r}\) denotes the \(\theta\)-polarized component of the transmit/receive antenna response averaged over all directions and can be expressed as

\[
\mu^\theta_{t/r} = \frac{1}{4 \pi} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \lambda^\theta_{t/r}(\theta, \phi) \sin \theta d\theta d\phi.  
\]

The \(\phi\)-component, \(\mu^\phi_{t/r}\) is defined analogously. The terms \((g^2 M)^k\) and \(M\) result from recursive inter-scatterer interactions and the depolarization effect of the scatterer to receiver edges. For simplicity, we ignore the delay spread due to different edge lengths in the propagation graph and consider only the mean delay, \(\mu_\tau\). In this paper we approximate the mean delay as the mean cord length divided by speed of light \(c\),

\[
\mu_\tau = \frac{4V}{cS},  
\]

where \(V\) and \(S\) are the volume and total surface area of the room, respectively. Similar approximations have been used in [29], [30] where the time of arrival of rays undergoing single reflection is taken as twice the mean delay in [21]. The power received at delay \(\tau\) after \(k = \tau/\mu_\tau\) interactions, therefore, reads

\[
P_t(\tau) = P_D(\tau) + \frac{g(2\tau/\mu_\tau)}{L(N_s - 1)P_{\text{vis}}} G_t^T M^{(1+\tau/\mu_\tau)} G_r P_t,  
\]

where the normalization factor \(L\) accounts for the power decay during the average period between transmission and first non line-of-sight (NLOS) arrival (i.e., single bounce arrivals). Approximating this arrival time by \(2\mu_\tau\), \(L\) is obtained from Friis’s law as

\[
L = \frac{8\pi \mu_\tau c}{\lambda},  
\]

where \(\lambda\) is the carrier wavelength. The LOS component of the power delay spectrum in (22) is here, approximated by

\[
P_D(\tau) = \begin{cases} 
\frac{P_G T G_r}{P_{\text{FS}}} \delta(\tau - \tau_{\text{LOS}}) & \text{for LOS propagation} \\
0 & \text{for NLOS propagation}, 
\end{cases}  
\]

where \(P_{\text{FS}} = 4\pi \tau_{\text{LOS}} c/\lambda\) denotes the free space power decay of the LOS component. Inserting the eigenvalue decomposition \(M = SAS^T\) into (22) yields

\[
P_t(\tau) = P_t (\frac{g(2\tau/\mu_\tau)}{L(N_s - 1)P_{\text{vis}}} G_t^T S A^{(1+\tau/\mu_\tau)} S^T G_r P_t,  
\]

with

\[
S = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix},  
\]

and

\[
\Lambda = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1-\gamma}{1+\gamma} \end{bmatrix}.  
\]

Substituting (19), (26) and (27) into (25) and performing some simplifications yield

\[
P_t(\tau) = \begin{cases} 
P_t \left( \frac{g(2\tau/\mu_\tau)}{2L(N_s - 1)P_{\text{vis}}} \left( 1 + \left( \frac{1-\gamma}{1+\gamma} \right)^{(1+\tau/\mu_\tau)} \right) \right) \\
+ \frac{g(2\tau/\mu_\tau)}{2L(N_s - 1)P_{\text{vis}}} \left( 1 - \left( \frac{1-\gamma}{1+\gamma} \right)^{(1+\tau/\mu_\tau)} \right) 
\end{cases}  
\]

The expression (28) relates the PDS to averaged antenna responses (and hence, the antenna cross-polar isolation) and parameters of the propagation graph. The first term of (28) is the LOS component of the PDS. The second and third terms are the co- and cross-polar components of the PDS, respectively. As seen in the last two terms, the decay of the PDS is controlled by the average reflection gain, \(g\) and polarization mixing parameter, \(\gamma\). However, the effect of \(\gamma\) vanishes with increasing delay. Note that the expression in (28) is valid for general polarimetric antenna responses.

1 Special Case: Lossless Antennas With Perfect Cross-Polar Isolation: We now assume for simplicity that the transmit and receive antennas are lossless and have perfect cross-polar isolation. For a lossless antenna, the principle of conservation of energy implies that \(\mu^\theta_{t/r} + \mu^\phi_{t/r} = 1\). Furthermore, with perfect cross-polar isolation, the co- and cross-polar averaged responses become one and zero, respectively. Consider transmission between a polarized antenna with \(G_t = [1 0]^T\) and receive antenna with \(G_r = [1 0]^T\).
in (25) gives the co- and cross-polar received power delay spectrum as

\[
P_{\text{co}}(\tau) = \frac{P_t}{P_{\text{FS}}} \delta(\tau - \tau_{\text{LOS}}) + \frac{g^2(2\tau/\mu_\tau)}{2L(N_s - 1)P_{\text{vis}}} \left(1 + \left(\frac{1-\gamma}{1+\gamma}\right)^{(1+\gamma/\mu_\tau)}\right),
\]

and

\[
P_{\text{cro}}(\tau) = \frac{g^2(2\tau/\mu_\tau)}{2L(N_s - 1)P_{\text{vis}}} \left(1 - \left(\frac{1-\gamma}{1+\gamma}\right)^{(1+\gamma/\mu_\tau)}\right),
\]

respectively. In the region where \(\tau \gg \mu_\tau\) (i.e., tail of the PDS), the co- and cross-polar PDS decay exponentially as

\[
P_{\text{co/cro}}(\tau) \approx \frac{g^2(2\tau/\mu_\tau)}{2L(N_s - 1)P_{\text{vis}}},
\]

The polarimetric PDS in (31) is independent of the polarization coupling parameter \(\gamma\) and shows that in the later part of the profile, the co- and cross-polar channels become approximately equal both in power level and decay rate. Based on (31), the decay rate of the PDS is then defined as

\[
\rho[\text{dB}] \approx \frac{20}{\mu_\tau} \log_{10}(g) \quad [\text{dB/s}].
\]

We observe that the decay rate of the PDS is controlled by the average reflection gain, \(g\) and the mean interaction delay, \(\mu_\tau\). The cross-polar power ratio, denoted here as \(\beta\) is obtained from (29) and (30) as

\[
\beta(\tau) = \frac{P_{\text{co}}(\tau)}{P_{\text{cro}}(\tau)} = \frac{P_t \delta(\tau - \tau_{\text{LOS}})}{P_{\text{FS}}P_{\text{cro}}(\tau_{\text{LOS}})} + \frac{1 + \left(\frac{1-\gamma}{1+\gamma}\right)^{(1+\gamma/\mu_\tau)}}{1 - \left(\frac{1-\gamma}{1+\gamma}\right)^{(1+\gamma/\mu_\tau)}}.
\]

For \(\tau \gg \mu_\tau\), the second term of (33) becomes one. For NLOS propagation, the first term in (33) equals zero. Since the first term only affects the power of the LOS component and hence, the power ratio at \(\tau_{\text{LOS}}\), we will ignore the LOS term throughout the remaining part of this paper. Fig. 3 shows an example of the approximate power delay spectrum and cross-polarization ratio with different model parameters. As predicted by (31), the co- and cross-polar power delay spectra approach each other with increasing delay and become nearly equal.

IV. MODEL CALIBRATION

To utilize the proposed model, specific values should be given to the parameters, \(\Theta = [g, N_s, P_{\text{vis}}, \gamma]\). Here, we calibrate the model by estimating these parameters from measurements of the channel transfer function. To this end, we derive a method of moment (MoM) (31) based estimator for the model parameters. We utilize the approximate expressions for the moments and ratio of moments of the channel impulse response in Section III-B and estimate the parameters by fitting estimated moments of the measured channel to the derived expressions.

![Fig. 3: Dependence of channel statistics on model parameters for a $3 \times 4 \times 3$ m$^3$ room. The LOS term is set to zero.](image)

A. Model Calibration Procedure based on MoM

To calibrate the model, we fit the estimates of the second moments of PDS and cross-polarization ratio to the expressions (29), (30) and (33). Since \(N_s\) and \(P_{\text{vis}}\) are not identifiable in the PDS and cross-polarization ratio, we therefore introduced the product \(\nu = (N_s - 1)P_{\text{vis}}\) as a parameter which leads to the non-linear least square problem:

\[
\hat{\Phi} = \arg \min_{\Phi} \| \Sigma^{\text{approx}}(\Phi) - \Sigma^{\text{meas}}(\Phi) \|_F, \quad \Sigma^{\text{meas}}(\Phi) = \left[ \begin{array}{c} P_{\text{co}}(\tau_1) \\
\vdots \\
P_{\text{cro}}(\tau_K) \\
\end{array} \right],
\]

where \(\Phi = [g, \gamma, \nu]^T\) is the model parameter vector, \(\Sigma\) denotes element-wise division and \(\Sigma^{\text{meas}}(\Phi)\) is the matrix

\[
\Sigma(\Phi) = \left[ \begin{array}{ccc} P_{\text{co}}(\tau_1) & P_{\text{cro}}(\tau_1) & \beta(\tau_1) \\
\vdots & \vdots & \vdots \\
P_{\text{co}}(\tau_K) & P_{\text{cro}}(\tau_K) & \beta(\tau_K) \\
\end{array} \right].
\]

In (34), \(\Sigma^{\text{approx}}\) denotes the approximate value of \(\Sigma\) obtained from (29), (30) and (33). The value of \(\Sigma\) estimated from measurements is denoted \(\Sigma^{\text{meas}}\) in (34).

Since the tail decay slope in (32) depend on \(g\) only, it is reasonable to solve for this parameter separately. Hence, we estimate the decay-rate \(\rho\) from the tail of the PDS of the measured data and solve (32) for \(\hat{g}\). Similarly, the cross-polar power ratio in (33) depend only on \(\gamma\), and \(\gamma\) can be estimated by fitting the tail of the measured co- and cross-polar ratio to (33) and solve for polarization mixing parameter \(\hat{g}\). Finally, we find \(\hat{\nu}\) by fitting the sum of the computed co- and cross-polar PDS to the sum of (29) and (30). Note that \(\nu\) is not affected by the independent variable, \(\tau\), in (29) and (30) and can be estimated by a linear fit as explained in the sequel.

Noting that the sum of (29) and (30) can be expressed as

\[
P(\tau) = P_{\text{co}}(\tau) + P_{\text{cro}}(\tau) = \frac{U(\tau)}{\nu},
\]

where
where $U(\tau)$ is

$$U(\tau) = \frac{1}{P_{FS}} \delta(\tau - \tau_{LOS}) + \frac{\mu^{2\tau}/\nu}{PL},$$

(37)

it is straightforward to show that least square estimate of $\nu$ is given as

$$\log_{10}(\hat{\nu}) = \frac{1}{K} \sum_{k=1}^{K} \log_{10}(U(\tau_k) - \log_{10}(P^{\text{meas}}(\tau_k))).$$

(38)

The final step in our calibration procedure involves estimation of $N_s$ and $P_{\text{vis}}$ from $\nu$. This can be achieved by selecting a value for either parameter and computing the other from $\nu$. While it is advantageous for computational complexity reasons to select a low value for $N_s$, choosing a reasonable value to reproduce the scattering in a particular environment may be difficult. We therefore, propose setting value of $P_{\text{vis}}$ in this paper. It is relatively straightforward to set values of $P_{\text{vis}}$ since probability values are bounded (i.e., $0 < P_{\text{vis}} \leq 1$) and relates intuitively to the density of objects in the room. Note that further work may be needed on characterizing the probability of visibility and determining these values for different propagation environments.

The complete calibration procedure is summarized in Algorithm 2 below.

Algorithm 2: Model Calibration Procedure Based on MoM

**Input:** Measured impulse response; $h(\tau)$ for co- and cross-polar channels

1. Estimate the PDS, $\hat{P}_c$ and $\hat{P}_{x}$ and cross-polarization ratio, $\beta$ from $h(\tau)$.
2. Estimate the decay rate, $\rho$ from $\hat{P}_c$ and solve (32) for $\hat{\gamma}$
3. Estimate of the polarization mixing parameter, $\hat{\gamma}$ by fitting $\beta$ to (33).
4. Find $\nu$ using (37) and (38).
5. Set a value for $P_{\text{vis}}$ with $0 < P_{\text{vis}} \leq 1$ and compute $N_s = \nu/P_{\text{vis}}$

**Output:** Model parameters: $\hat{\Theta} = [\hat{\gamma}, N_s, P_{\text{vis}}, \hat{\gamma}]$

**B. Verification of Approximate Polarimetric Power Delay Spectrum**

We compare predictions of the power delay profile and cross-polarization ratios from the approximate expressions to those obtained from the graph model. We consider two scenarios in the evaluation:

- **Graph Model I:** Transmitter and receiver locations are fixed and equal for each realization of the propagation graph, and
- **Graph Model II:** Transmitter and receiver locations are random and drawn uniformly within the room for each channel realization.

The estimated statistics are obtained via averaging over 1000 Monte Carlo realizations of the propagation graph with the parameters in Table III. As seen in Fig. 4, the approximate PDS has very good agreement with the simulated PDS from the model for the two scenarios. The XPR plots in Fig. 4 also shows that the predicted and simulated cross-polarization delay profile exhibits very good agreement with a difference less than 1 dB over the entire delay values shown.

**C. Model Calibration Performance**

In order to evaluate the performance of the proposed calibration procedure, we first test the method on simulated data before applying the procedure on the measured data sets. We consider an in-room scenario with parameters in Table III and different combinations of the model parameters. The number of estimates of the PDS utilized in the calibration is set to $K = 200$ with $\tau_1 = 7.75\, \text{ns}$ and $\tau_K = 57.75\, \text{ns}$. The true and estimated parameters are presented in Table IV. The probability of visibility which is chosen and number of scatterers obtained from $\nu$ are included in the table for completeness. As shown in Table IV all model parameters are accurately estimated with calibration error less than 3% for all parameter values. Thus, we consider the procedure to be sufficiently accurate to calibrate the model.

**V. MEASUREMENT DATASETS**

The measurement datasets used for calibration and validation of the polarized propagation graph model include 60 GHz...
TABLE II: Transmitter and receiver locations. The height of the transmitter and receiver are 2.35 m and 1.85 m, respectively for M1 and 2.00 m and 2.50 m, respectively for M2.

<table>
<thead>
<tr>
<th>Meas.</th>
<th>LOS</th>
<th>NLOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>I.</td>
<td>[0.86, 2.25]</td>
<td>[1.88, 1.31]</td>
</tr>
<tr>
<td>II.</td>
<td>[0.85, 2.16]</td>
<td>[1.89, 1.48]</td>
</tr>
<tr>
<td>III.</td>
<td>[0.95, 2.34]</td>
<td>[2.51, 2.03]</td>
</tr>
<tr>
<td>IV.</td>
<td>[0.90, 2.30]</td>
<td>[2.07, 1.78]</td>
</tr>
</tbody>
</table>

M2

<table>
<thead>
<tr>
<th>Meas.</th>
<th>LOS</th>
<th>NLOS</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td>[0.60, 6.64]</td>
<td>[3.69, 7.62]</td>
</tr>
<tr>
<td>II.</td>
<td>[0.60, 6.64]</td>
<td>[3.73, 6.41]</td>
</tr>
<tr>
<td>III.</td>
<td>[0.60, 6.64]</td>
<td>[3.82, 5.20]</td>
</tr>
<tr>
<td>IV.</td>
<td>[0.60, 6.64]</td>
<td>[3.69, 3.95]</td>
</tr>
</tbody>
</table>

MIMO measurements [32] and 15 GHz multiple-input-single-output (MISO) measurements [33] obtained from measurement campaigns conducted at Lund University, Sweden. In the following, we present summary of the measurements and settings required for the propagation graph model. Throughout the rest of this paper, we will refer to the 60 GHz MIMO and 15 GHz MISO measurements as M1 and M2, respectively. A summary of measurement settings are presented in Table III.

A. 60 GHz Measurement (M1)

The 60 GHz measurements comprised of four LOS and four NLOS datasets obtained in a small meeting room. For each measurement location, the transmitter and receiver has a 5 × 5 virtual dual polarized rectangular array in the horizontal and vertical plane, respectively. The transmit virtual arrays are obtained by moving the virtual element at a regular interval of 5 mm along the y- and z-axis from the positions giving in Table I. At the receiver, the virtual element is moved along the x- and y-axis to form the virtual array. The virtual arrays emulate a 25 × 25 dual polarized MIMO system with 50 × 50 antenna ports. The measurement setup and virtual array locations are shown in Fig. 5. Detailed description of the measurement can be found in [32].

B. 15 GHz Measurement (M2)

The 15 GHz transfer functions were measured using virtual MISO system with a single antenna at the receiver and 100-element virtual array at the transmitter in a 6 × 10 × 3, m³ conference room [33]. In the measurements, the virtual array was placed at different locations. LOS and NLOS measurements from the four receiver locations in Table II are used in this work. The height of the transmitter and receiver are 2.00 m and 2.50 m, respectively. Detailed description of the measurement can be found in [33].

VI. MODEL VALIDATION

In this section, we validate the proposed model and approximate expressions using data from the measurements described in section V. We follow the cross-validation procedure summarized in Fig. 6. The measurement data is divided into two groups for model calibration and validation, respectively. For M1, datasets NLOS I, NLOS II and LOS I representing half of the available data are used for model calibration with the remaining half (i.e. NLOS IV, LOS II and LOS IV) used for validating the model. Similarly, datasets from M2 are grouped into calibration (LOS I, LOS II, NLOS II and NLOS IV) and validation (LOS II, LOS IV, NLOS I and NLOS III) data sets.

The model parameters obtained from the calibration procedure for both measurements are presented in Table IV. Here, we set a high value for the probability of visibility (i.e., $P_{\text{vis}} = 0.9$) since the measurements were conducted in empty rooms. As can be observed from Fig. 7, the measured PDS and used at the transmitter was a 10 × 10 antenna array and the receiver has a single monopole antenna. The transmitter was placed at a fixed location in the room and the receiver was placed at different locations. LOS and NLOS measurements from the four receiver locations in Table II are used in this work. The height of the transmitter and receiver are 2.00 m and 2.50 m, respectively. Detailed description of the measurement can be found in [33].

TABLE IV: Model parameter estimates obtained from the calibration datasets.

<table>
<thead>
<tr>
<th>Meas.</th>
<th>$g$</th>
<th>$\gamma$</th>
<th>$P_{\text{vis}}$</th>
<th>$N_{\text{cal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>0.64</td>
<td>0.06</td>
<td>0.90</td>
<td>11</td>
</tr>
<tr>
<td>M2</td>
<td>0.65</td>
<td>0.26</td>
<td>0.90</td>
<td>18</td>
</tr>
</tbody>
</table>

TABLE III: Measurement settings for M1 and M2.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>M1</th>
<th>M2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room size</td>
<td>$3 \times 4 \times 3$ m³</td>
<td>$6 \times 10 \times 3$ m³</td>
</tr>
<tr>
<td>Transmitter height</td>
<td>2.35 m</td>
<td>2.00 m</td>
</tr>
<tr>
<td>Receiver height</td>
<td>1.85 m</td>
<td>2.50 m</td>
</tr>
<tr>
<td>Frequency range</td>
<td>58 GHz – 62 GHz</td>
<td>14.5 GHz – 15.5 GHz</td>
</tr>
<tr>
<td>Number of freq. samples</td>
<td>801</td>
<td>801</td>
</tr>
</tbody>
</table>
Fig. 7: Measured calibration data and theoretical PDS and cross-polarization ratio with model parameters in Table IV.

(a) M1.

(b) M2.

Fig. 8: Measured and averaged simulated polarimetric PDS and XPR for the M1 validation datasets.

Fig. 9: Measured and averaged simulated polarimetric PDS and XPR for the M2 validation datasets.

cross-polarization ratio agree closely with the predicted values at the estimated model parameters for both measurements.

Fig. 8 shows that the power level and tail decay of the PDS for M1 measurements are accurately predicted by the model as well as the theoretical approximation. Similar agreements between the validation data, propagation graph and the approximate model are seen in the PDS plots in Fig. 9 for M2 validation data. We observe in Fig. 8 and Fig. 9 that the measured XPR delay profile exhibit similar trends as predicted ratios. A transition from a region of decreasing polarization ratio to a region with nearly constant ratio is observed.

The stochastic graph model cannot be expected to represent all features of the measured instantaneous power delay profiles, but as we have seen, to agree well in terms of mean values. Nonetheless, we now compare single realizations of the model measurements in order to evaluate how well the model represents the behaviour of the instantaneous co- and cross-polar power delay profiles. Fig. 10 reports the co- and cross-polar power delay profile for three realizations of the propagation graph along with approximation and measured power delay profile for the M1 dataset. As can be observed from Fig. 10, the power level and decay rate of the measured channel are well predicted by the model except for few spikes in the measurements that were not captured by the model. A plausible explanation for this is that these few peaks may be due to the presence of very strong reflections from objects in the room which are ignored in the model. We further remark that exact reproduction of the measured profile from the graph model may be possible by using a detailed map and information on the materials of the environment for constructing the propagation graph. This is, however, outside the scope of the present contribution.

VII. DISCUSSION

The propagation graph model presented in this paper provides a simple method for simulating the transfer function as
well as the impulse response of the polarized channel. Stochastic implementation of the model requires only four real valued model parameters (i.e., number of scatterers, reflection gain, probability of visibility and polarization mixing parameter) in addition to basic geometric parameters such as dimensions of the scattering region (i.e., room dimensions for the in-room channel considered in the simulations) and location of transmitter and receiver to accurately predict the polarimetric power delay spectrum of the channel. The model has relatively low complexity in terms of both computational cost and the number of model parameters compared to other models for polarimetric channels. For example, spatial channel models (see e.g., [6], [34]) typically require characterizing parameters of the distribution of a large number of multipath components and/or clusters. It should be noted that the propagation graph model also allows a deterministic approach for generating the channel impulse response. In this case, detailed description of the environment, obtained from a map of the environment and/or an initial ray tracing step may be used to construct the adjacency sub-matrices for the propagation graph.

The calibration results for the two measured rooms considered showed nearly equal values for the reflection gain. This appears reasonable from a physical point of view, since both rooms are in the same building and most probably made of similar materials. We therefore, expect that regardless of room sizes and transmission frequency, the reflection gain, $g$, will be the same for rooms made of similar materials. With the same value of probability of visibility, estimated number of scatterers is higher for the medium sized room than the small room. This implies that more scatterers are needed to reproduce channel effects in larger rooms. The polarization coupling parameter, $\gamma$ obtained from the calibration is observed to be larger for M2. While this may be due to the increased size and/or difference in frequency, other factors such as polarimetric antenna properties, height and orientation of the antenna may result in significant change in the polarization behaviour of the channel and hence, the coupling parameter. Further study is needed to characterize the dependence of this model parameter on frequency as well as geometrical and environmental effects.

The observation that the co- and cross-polar channels exhibit different decay rates agreed with our measurements. The cross-polar power ratio is observed to be decreasing with delay in the early part of the PDS and becoming nearly constant in the late part of the PDS. This is intuitive since no power is created in the propagation environment and as power is leaked from one polarization state to an orthogonal state during interaction with scatterers, the ratio between the co- and cross-polar channels decreases with increasing number of interactions. This trend may also be viewed as a transition of the propagating signal from a fully polarized state to a partially and/or non-polarized state. While there has been very limited studies on the dependence of XPR on delay in recent times, similar observations have been reported in [35], [36]. While analyzing polarimetric channel measurements at 1800 MHz in [35], the authors observed that the ratio of co- and cross-polar channels varies over time. Similarly, it was found in [36] that the co- and cross-polar channels exhibit different decay constants. However, the cross-polarization ratio was shown to be increasing with delay for the macrocellular environment considered. This contrasting observation was noted in [6] as surprising. For the same macrocellular environment, the cross-polarization ratio is modelled as a decreasing function of delay in the 3GPP model [7]. In a recent study based on measurements at 63 GHz, the cross-polarization ratio is found to be decreasing with increasing excess loss of the propagation paths [37]. This agree with our observation that the ratio decreases with delay, since propagation paths with longer delay are more likely to have higher excess loss with respect to free space.

VIII. CONCLUSION

We have presented a propagation graph based model for polarized wireless channels in this paper. We also derived approximate closed form expressions for the power delay spectrum and cross polarization ratio of the indoor channel via the propagation graph formalism. A method of moments procedure for calibrating the graph model using measured data has also been presented. Our results showed that both graph model and theoretical approximation predicts accurately the power level and tail decay of the measured power delay profile for both co- and cross-polar channels. The co- and cross-polar channels decay exponentially with different and equal decay rates in the early and later parts of the power delay spectrum, respectively. We observed that the measured
cross polarization ratio as a function of delay exhibit similar trend as that obtained via simulations from the model and theoretical approximations. A transition from polarized to partially polarized and/or unpolarized state is observed in the ratio.

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REFERENCES