A Voltage Modulated DPC Approach for Three-Phase PWM Rectifier

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Abstract—In this paper, a voltage modulated direct power control for three-phase pulse-width modulated rectifier is proposed. With the suggested method, the differential equations describing the rectifier dynamics are changing from a linear time-varying system into a linear time-invariant one. In this way, the conventional feedback and feedforward controllers are applicable for the independent control of active and reactive powers. The proposed method is guaranteed that the closed system is globally exponentially stable. A feedback linearization method is also employed for generating the active power reference of inner loops. Finally, some experimental tests are conducted to verify its effectiveness.

Index Terms—Exponential stability, linear time-invariant, rectifier, three-phase systems.

I. INTRODUCTION

Three-phase rectifiers with the pulse-width modulation (PWM) are widely employed in different applications, particularly in interfacing distributed generation systems, drives of electrical motors, uninterruptible power supplies, etc. [1]. The control objectives for these applications are to maintain the DC-link voltage to a certain reference, supply a desired reactive power, and draw grid currents with the lowest possible harmonic distortion. The conventional control of the PWM rectifier is designed in a synchronous rotating reference frame, and a decoupled proportional-integral (PI) controller is applied for separately controlling d–q axes currents [2]. However, it suffers from a slow dynamical response.

To deal with such problem, different control methods have been designed [3]–[6]. Among them, methods based on the direct power control (DPC) have received great attention because it achieves the direct control of active and reactive power without using inner-loop current regulators [3]. A very brief review of recent advances in controlling rectifiers using the DPC method is presented in what follows.

A DPC technique based on the space vector modulation is proposed [5]. This control method, which does not require the extraction of positive and negative sequence components, results in low-THD line currents and eliminates double-frequency ripples under unbalanced grid conditions. In [7] and [8], a novel definition of reactive power has been applied in DPC to achieve an active power oscillation cancellation automatically with a simple structure. Providing a constant active power and sinusoidal grid currents even under an unbalanced grid are other advantages of this method. Recently, a model predictive control (MPC) based DPC technique has been applied to power converters [9]. The MPC-based DPC selects a voltage vector sequence and calculates duty cycles in every sampling period, which results in a constant switching frequency [10], [11]. The problem is that an incorrect voltage sequence selection may adversely affect the converter performance [12], [13]. MPC takes advantage of the discrete nature of the power converter, but the computation time is a critical issue.

In order to improve the DC-bus voltage dynamic performance with the consideration of the mismatched current or power disturbances, feedforward control techniques have been researched [14]–[17]. To improve reliability and decrease the cost of the load current sensor on the DC bus, load current observers are designed to identify the load current disturbance [18]–[20]. Recently, an extended state observer (ESO) based second-order sliding mode is proposed to improve the disturbance rejection ability [6]. Also, a proportional controller based ESO for the DC-link regulation is proposed to achieve the fast dynamics and high robustness. However, the nonlinear analysis and tuning of ESO are a bit complicated [21].

The motivation of this paper is to design a robust yet simple control strategy for the three-phase PWM rectifier. A voltage modulated DPC method is proposed, which has the following key features.

1) It changes the differential equations describing the dynamics of the three-phase PWM rectifier into a linear time-invariant (LTI) system;
2) It offers a simple way to design a traditional control method (feedforward and PI feedback) to obtain both a fast dynamic response and good steady-state performance without a phase-locked loop (PLL);
3) It is guaranteed that the closed system is globally exponentially stable;
4) It can be easily analyzed and designed by using conventional linear control techniques.

Finally, its effectiveness is verified through a real power converter prototype.

The rest of the paper is organized as follows. In Section III, the modeling of AC and DC side of the rectifier system...
is presented. Section III describes the controller design and Section IV presents the experimental results with a 2.2-kW-VSC system. Finally, the conclusions are given in Section V.

II. OVERVIEW AND MODELING OF PWM RECTIFIER

A. AC-Side Modeling

Assuming the grid voltage is balanced, the grid voltage can be described as follows [22]:

\[ v_s = R_s i_s + L_s \frac{di_s}{dt} + u, \]  

(1)

where \( v_s, i_s, u \) are the grid voltage vector, the input current vector, and the rectifier voltage vector, respectively. \( L_s \) and \( R_s \) are the filter inductance and resistance, respectively. We define the instantaneous active and reactive powers in the \( \alpha-\beta \) frame as follows:

\[ P = \frac{3}{2} (v_{s\alpha}i_{s\alpha} + v_{s\beta}i_{s\beta}), \]

\[ Q = \frac{3}{2} (v_{s\alpha}i_{s\beta} - v_{s\beta}i_{s\alpha}), \]  

(2)

where \( P \) and \( Q \) are the instantaneous output active and reactive powers, respectively. \( v_{s\alpha} \) and \( v_{s\beta} \) indicate the grid voltages, and \( i_{s\alpha} \) and \( i_{s\beta} \) denote the injected currents in the \( \alpha-\beta \) frame. By differentiating (2) with respect to time, the instantaneous active and reactive powers variations can be expressed as follows:

\[ \frac{dP}{dt} = \frac{3}{2} \left( i_{s\alpha} \frac{dv_{s\alpha}}{dt} + v_{s\alpha} \frac{di_{s\alpha}}{dt} + i_{s\beta} \frac{dv_{s\beta}}{dt} + v_{s\beta} \frac{di_{s\beta}}{dt} \right), \]

\[ \frac{dQ}{dt} = \frac{3}{2} \left( i_{s\alpha} \frac{dv_{s\beta}}{dt} + v_{s\alpha} \frac{di_{s\beta}}{dt} - i_{s\beta} \frac{dv_{s\alpha}}{dt} - v_{s\beta} \frac{di_{s\alpha}}{dt} \right). \]  

(3)

By assuming a nondistorted grid, the following relationship could be obtained as

\[ v_{s\alpha} = V_s \cos(\omega t), \]

\[ v_{s\beta} = V_s \sin(\omega t), \]  

(4)

where \( V_s \) and \( \omega \) are the magnitude and angular frequency of the grid voltage, respectively. By differentiating (4), the grid voltage variations can be expressed as follows:

\[ \frac{dv_{s\alpha}}{dt} = -\omega V_s \sin(\omega t) = -\omega v_{s\beta}, \]

\[ \frac{dv_{s\beta}}{dt} = \omega V_s \cos(\omega t) = \omega v_{s\alpha}. \]  

(5)

Consequently, the dynamics of the instantaneous active and reactive powers could be obtained using from (1) to (5) as follows:

\[ \frac{dP}{dt} = \frac{R_s}{L_s} P - \omega Q + \frac{3}{2L_s} (v_{s\alpha} u_{\alpha} + v_{s\beta} u_{\beta}) - \frac{3}{2L_s} V_s^2, \]

\[ \frac{dQ}{dt} = \omega P - \frac{R_s}{L_s} Q + \frac{3}{2L_s} (v_{s\beta} u_{\alpha} - v_{s\alpha} u_{\beta}), \]  

(6)

where \( u_{\alpha} \) and \( u_{\beta} \) indicate the rectifier voltages in the \( \alpha-\beta \) frame. Notice that (6) describes a time-varying system because the grid voltages multiply the rectifier voltages.

B. DC-Link Modeling

By neglecting the converter losses, the DC-link capacitor power variation can be expressed as

\[ \frac{dV_{dc}}{dt} = P_i - P_l, \]  

(7)

where \( P_c, P_i, \) and \( P_l \) are the capacitor power, injected power form AC side, and consumed power by the load, respectively. \( C \) and \( V_{dc} \) are the DC-link capacitor and voltage, respectively. For the sake of simplicity, a resistive load is considered. Therefore, \( P_l \) can be expressed as

\[ P_l = V_{dc} I_{dc}, \]  

(8)

where \( I_{dc} \) is the current flowing into the DC load. Substituting (8) into (7) yields

\[ \frac{dV_{dc}}{dt} = \frac{P_c - 1}{C V_{dc}} - \frac{1}{C} I_{dc}. \]  

(9)

Notice that (9) is a nonlinear system.

III. CONTROLLER DESIGN FOR PWM RECTIFIER

To simplify the controller’s design, an LTI system is needed as a system described in the \( d-q \) frame. To this end, we find a relationship between DPC model and system model in the \( d-q \) frame.

A. Voltage Modulated DPC

Let us define voltage modulated regulation (VMR) inputs as

\[ u_P = v_{s\alpha} u_{\alpha} + v_{s\beta} u_{\beta}, \]

\[ u_Q = -v_{s\beta} u_{\alpha} + v_{s\alpha} u_{\beta}. \]  

(10)

Notice that the VMR inputs are constant since the relationship between the VMR inputs and the rectifier voltages, as shown below, is the Park transformation.

\[ \begin{bmatrix} u_P \\ u_Q \end{bmatrix} = V_s \begin{bmatrix} \cos(\omega t) & \sin(\omega t) \\ -\sin(\omega t) & \cos(\omega t) \end{bmatrix} \begin{bmatrix} u_{\alpha} \\ u_{\beta} \end{bmatrix}. \]

(11)

Park Transformation

With the VMR inputs, the dynamics are transformed into an LTI system. To eliminate the coupling terms, the new control inputs are designed as follows:

\[ u_P = \frac{2L_s}{3} (\omega Q + v_P) + V_s^2, \]

\[ u_Q = \frac{2L_s}{3} (\omega P - v_Q), \]  

(12)

where \( v_P \) and \( v_Q \) are the new control inputs. With the VMR inputs, the dynamics (13) are transformed into a simple LTI system as follows:

\[ \frac{dP}{dt} = \frac{R_s}{L_s} P + v_P, \]

\[ \frac{dQ}{dt} = \frac{R_s}{L_s} Q + v_Q. \]  

(13)
Finally, using the inversion of (11), the original control inputs can be calculated as follows:

\[
\begin{align*}
    u_\alpha &= v_{\beta}\alpha u_P - v_{\beta}\alpha u_Q, \\
    u_\beta &= v_{\beta}\alpha u_P + v_{\alpha}\beta u_Q.
\end{align*}
\]

(14)

A controller for the active power can be constructed as shown in Fig. 1. A similar controller may be used for \( Q \) component.

Notice that, for \( C(s) \), various control methods could be designed to satisfy the system’s requirements. In this paper, we design PI controllers for both components,

\[
\begin{align*}
    \nu_P &= K_{i,P}(P^* - P) + K_{i,P} \int (P^* - P)dt, \\
    \nu_Q &= K_{i,Q}(Q^* - Q) + K_{i,Q} \int (Q^* - Q)dt,
\end{align*}
\]

where \( P^* \) and \( Q^* \) are the references of active and reactive powers, respectively. If the controller gains, \( K_{i,P}, K_{i,Q}, K_{p,P}, \) and \( K_{i,Q} \) are designed as positive values, the closed-loop system remains exponentially stable. Substituting (15) into (13), the closed loop system can be obtained as follows:

\[
\begin{align*}
    \frac{dP}{dt} &= -\frac{R_s}{L_s}P + K_{p,P}(P^* - P) + K_{i,P} \int (P^* - P)dt, \\
    \frac{dQ}{dt} &= -\frac{R_s}{L_s}Q + K_{p,Q}(Q^* - Q) + K_{i,Q} \int (Q^* - Q)dt.
\end{align*}
\]

(16)

Differentiating (16) with respect to time yields

\[
\begin{align*}
    \dot{P} &= -\frac{R_s}{L_s}\dot{P} + K_{p,P}(\dot{P}^* - \dot{P}) + K_{i,P}(\dot{P}^* - \dot{P}), \\
    \dot{Q} &= -\frac{R_s}{L_s}\dot{Q} + K_{p,Q}(\dot{Q}^* - \dot{Q}) + K_{i,Q}(\dot{Q}^* - \dot{Q}).
\end{align*}
\]

(17)

Applying the Laplace transform to (17) gives

\[
\begin{align*}
    s^2P &= -\frac{R_s}{L_s}sP + K_{p,P}(sP^* - sP) + K_{i,P}(sP^* - sP), \\
    s^2Q &= -\frac{R_s}{L_s}sQ + K_{p,Q}(sQ^* - sQ) + K_{i,Q}(sQ^* - sQ).
\end{align*}
\]

(18)

or equivalently

\[
\begin{align*}
    P(s) &= \frac{K_{p,P}s + K_{i,P}}{s^2 + \left(\frac{R_s}{L_s} + K_{p,P}\right)s + K_{i,Q}}, \\
    Q(s) &= \frac{K_{p,Q}s + K_{i,Q}}{s^2 + \left(\frac{R_s}{L_s} + K_{p,Q}\right)s + K_{i,Q}}.
\end{align*}
\]

(19)

The characteristic polynomial of both these transfer functions, which describe the closed-loop dynamics of the active and reactive powers, is as

\[
\begin{align*}
    \omega_n^2 &= 2\zeta\omega_n, \\
    K_{i,P} &= \omega_n^2, \\
    K_{i,Q} &= \omega_n^2, \\
    K_{p,P} &= 2\zeta\omega_n - \frac{R_s}{L_s}, \\
    K_{p,Q} &= 2\zeta\omega_n - \frac{R_s}{L_s}.
\end{align*}
\]

(20)

Fig. 2 shows the step response of the switched model and averaged model. Notice that the closed-loop system is stable with proper controller gains, and then we can conclude that the closed-loop system is globally exponentially stable since it is an LTI system.
Inverter load

B. DC-Link Voltage Control

The main objective of the DC-link voltage is to regulate a constant reference value. Neglecting the losses, the injected power is designed as follows:

\[ P_\text{dc} = P_\text{in} + CV \cdot \nu_{\text{dc}}, \]  \hspace{1cm} (22)

where \( \nu_{\text{dc}} \) is a new control input. Note that \( CV \cdot \nu_{\text{dc}} \) is equal to the power of the DC-link capacitor, \( P_C \). The DC-link dynamics are changed into a simple LTI system as follows:

\[ \frac{dV_{\text{dc}}}{dt} = \nu_{\text{dc}}. \]  \hspace{1cm} (23)

For (23), various control methods can be applied to maintain a constant voltage. In this paper, a conventional PI DC-link controller is applied.

\[ \nu_{\text{dc}} = K_{p,\text{dc}} (V_{\text{dc}}^* - V_{\text{dc}}) + K_{i,\text{dc}} \int (V_{\text{dc}}^* - V_{\text{dc}}) dt, \]  \hspace{1cm} (24)

where \( V_{\text{dc}}^* \) is the reference of the DC voltage, and \( K_{p,\text{dc}} \) and \( K_{i,\text{dc}} \) are controller gains (see Fig. 3). Notice that the generated control input, \( P_\text{dc} \), is the reference of the active power of the AC side. Finally, the whole block diagram of the proposed method is described as shown in Fig. 4. The DC-link controller enables the DC voltage to maintain a constant value, and the output of the DC-link controller, \( P^* \), goes to the active power controller.

IV. EXPERIMENTAL RESULTS

The effectiveness of the proposed method is verified by using a VSC (three-leg three-phase converter with an \( L \) filter), as shown in Fig. 5(a). The rectifier (Danfoss FC302 VLT 2.2kVA [24]) is connected to a programmable Grid Simulator, and two series-connected 2200\( \mu \)F/385V aluminum electrolytic capacitors in parallel with the DC-link. At the DC-link, the resister is connected as a DC load and another VSC with an AC load is emulated as an inverter load, as shown in Fig. 5(b). The detailed parameters used in the experiment are listed in Table I. The performance of the proposed method is compared with the traditional PI-based DC-link voltage controller and the PI plus feedforward method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase-to-phase RMS voltage</td>
<td>208</td>
<td>V</td>
</tr>
<tr>
<td>DC-link voltage, ( V_{\text{dc}} )</td>
<td>500</td>
<td>V</td>
</tr>
<tr>
<td>Filter inductor, ( L_s )</td>
<td>3.6</td>
<td>mH</td>
</tr>
<tr>
<td>Filter inductor, ( R_s )</td>
<td>0.1</td>
<td>( \Omega )</td>
</tr>
<tr>
<td>DC-link capacitor, ( C )</td>
<td>1100</td>
<td>( \mu )F</td>
</tr>
<tr>
<td>Switching frequency, ( f_s )</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>DC load, ( R_l )</td>
<td>230</td>
<td>( \Omega )</td>
</tr>
</tbody>
</table>
A. DC Load

As the first test, a disturbance is generated by connecting a 230 Ω resistor in the DC link. Fig. 6 shows the time response of the PI method, the PI plus feedforward method, and the proposed technique. It can be observed that the DC-link voltage and current using the proposed method has the smallest overshoot and fastest convergence time compared to other methods. This fact can be better visualized in Fig. 7, which shows the DC-link voltage response of all methods at the same time. It should be mentioned here that the simple PI method has the worst performance in this test.

B. Parameter Uncertainties

To test the robustness of the proposed controller to parameter uncertainties, its performance is evaluated in the presence of the inductance variation of the filter and capacitance variation of the DC-link capacitor. Fig. 8(a), (b), (c), (g), and (h) show the performance when the inductance of the filter in the control algorithm is set to 75% of the original value. Compared with Fig. 6, the tracking and steady-state performance of the proposed controller are not affected. Fig. 8(d), (e), (f), (i), and (j) show the performance when the capacitance of the DC-link capacitor in the control algorithm is set to 75% of the original value. The convergence time is a bit shorter than that without parameter uncertainty. Consequently, these mismatches have a small effect on the performance of the proposed controller. In addition, we insert 0.7%, 5th and 0.7%, 7th harmonics to the grid voltage by using the grid simulator to emulate the real grid. The DC load is suddenly connected to the DC link, in Fig. 9. The THD of the output currents is 2.5% when the THD of the grid voltage is 1.0%, as shown in Fig. 9(a) and (b). The tracking and steady-state performance are not affected by the harmonics, as shown in Fig. 9(c) and (f). Consequently, we can expect that the proposed method will be working well in the real grid.

C. Inverter Load

We also test a disturbance generated by connecting an inverter with an AC load in the DC link. At 0.5s, the inverter is suddenly connected to the DC link as shown in Fig. 10(e). Using the proposed method, the overshoot of the DC voltage is approximately 25V and the settling time is 0.2s. We can conclude that the proposed method is robust to the inverter load as well.

V. CONCLUSION

The proposed VM-DPC for the PWM rectifier is easy to control the active and reactive powers independently by using feedforward and feedback controllers. Experimental results show that the proposed method has not only fast transient response but also good steady-state performance. Under the proposed method, it is not hard to analyze and design for practicing engineers, since the system is changed into an LTI one. We show the proposed method is robust to the disturbance with the experiment since the closed-loop system is exponential stable.

REFERENCES


Fig. 6. Transient response and steady-state performance of the system under step change of the DC load. PI method: (a) DC voltage, (b) DC current, (c) injected current, (d) active power, (e) reactive power; PI plus feedforward method: (f) DC voltage, (g) DC current, (h) injected current, (i) active power, (j) reactive power; Proposed method: (k) DC voltage, (l) DC current, (m) injected current, (n) active power, (o) reactive power.

Fig. 7. Comparison of DC voltage.

Fig. 8. Transient response and steady-state performance of the system under step change of the DC load when parameters of the controller are mismatched. 75% L: (a) DC voltage, (b) DC current, (c) injected current, (g) active power, (h) reactive power; 75% C: (d) DC voltage, (e) DC current, (f) injected current, (i) active power, (j) reactive power.

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Fig. 9. Transient response of the system with the proposed method under step change of the DC load when the grid voltage has 5th and 7th harmonics. (a) voltage spectrum (THD=1.0%), (b) currents spectrum (THD=2.5%), (c) DC voltage, (d) DC current, (e) injected current, (f) active power, (g) reactive power.

Fig. 10. Performance of the proposed method when the inverter is connected. (a) DC voltage, (b) DC current, (c) injected current, (d) active power of rectifier, (e) active power of the inverter, (f) output current of the inverter.

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