A Decentralized Robust Model for Optimal Operation of Distribution Companies with Private Microgrids

Mohiti, Maryam; Monsef, Hassan; Anvari-Moghaddam, Amjad; Guerrero, Josep M.; Lesani, Hamid

Published in:
International Journal of Electrical Power & Energy Systems

DOI (link to publication from Publisher):
10.1016/j.ijepes.2018.09.031

Publication date:
2019

Document Version
Accepted author manuscript, peer reviewed version

Citation for published version (APA):
Mohiti, M., Monsef, H., Anvari-Moghaddam, A., Guerrero, J. M., & Lesani, H. (2019). A Decentralized Robust Model for Optimal Operation of Distribution Companies with Private Microgrids. *International Journal of Electrical Power & Energy Systems*, *106*, 105-123. https://doi.org/10.1016/j.ijepes.2018.09.031

A Decentralized Robust Model for Optimal Operation of Distribution Companies with Private Microgrids

Maryam Mohiti[1], Hassan Monsef[1], Amjad Anvari-moghaddam[2], Josep Guerrero[2], Hamid Lesani[1]

[1]Department of Electrical and Computer Engineering, University of Tehran (UT), Iran.

[2]Department of Energy Technology, Aalborg University (AAU), Denmark.

m.mohiti@ut.ac.ir, hmonsef @ut.ac.ir, aam@et.aau.dk, joz@et.aau.dk, Lesani@ut.ac.ir

Abstract

With the integration of microgrids (MGs) in future distribution networks (DNs), it is essential to develop a practical model for the distribution company (DISCO). Optimal operation of MGs is not generally consistent with DISCO, especially when they are operated by private owners. To this end, a decentralized robust model for optimal operation of DISCO with private MGs (PMGs) is proposed in this paper. The objective is to minimize the total operation cost of the system including DN and PMGs through coordinated operation of them. The enforced operational uncertainties are handled using an adaptive robust optimization (ARO) approach, enabling the operators of DISCO and PMGs to adjust different conservation levels during operating horizon. To respect the ownership of PMGs, a decentralized algorithm based on alternating direction method of multipliers (ADMM) is proposed to efficiently solve the resulting ARO model in which the operating problems of DISCO and PMGs are optimized independently. Case studies of a test system including modified IEEE 33-bus distribution network with three PMGs is used to demonstrate the effectiveness of the proposed model.

Keywords: Alternating direction method of multipliers (ADMM), decentralized model, distribution company (DISCO), multi-microgrid optimal operation.

Nomenclature

Indices and Sets

d	Index of battery of interruptible load.
$MG_{(n)}$	Set of DGs connected to bus n.
e	Index of battery energy storage.
i	Index of microgrid.
$IL_{(n)}$	Set of interruptible loads connected to bus *n*.
j	Index of DG.
k	Index of ADMM iteration.
n, m	Index of distribution network buses.
$MG_{(n)}$	Set of microgrids connected to bus *n*.
r	Index of renewable resource.
s	Index of step of price-quantity offer.
t	Index of time.
w / pv	Index of wind turbine/photovoltaic.

Parameters

α, β, γ	Cost function coefficients of DG.
$\alpha', \beta', \lambda'$	Cost function coefficients of micro turbine.
C^{PV} / C^{WT}	Cost of maintenance and operation of photovoltaic/wind turbine.
C_{OM}^{BS}	Maintenance cost of battery energy storage.
$\varepsilon_{(i,t)}$ / $\varepsilon'_{(i,t)}$	Self-elasticity of responsive load indicating its variation during hour t to the price during that hour/ cross-elasticity of responsive load indicating its variation during hour *t* to hour *t'*.
δ	Offer of interruptible load at each step.
η^{Bd} / η^{Bc}	Discharge/charge efficiency of battery energy storage.
Γ_0 / $\Gamma_{(t)}$	Uncertainty budget of wholesale price/ renewable generation.
I_{sub}	Current flow allowed at substation.
k_{SOC}	A scalar parameter related to minimum state of charge of battery.
L_0^{MG}	Initial demand of responsive load in microgrids.
Lp / Lq	Active/reactive load demand at each bus.
Lp^{DN} / Lq^{DN}	Active/reactive load demand at each bus of distribution network.
π_0^{MG}	Based electricity price offered to microgrids.
π^{IL}	Price of interruptible load at each step.
π^{WS}	Forecasted price of wholesale market.
r_L / x_L	Resistance/reactance of feeders of distribution network.
SDC	Shut-down cost of DG.

u	Commitement state of DG.
SUC	Start-up cost of DG.
u^{ON} / u^{OFF}	Start-up/shut-down status of DG.
UR / DR	Ramp up/down of DG.
UT / DT	Minimum up/down time of DG.
v, η, yp, yw	Auxiliary variables in robust approach.
ζ^{WS} / ζ^{REN}	Degree of wholesale price/ renewable generation uncertainty.
Variables	
bsc / bsd	Charge/ discharge status of battery energy storage.
CIL	Compensation cost of interruptible loads.
CMG	Operation cost of microgrid.
l / v	Auxiliary variables introduced in the AC power flow equations.
L^{MG}	Responsive load demand of microgrids.
P^{Bd} / P^{Bc}	Discharge/charge power of battery energy storage.
P^{DG} / Q^{DG}	Active/reactive power generation of DGs.
P^{flow} / Q^{flow}	Active/reactive power flow thorough feeders of distribution network.
P^{IL} / Q^{IL}	Active/reactive power interuption of interruptible load.
$P^{MG-\mathrm{Im}}$ / P^{MG-Exp}	Power imported/exported from distribution network to microgrid.
P^{MT}	Power generation of micro-turbine.
P^{REN}	Total power generation of wind turbines and photovoltaic systems.
P^{WT} / P^{PV}	Power generation of wind turbine/ photovoltaic system.
P^{WS} / Q^{WS}	Active/reactive exchanged power with wholesale market.
π^{MG}	Time varying prices offered by microgrids to their responsive loads
SOC	State of charge of battery energy storage.
V	Voltage of distribution network busses.
Symbol	
$\overline{(\bullet)}$ / $\underline{(\bullet)}$	Maximum/minimum bounds of $(\bullet)$.
$(\bullet)$	Deviation of $(\bullet)$ from forecasted value.
$(\bullet)$	Uncertain value of $(\bullet)$.

1. Introduction

In deregulated power systems, a distribution company (DISCO) as private entity involves in the wholesale market to procure electricity needs of its customers within their territories. Recently, integration of distributed energy resources (DERs) (e.g. micro turbines, wind turbines,

photovoltaic systems, and battery energy storage systems) into distribution networks (DN) has enabled the DISCO to perform a bi-directional power exchange, i.e. selling or purchasing power, with wholesale market and DERs. To operate the DISCO more efficiently, integrated DERs into the DN could be clustered in form of microgrids (MGs). MG is a cluster of DERs that supply electricity to the load in a localized area on the DN. MG may be operated by private owners and share different interests. The private MG (PMG) is an independent entity which negotiates with the DISCO and schedule tits DERs with the purpose of achieving higher profits while technical limits are not sacrificed. Therefore, coordinated operation of DN and multiple PMGs is challenging for the DISCO, especially when the uncertainties of renewable generations and wholesale market are considered.

A lot of studies have been focused on the optimal operation of DISCO and MGs considering economic and technical issues. In [1], a two-level decision making model for a DISCO in day-ahead electricity market is proposed considering interruptible loads (ILs) and distribution generations (DGs). In [2], a stochastic framework for short-term operation of DISCO is presented considering day-ahead and real-time markets. Authors of [3], integrate price-based demand response (DR) in short-term operation model of a DISCO. A real-time procurement strategy for a DISCO with DR aggregators is presented in [4] to maximize the profit of the company. In [5], a two-level decision-making scheme for a DISCO is proposed in which a competition is formed between DISCOs to purchase power from day-ahead market . In [6], an energy and reserve scheduling model is presented for optimal operation of MGs considering renewable generation and DR programs. The proposed model is formulated as a two-stage stochastic programing optimization problem. Authors of [7] have presented an optimal bidding strategy for MGs in both day-ahead and real-time market using robust/ stochastic optimization

method. In [8], an agent based model is used for energy management of MGs and uncertainties of wind power generation and energy consumption are modeled in form of prediction intervals. Authors of [9] have proposed an optimal schedule for MGs using chance-constrained method in which islanding constraints are considered.

With high integration of DERs into DNs, it is expected that multiple MGs are organized in DNs. In the literature, several models have been proposed to operate multiple MGs systems optimally. Authors of [10] present a stochastic energy management system in which daily optimal scheduling of networked MGs are performed. In this paper, a centralized operator is the main entity responsible for coordination of MGs. In [11], a distributed robust energy management system scheme for multiple interconnected MGs is developed to minimize total operation cost of MGs. Based on [11], authors of [12] have presented an optimal energy and reserve scheduling model for a system of multiple MGs. In [13], a centralized model is proposed for optimal operation of interconnected MGs in which network reliability is considered. Reference [14] solves the optimal power dispatch problem of multiple MG system in which total cost of power generation in each MG as well as the total cost of exchanged power between MGs and main grid is minimized. A stochastic model has been adopted to model uncertainties in load and renewable energy sources.

The above mentioned studies assumed that MGs trade power together without considering the role of DISCO. However, since MGs are connected to DN through electrical network and exchange power with it, the operating strategies of DISCO and MGs effect on the operation cost of each other. Therefore, the operation of DISCO and MGs needs to be coordinated during the operating horizon. In [15], a multi-objective bi-level optimal operation model for DN with grid-connected MGs is explained. In this paper the uncertainties are ignored and a genetic algorithm

is used to optimize the proposed model. In [16] an agent-based energy management system is introduced to schedule a DN with MGs considering a transactive market. In this study, DR actions and energy storage devices are utilized to manage energy imbalances of MGs. In [17], a system of systems model is proposed for hourly operation scheduling of active DNs including multiple MGs. In the proposed model, active DN and MGs are considered autonomous systems, which are coordinated using a hierarchical optimization algorithm.

In DNs with multiple MGs usually they are private entities with different interests; thus, the optimal operation of whole system including a DISCO and PMGs becomes more complicated. In this regard, centralized coordination model may violate the ownership of entities. Meanwhile, centralized models may increase computational complexity of operating model and possibility of congestion in communication networks. Furthermore, in DN with multiple MGs operation uncertainties can be intensified especially if MGs are located in the same metrological area. The above mentioned studies have considered the operation uncertainties using the scenario based methods which their optimality depend on the accuracy of the probability distribution function (PDF) and the number of scenarios. Insufficient historic data could lead to inaccurate fitted PDFs and non-optimal results. Furthermore, as the number of scenarios increases the computational burden of the problem increases significantly [18]. To address these issues, robust optimization (RO) method is proposed in which only the lower and upper band of uncertain variables are required for evaluation of their effects. Author of [19] suggested a RO optimization model for multi microgrids with uncertainties in renewable energy sources and load, to minimize the operation cost of the multiple MG system. However, in this paper multi-MGs are considered as a unified system. Reference [20] has proposed a two stage RO optimization method which has considered tie-line disconnection uncertainty in addition to generation and load uncertainty.

Although, this study has used RO approach to address the operational uncertainties but the ownership of PMGs and interaction between DISCO and PMGs are ignored.

To fulfill these gaps, this paper presents a decentralized robust model for optimal operation of a DISCO including multiple PMGs aiming at minimizing the total cost of the system. The uncertainties of wholesale market price and renewable generation are handled via RO approach, enabling the operators of DISCO and PMGs to evaluate different levels of uncertainty and conservation during operation horizon. Moreover, a decentralized solution algorithm using alternative direction method of multipliers (ADMM) is introduced to respect the ownership of PMGs and reduce the complexity of the developed RO based model. In this regards, the operating problem of DISCO and PMGs are solved separately in an iterative manner. The major contributions of this paper are as follow:

- Considering the interaction between a DISCO and multiple PMGs.
- Proposing a model for coordinated operation of a DISCO and multiple PMGs in which the financial benefits and technical constraints are both met for all entities,
- Applying an adaptive RO approach to cope with uncertainty of wholesale market price and renewable generation,
- Introducing a decentralized solution algorithm based on the ADMM method to respect the ownership of PMGs and handle the computational burden of the proposed model.
- Proposing a Monte-Carlo simulation based after-the-fact analysis to evaluate performance of proposed model.

The remainder of this paper is organized as follows. A generic description of the proposed model is provided in Section II. A centralized deterministic model for coordinated operation of DISCO and PMGs is introduced in Section III. Then, the RO approach is presented, and,

subsequently, with considering the uncertainties of renewable generation and wholesale market price, the robust counterpart of the introduced model is obtained. To preserve the ownership of PMGs and reduce the computational burden, the developed RO model is recast into a decentralized one using the ADMM method in Section IV. Section V discusses the results and Section VI concludes the paper.

2. General Description of the proposed model

A graphical description of the system is shown in Fig. 1. As can be seen, this model consists of two independent layers. The first layer is related to the decision making of DISCO. In this layer DISCO minimizes the total cost of the system including DN and PMGs by coordinated operation of them while considering the uncertainties associated with renewable generation and wholesale market price. To this end, DISCO determines optimal scheduling of DG units, invocation of interruptible loads (ILs), trading strategy with the wholesale market, and dispatch strategies of PMGs containing the amount of exchanging power with PMGs and their prices during operation horizon. The decision makers of the second layer are PMGs. In this layer, each PMG receives dispatch strategies via two-way communication infrastructure from DISCO. Then, the PMGs optimize their own operation, simultaneously, and inform the DISCO of their exchanging power with DN. The conflicts of exchanging power between DISCO and PMGs are resolved using the ADMM method in an iterative procedure.

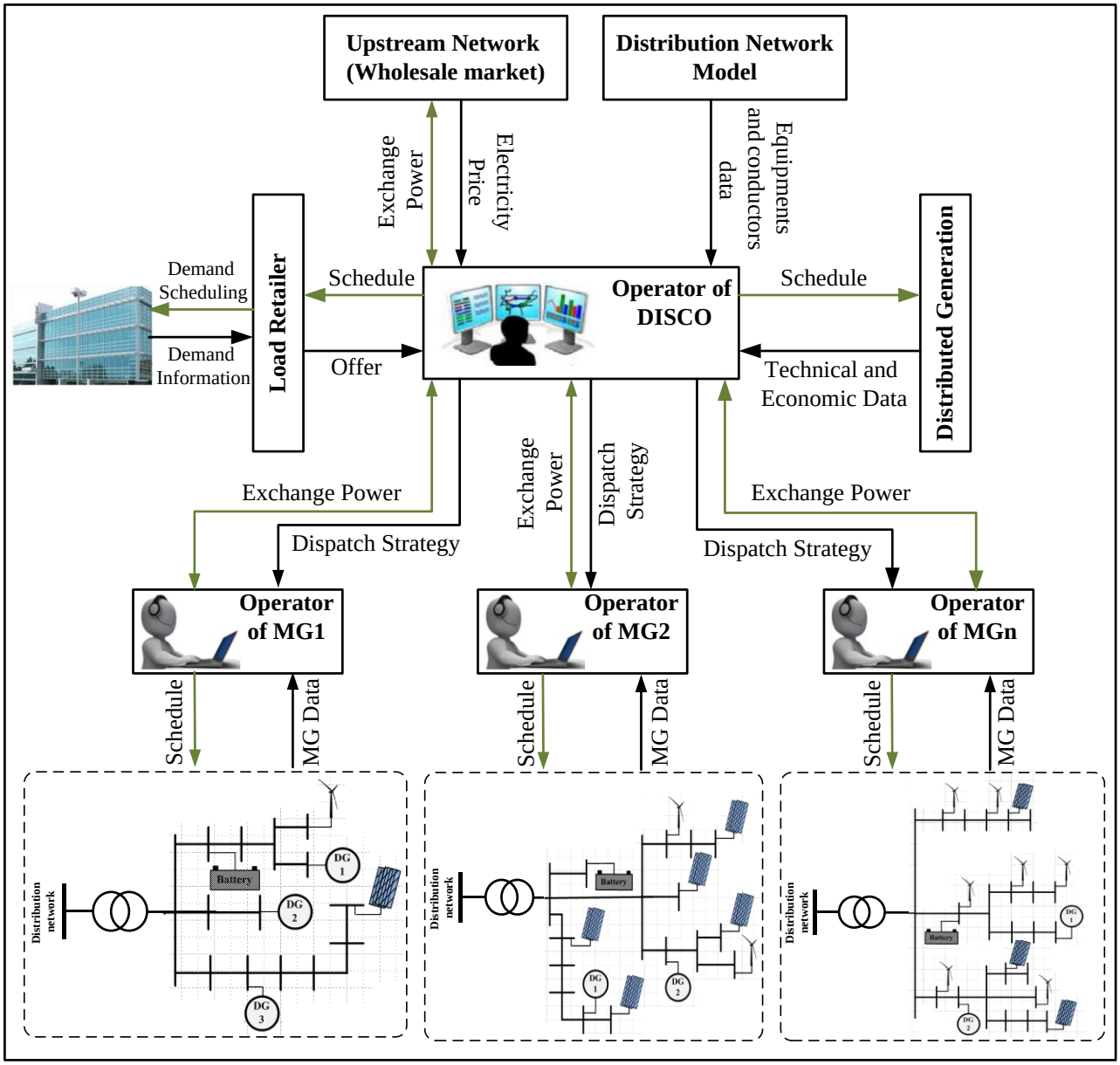

Fig. 1. Graphical description of the system

3. Problem Formulation

In this section, the centralized deterministic model is described. In this model, PMGs send their data to DISCO as the main entity responsible for optimal operation of the system including DN and PMGs. Then, the operational uncertainties related to renewable generation and wholesale market price are considered and the robust counterpart of the introduced model is obtained.

In the following the problem formulation including objective function and constraints are presented.

3.1 Objective Function

The objective function is to minimize the total cost of system which is formulated as follows:

$$\begin{aligned} & Min \sum_{t \in T} \pi_{(t)}^{WS} P_{(t)}^{WS} \\ & + \sum_{t \in T} \sum_{j \in DG} \left\{ SUC_{(j)} u_{(j,t)}^{ON} + \left(\alpha_{(j)} u_{(j,t)} + \beta_{(j)} P_{(j,t)}^{DG} + \gamma_{(j)} P_{(j,t)}^{DG\,2} \right) + SDC_{(j)} u_{(j,t)}^{OFF} \right\} \\ & + \sum_{t \in T} \sum_{d \in IL} CIL_{(d,t)} + \sum_{t \in T} \sum_{i \in MG} CMG_{(i,t)} \end{aligned} \tag{1}$$

The first term represents the cost of energy exchanged with the wholesale market. The second term is the operating cost of DGs which includes the start-up cost, fuel cost, and shut-down cost, respectively. The compensation cost of ILs is represented in the third term. Finally, the operation cost of PMGs is indicated in the last term. It should be noted that the non-linearity of DGs' fuel cost could be alleviated using piecewise linear approximation method [21].

3.2 Constraints

The DISCO should consider a set of financial and technical constraints which are described in the following.

3.2.1 Power balance constraints

The complex power flow equations associated with bus n of the DN can be described as follows [22]:

$$Lp_{(n)} = P_{(n,m)}^{flow} - r_{L(n,m)} \times l_{(n,m)} - \sum_{\kappa \in (n,\kappa)} P_{(n,\kappa)}^{flow} \tag{2}$$

$$Lq_{(n)} = Q_{(n,m)}^{flow} - x_{(n,m)} \times l_{(n,m)} - \sum_{\kappa \in (n,\kappa)} Q_{(n,\kappa)}^{flow} \tag{3}$$

$$v_{(n)} = v_{(m)} - 2\left(r_{(n,m)} \times P_{(n,m)}^{flow} + x_{(n,m)} \times Q_{(n,m)}^{flow}\right) + \left(r_{(n,m)}^{2} + x_{(n,m)}^{2}\right) \times l_{(n,m)} \tag{4}$$

$$l_{(n,m)} = \frac{P_{(n,m)}^{flow\ 2} + Q_{(n,m)}^{flow\ 2}}{v_{(m)}} \tag{5}$$

Where, $l_{(n,m)} = \left|I_{(n,m)}\right|^2$ and $v_{(n)} = \left|V_{(n)}\right|^2$. Accordingly, the active and reactive power balance constraints at bus n of DN and at hour t could be written as follow:

$$\begin{aligned} & Lp_{(n,t)}^{DN} - P_{(n=1,t)}^{WS} - \sum_{j \in DG_{(n)}} P_{(j,t)}^{DG} - \sum_{d \in IL_{(n)}} P_{(d,t)}^{IL} - \sum_{i \in MG_{(n)}} P_{(i,t)}^{MG} \\ & = P_{(n,m,t)}^{flow} - r_{(n,m)} \times l_{(n,m,t)} - \sum_{\kappa \in (n,\kappa)} P_{(\kappa,m,t)}^{flow}; \quad \forall n,m,t \end{aligned} \tag{6}$$

$$\begin{aligned} & Lq_{(n,t)}^{DN} - Q_{(n=1,t)}^{WS} - \sum_{j \in DG_{(n)}} Q_{(j,t)}^{DG} - \sum_{d \in IL_{(n)}} Q_{(d,t)}^{IL} \\ & = Q_{(n,m,t)}^{flow} - x_{(n,m)} \times l_{(n,m,t)} - \sum_{\kappa \in (n,\kappa)} Q_{(\kappa,m,t)}^{flow}; \quad \forall n,m,t \end{aligned} \tag{7}$$

In (7), $P_{(i,t)}^{MG}$ is the net active power exchanged between DN and PMGs which is represented as follows:

$$P_{(i,t)}^{MG} = P_{(i,t)}^{MG-Ex} - P_{(i,t)}^{MG-Im}; \quad \forall i,t \tag{8}$$

A positive (negative) value of $P_{(i,t)}^{MG}$ indicates that power is exported (imported) from (to) the PMGs to (from) the DN. The exchanged power is be limited by:

$$\underline{P_{(i,t)}^{MG}} \le P_{(i,t)}^{MG} \le \overline{P_{(i,t)}^{MG}}; \quad \forall i,t \tag{9}$$

It should be mentioned that the quadratic terms in power flow equations can be linearized using the piecewise linear approximation method as explained in [22].

3.2.2 Network security constraints

In order to ensure the safe operation of DN, technical constraints are considered as follows:

$$\underline{v_{(n)}} \le v_{(n,t)} \le \overline{v_{(n)}}; \quad \forall n,t \tag{10}$$

$$l_{(n,m,t)} \le \overline{l_{(n,m)}}; \quad \forall m,n,t \tag{11}$$

Equation (10) guarantees an authorized voltage level for all buses of the DN within prescribed values during the operating horizon. Feeder current limits are also taken into account in equation (11). It should be mentioned that the substation of DN represents a controlled voltage bus. Moreover, limited capacity of substation transformers imposes an upper-bounded on the exchanged power with the upstream grid. Therefore, the following additional constraints are considered when a substation is connected to bus 1 of the distribution network:

$$v_{(n=1,t)} = 1; \quad \forall t \tag{12}$$

$$l_{(n=1,m=2,t)} \le \overline{I^2_{sub}}; \quad \forall m,t \tag{13}$$

3.2.3 DG unit constraints

To guarantee the safe operation of DGs the following constraints are considered [22]:

$$\underline{P^{DG}_{(j)}} u_{(t,j)} \le P^{DG}_{(t,j)} \le \overline{P^{DG}_{(j)}} u_{(t,j)}; \quad \forall j,t \tag{14}$$

$$P^{DG}_{(t,j)} - P^{DG}_{(t-1,j)} \le UR_{(j)} \left(1 - u^{ON}_{(t,j)}\right) + \underline{P^{DG}_{(j)}} u^{ON}_{(t,j)}; \quad \forall j,t \tag{15}$$

$$P^{DG}_{(t-1,j)} - P^{DG}_{(t,j)} \le DR_{(j)} \left(1 - u^{OFF}_{(t,j)}\right) + \underline{P^{DG}_{(j)}} u^{OFF}_{(t,j)}; \quad \forall j,t \tag{16}$$

$$\sum_{h=t}^{t+UT(j)-1} u_{(h,j)} \ge UT_{(j)} u^{ON}_{(t,j)}; \quad \forall j,t \tag{17}$$

$$\sum_{h=t}^{t+DT(j)-1} (1 - u_{(h,j)}) \ge DT_{(j)} u^{OFF}_{(t,j)}; \quad \forall j,t \tag{18}$$

$$u_{(t+1,j)} - u_{(t,j)} \le u^{ON}_{(t+1,j)}; \quad \forall j,t \tag{19}$$

$$u_{(t,j)} - u_{(t+1,j)} \le u^{OFF}_{(t+1,j)}; \quad \forall j,t \tag{20}$$

$$u_{(t+1,j)} - u_{(t,j)} = u^{ON}_{(t+1,j)} - u^{OFF}_{(t+1,j)}; \quad \forall j,t \tag{21}$$

The capacity limit of DGs is given by (14). Constraints (15) and (16) represent the ramp up and ramp down capabilities of DGs, respectively. Moreover, the minimum up time and down time of DGs are considered in (17) and (18), respectively. To prevent conflicted situations in the status of DGs, constraints (19) to (21) are incorporated. It should be mentioned that the constraints of (14) to (21) should be only considered for non-renewable DGs, e.g. diesel generators and micro-turbines. The renewable based DGs, e.g. wind turbines and photovoltaic systems, are non-dispatchable and inject all their power production to the DN.

3.2.4 IL constraints

In this paper, IL is adopted in which certain customers reduce their consumption. The candidate customers submit a step-wise price-quantity offer to the DISCO, as formulated in (22) to (25). Each package includes the amount of curtailed load and their offered price. While the offers are accepted from the DISCO, ILs are called to reduce their load and receive the conservation cost according to (26) [22].

$$\underline{\Delta_{(d)}} \le \delta_{(d,t,s)} \le \Delta_{(d,s)}; \quad \forall d,t,s \tag{22}$$

$$0 \le \delta_{(d,t,s)} \le \left(\Delta_{(d,s)} - \Delta_{(d,s-1)}\right); \quad \forall d,t,s \tag{23}$$

$$P^{IL}_{(d,t)} = \sum_{s \in NS} \delta_{(d,t,s)}; \quad \forall d,t \tag{24}$$

$$\underline{\Delta_{(d)}} \le P^{IL}_{(d,t)} \le \overline{\Delta_{(d)}}; \quad \forall d,t \tag{25}$$

$$CIL_{(d,t)} = \sum_{s \in NS} \pi^{IL}_{(d,s)} \delta_{(t,d,s)}; \quad \forall d,t \tag{26}$$

3.2.5 PMG operation constraints

The operating cost of PMGs could be formulated as follows:

$$CMG_{(i,t)} = \sum_{j \in MG_{(i)}} \left\{ \alpha'_{(j)} + \beta'_{(j)} P^{MT}_{(j,t)} \right\} + \sum_{w \in MG_{(i)}} C^{WT} P^{WT}_{(w,t)} + \sum_{pv \in MG_{(i)}} C^{PV} P^{PV}_{(pv,t)} + \sum_{e \in MG_{(i)}} \left\{ C^{BS}_{OM} k_{\underline{SOC}} \left(1 - \underline{SOC}\right) \right\} SOC_{(e,t)}; \quad \forall i,t \tag{27}$$

The first term is the fuel cost of micro-turbines. The second and third terms are maintenance cost of wind turbines and photovoltaic systems, respectively. The battery degradation cost due to frequent charge/discharge is shown in the fourth term. Also, power balance in MGs is as follow:

$$P^{MG\_Im}_{(i,t)} + \sum_{j \in MG_{(i)}} P^{MT}_{(j,t)} + \sum_{w \in MG_{(i)}} P^{WT}_{(w,t)} + \sum_{pv \in MG_{(i)}} P^{PV}_{(pv,t)} + \sum_{e \in MG_{(i)}} P^{Bd}_{(e,t)} - \sum_{e \in MG_{(i)}} P^{Bc}_{(e,t)} - P^{MG\_Ex}_{(i,t)} = L^{MG}_{(i,t)}; \quad \forall i,t \tag{28}$$

It is assumed that the consumers of PMGs are price responsive and their behavior with respect to prices could be modeled as follow [18]:

$$L^{MG}_{(i,t)} = L^{MG}_{0(i,t)} \left(1 + \varepsilon_{(i,t)} \frac{\left[\pi^{MG}_{(i,t)} - \pi^{MG}_{0(i)} \right]}{\pi^{MG}_{0(i)}} + \sum_{\substack{t' \in T \\ t' \neq t}} \varepsilon'_{(i,t)} \frac{\left[\pi^{MG}_{(i,t')} - \pi^{MG}_{0(i)} \right]}{\pi^{MG}_{0(i)}} \right); \quad \forall i,t \tag{29}$$

$$\overline{L^{MG}_{(i,t)}} \leq L^{MG}_{(i,t)} \leq \underline{L^{MG}_{(i,t)}}; \quad \forall i,t \tag{30}$$

$$\pi^{MG}_{(i,t)} \leq \overline{\pi^{MG}_{(i,t)}}; \quad \forall i,t \tag{31}$$

Equation (29) represents how much consumers utilize electricity to achieve minimum bill during operating horizon. The electricity consumption of each customer should be in the specific interval, which is considered by (30). Constraint (31) imposes a cap for the sale price offered to customers.

The technical constraints of micro-turbines are the same as (14)-(21). Meanwhile, the constraints of battery energy storage (BES) are presented as follow:

$$0 \le P_{(e,t)}^{Bc} \le bsc_{(e,t)} \overline{P_{(e)}^{Bc}}; \ \ \forall e,t \tag{32}$$

$$0 \le P_{(e,t)}^{Bd} \le bsd_{(e,t)} \overline{P_{(e)}^{Bd}}; \ \ \forall e,t \tag{33}$$

$$bsc_{(e,t)} + bsd_{(e,t)} \le 1; \ \ \forall e,t \tag{34}$$

$$SOC_{(e,t)} = SOC_{(e,t-1)} - P_{(e,t)}^{Bd} \Delta t \big/ \eta_{(e)}^{Bd} + P_{(e,t)}^{Bc} \Delta t \, \eta_{(e)}^{Bc}; \ \ \forall e,t \tag{35}$$

$$\underline{SOC_{(e)}} \le SOC_{(e,t)} \le \overline{SOC_{(e)}}; \ \ \forall e,t \tag{36}$$

The charging and discharging power limits are imposed by (32) and (33). Simultaneous charging and discharging is avoided by (34). The battery state of charge (SOC) is defined in (35) and the limit of SOC is enforced by (36).

3.3 Robust Model

In the above described model it is assumed that the wholesale market price and renewable generation are perfectly forecast and do not violate at the operating time. However, due to the lack of accurate forecasting methods, wholesale market price and renewable generation may change during the operation horizon causing critical financial and technical challenges for DISCO and PMGs. To cope with these uncertainties, RO approach is introduced in this section and then, the robust counterpart of the described model is presented.

3.3.1 RO approach

The quadratic terms in power flow equations and objective function can be linearized using the piecewise linear approximation method as explained in [22]. Therefore, equations (1)-(36) form a standard mixed integer linear programming (MILP) problem as follow [23]:

$$\min_{x_j \forall j} \sum_{j=1}^{n} \tilde{c}_{(j)} x_{(j)} \tag{37}$$

Subject to:

$$\sum_{j=1}^{n} \tilde{a}_{(i,j)} x_{(j)} \le b_{(i)}; \quad \forall i = 1,\ldots,NC \tag{38}$$

$$\underline{x_{(j)}} \le x_{(j)} \le \overline{x_{(j)}}; \quad \forall j = 1,\ldots,ND \tag{39}$$

Where, $\tilde{c}_{(j)}$ and $\tilde{a}_{(i,j)}$ are the coefficients of the objective function and constraints, respectively, which could be represented as follows:

$$\tilde{c}_{(j)} \in [c_{(j)} - \hat{c}_{(j)}, c_{(j)} + \hat{c}_{(j)}]; \quad \forall j \tag{40}$$

$$\tilde{a}_{(i,j)} \in \left[a_{(i,j)} - \hat{a}_{(i,j)}, a_{(i,j)} + \hat{a}_{(i,j)}\right]; \quad \forall i, j \tag{41}$$

If $\tilde{c}_{(j)}$ and $\tilde{a}_{(i,j)}$ be the uncertain variables, the values of $\hat{c}_{(j)}$ and $\hat{a}_{(i,j)}$ determine the maximum range of their deviations, respectively; elsewise, they are zero.

The robust counterpart of (37) to (42) could be formulated as follow [23]:

$$\min_{x_j \forall j} \sum_{j=1}^{n} c_{(j)} x_{(j)} + \max_{\left\{S_0 \cup \{t_0\} \middle| S_0 \subseteq J_0, |S_0| = \lfloor \Gamma_0 \rfloor, t_0 \in J_0 \setminus S_0\right\}} \left\{ \sum_{j \in S_0} \hat{c}_{(j)} \left| x_{(j)} \right| + \left(\Gamma_0 - \lfloor \Gamma_0 \rfloor\right) \hat{c}_{(t_0)} \left| x_{(t_0)} \right| \right\} \tag{42}$$

Subject to:

$$\sum_{j=1}^{n} a_{(i,j)} x_{(j)} + \max_{\left\{S_i \cup \{t_i\} \middle| S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\right\}} \left\{ \begin{array}{l} \sum_{j \in S_i} \hat{a}_{(i,j)} \left| x_{(j)} \right| \\ + \left(\Gamma_i - \lfloor \Gamma_i \rfloor\right) a_{(t_i)} \left| x_{(t_i)} \right| \end{array} \right\} \le b_i; \quad \forall i = 1,\ldots,NC \tag{43}$$

$$\underline{x_{(j)}} \le x_{(j)} \le \overline{x_{(j)}}; \quad \forall j = 1,\ldots,ND \tag{44}$$

The RO approach solves the worst-case problem and derives the optimal solutions which are immunized against all the uncertain variables. However, it is exposed to over-conservatism which could be resolved with defining the uncertainty budgets to control the degree of

conservation. In this regard, Γ_0 and Γ_i specify the conservatism degree for the objective function and *i*th constraint, respectively. Note that, Γ_0 and Γ_i could adopt different values in the intervals $\left[0,|J_0|\right]$ and $\left[0,|J_i|\right]$, respectively, where $J_0=\left\{j\,|\hat{c}_{(j)}>0\right\}$ and $J_i=\left\{j\,|\hat{a}_{(i,j)}>0\right\}$ [23].

The RO problem of (42) to (44) is nonlinear which can be recast as a MILP problem using duality theory and linearization procedures as follow [23]:

$$Min\sum_{j=1}^{n}c_{(j)}x_{(j)}+\Gamma_0\upsilon_0+\sum_{j\in J_0}\eta_{0(j)} \tag{45}$$

$$\sum_{j=1}^{n}a_{(i,j)}x_{(j)}+\Gamma_i\upsilon_i+\sum_{j\in J_i}\eta_{(i,j)}\le b_i;\quad \forall i=1,\ldots,NC \tag{46}$$

$$\upsilon_0+\eta_{0(j)}\ge\hat{c}_{(j)}yp_{(j)};\quad \forall j\in J_0 \tag{47}$$

$$\upsilon_i+\eta_{(i,j)}\ge\hat{a}_{(i,j)}yp_{(j)};\quad \forall j\in J_i, i=1,\ldots,NC \tag{48}$$

$$-yp_{(j)}\le x_{(j)}\le yp_{(j)};\quad \forall j=1,\ldots,ND \tag{49}$$

$$\underline{x_{(j)}}\le x_{(j)}\le\overline{x_{(j)}};\quad \forall j=1,\ldots,ND \tag{50}$$

$$\eta_{0(j)}\ge 0;\quad \forall j\in J_0 \tag{51}$$

$$\eta_{(i,j)}\ge 0;\quad \forall j\in J_i, i=1,\ldots,NC \tag{52}$$

$$yp_{(j)}\ge 0;\quad \forall j=1,\ldots,ND \tag{53}$$

$$\upsilon_0\ge 0 \tag{54}$$

$$\upsilon_i\ge 0;\quad \forall i=1,\ldots,NC \tag{55}$$

3.3.2 Uncertainties handling using RO approach

In robust optimization approach an uncertainty set is defined for each uncertain variable. Therefore, in the proposed robust based model, two uncertainty sets are consider to handle the uncertainties of renewable generation and wholesale market price as follow:

$$\tilde{\pi}^{WS}_{(t)} \in \left[\pi^{WS}_{(t)} - \xi^{WS}_{(t)} \pi^{WS}_{(t)}, \pi^{WS}_{(t)} + \xi^{WS}_{(t)} \pi^{WS}_{(t)} \right]; \ \forall t \tag{56}$$

$$\tilde{P}^{REN}_{(r,t)} \in \left[P^{REN}_{(r,t)} - \xi^{REN}_{(r,t)} P^{REN}_{(r,t)}, P^{REN}_{(r,t)} + \xi^{REN}_{(r,t)} P^{REN}_{(r,t)} \right]; \quad \forall r,t \tag{57}$$

In this regards, the worst realization over the uncertain variables, i.e. renewable generation and wholesale price, is determined and then, the optimized solution is calculated. It should be noted that for easy presentation, the power generation of renewable resources (i.e., WTs and PVs) are shown by $P^{REN}_{(r,t)}$.

Considering the mentioned uncertainties, the robust problem of (1)-(36) is recast in MILP format as follows. As mentioned the quadratic terms in power flow equations and objective function can be linearized using the piecewise linear approximation method.

$$\begin{aligned} Min \sum_{t \in T} \pi^{WS}_{(t)} P^{WS}_{(t)} + \Gamma_0 \upsilon_0 + \sum_{t \in T} q_{0(t)} + \sum_{t \in T} \sum_{j \in DG} \Big\{ SUC_{(j)} u^{ON}_{(j,t)} \\ + \left(\alpha_{(j)} u_{(j,t)} + \beta_{(j)} P^{DG}_{(j,t)} + \gamma_{(j)} P^{DG\,2}_{(j,t)} \right) + SDC_{(j)} u^{OFF}_{(j,t)} \Big\} \\ + \sum_{t \in T} \sum_{d \in IL} CIL_{(d,t)} + \sum_{t \in T} \sum_{i \in MG} CMG_{(i,t)} \end{aligned} \tag{58}$$

$$P^{MG}_{(i,t)} = P^{MG\_Ex}_{(i,t)} - P^{MG\_Im}_{(i,t)}; \ \forall i,t \tag{59}$$

$$(6),(7),(9)-(27) \tag{60}$$

$$\begin{aligned} P^{MG\_Im}_{(i,t)} + \sum_{r \in MG_{(i)}} P^{REN}_{(r,t)} + \sum_{j \in MG_{(i)}} P^{MT}_{(j,t)} + \sum_{e \in MG_{(i)}} P^{Bd}_{(e,t)} \\ + \Gamma_{(t)} \upsilon_{(t)} + \sum_{r \in MG_{(i)}} q_{(r,t)} - \sum_{e \in MG_{(i)}} P^{Bc}_{(e,t)} - P^{MG\_Ex}_{(i,t)} = L^{MG}_{(i,t)}; \ \forall i,t \end{aligned} \tag{61}$$

$$(29)-(36) \tag{62}$$

$$\upsilon_0 + q_{0(t)} \geq \delta_{(t)}^{WS} yp_{(t)}; \quad \forall t \tag{63}$$

$$\upsilon_{(t)} + q_{(r,t)} \geq \delta_{(r,t)}^{REN} yr_{(r,t)}; \quad \forall r,t \tag{64}$$

$$P_{(t)}^{WS} \leq yp_{(t)}; \ \forall t; \quad \upsilon_0 \geq 0; \ q_{0(t)} \geq 0; \ \forall t \tag{65}$$

$$P_{(r,t)}^{REN} \leq yr_{(r,t)}; \ \forall r,t; \quad \upsilon_{(r)} \geq 0; \ \forall r; \ q_{(r,t)} \geq 0; \ \forall r,t \tag{66}$$

The objective function includes N_T uncertain variables which are wholesale market prices. Therefore, $|J_0|$ is equal to 24 and Γ_0 could adopt different values in interval $[0,24]$. Moreover, hourly renewable generation in each PMG includes only one uncertain variable. Hence, $|J_{(t)}|$ is equal to 1 and $\Gamma_{(t)}$ could adopt different values in interval $[0,1]$. When Γ_0 and $\Gamma_{(t)}$ adopt their largest values (i.e. 24 and 1, respectively), the optimal operation of DN and PMGs is immunized against the worst case of renewable generation and wholesale price.

4. Decentralized Robust Model

The RO problem of (58)-(66) is in a MILP format which guarantees the global optimal solution. However, as the operating problem of DISCO and PMGs are coupled through (59), they cannot be optimized separately. Therefore, the ownership of PMGs is not respected. Also, computational burden of the proposed RO problem grows rapidly as the number of PMGs increases in the DN. To address these issues, a fast convergence algorithm based on ADMM method is utilized which enables the operating problems of DISCO and PMGs to be optimized in a decentralized manner.

4.1 ADMM method

A convex optimization problem could be solved by ADMM method in the following separable format [24]:

$$\underset{x \in X, z \in Z}{Min} \; f(x) + g(z) \tag{67}$$

Subject to:

$$Ax + Bz = c \tag{68}$$

The augmented Lagrangian function of (67) and (68) is denoted by (69) in which λ represent the Lagrangian multiplier vector corresponding to constraint (68), $\rho > 0$ is a penalty factor, and $\|.\|_2$ is l_2-norm of vector. ADMM method consists of the iteration procedure of (69) to (72), where k is the ADMM iteration index [24]. Therefore, the variables x and y are optimized separately in (70) and (71); which makes ADMM an effective method for decentralized optimization.

$$\underset{x \in X, z \in Z}{Min} \; L_\rho(x, z, \lambda) = f(x) + g(z) + \lambda^T (Ax + Bz - c) + \left(\rho/2\right)\|Ax + Bz - c\|_2^2 \tag{69}$$

$$x(k+1) = \underset{x \in X}{\arg\min} \, L_\rho(x, z(k), \lambda(k)) \tag{70}$$

$$z(k+1) = \underset{z \in z}{\arg\min} \, L_\rho(x(k+1), z, \lambda(k)) \tag{71}$$

$$\lambda(k+1) = \lambda(k) + \rho(Ax(k+1) + Bz(k+1) - c) \tag{72}$$

The convergence criteria of ADMM method is defined based on the primal residual which is denoted as follows [24]:

$$\|\lambda(k+1) - \lambda(k)\|_2 \le \varepsilon_{thr} \tag{73}$$

4.2 Detailed formulation

In the RO problem of (58) to (66), x includes $\left\{P_{(t)}^{WS}, P_{(j,t)}^{DG}, u_{(j,t)}, u_{(j,t)}^{ON}, u_{(j,t)}^{OFF}, P_{(d,t)}^{IL}, \upsilon_0, q_{0(t)}, yp_{(t)}\right\}$ and y involves $\left\{P_{(i,t)}^{MG\_Ex}, P_{(i,t)}^{MG\_Im}, P_{(r,t)}^{REN}, P_{(j,t)}^{MT}, P_{(e,t)}^{Bd}, P_{(e,t)}^{Bc}, \upsilon_{(r)}, q_{(r,t)}, yr_{(t,t)}\right\}$. Thus, set X includes constraints (60), (63), and (65) while set Z captures constraints (61), (62), (64), and (66).

Moreover, constraint (59) relates to (68) of ADMM method. So, the augmented Lagrangian function which presents the decentralized model is as follows:

$$
\begin{aligned}
&\underset{x\in X,z\in Z}{Min}\ L_{\rho}\left(x,z,\lambda\right)=\sum_{t\in T}\pi_{(t)}^{WS}P_{(t)}^{WS}+\Gamma_{0}\upsilon_{0}+\sum_{t\in T}q_{0(t)}\\
&+\sum_{t\in T}\sum_{j\in DG}\left\{SUC_{(j)}u_{(j,t)}^{ON}+\left(\alpha_{(j)}u_{(j,t)}+\beta_{(j)}P_{(j,t)}^{DG}+\gamma_{(j)}P_{(j,t)}^{DG\,2}\right)\right.\\
&\left.+SDC_{(j)}u_{(j,t)}^{OFF}\right\}+\sum_{t\in T}\sum_{d\in IL}CIL_{(d,t)}+\sum_{t\in T}\sum_{i\in MG}CMG_{(i,t)}\\
&+\sum_{t\in T}\sum_{i\in MG}\lambda_{(i,t)}\left(P_{(i,t)}^{MG}-\left(P_{(i,t)}^{MG\_Ex}-P_{(i,t)}^{MG\_Im}\right)\right)\\
&+\frac{\rho}{2}\sum_{t\in T}\sum_{i\in MG}\left(P_{(i,t)}^{MG}-\left(P_{(i,t)}^{MG\_Ex}-P_{(i,t)}^{MG\_Im}\right)\right)^{2}
\end{aligned}
\tag{74}
$$

As mentioned, (74) is separable and the operating problems of DISCO and PMGs can be optimized in a decentralized manner. In this way, the ownership of PMGs is preserved while reducing the computational burden of DISCO. The iterative solution algorithm based on the ADMM method is shown in Fig. 2 which could be summarized in the following steps:

Step1) Set the iteration index $k=0$; select the values of penalty factor ρ and threshold convergence criteria ε_{thr}. Choose initial values for offered values of exchanged power and prices which are determined by the DISCO, i.e. $\lambda_{(i,t)}$ and $P_{(i,t)}^{MG}$, respectively. It should be mentioned that the initial values of $\lambda_{(i,t)}\left(0\right)$ and $P_{(i,t)}^{MG}\left(0\right)$ are set to be 0.

Step 2) Each PMG optimizes its operation by solving the following MIQP problem:

$$
\begin{aligned}
z\left(k+1\right) &= \arg\min_{z} \sum_{j\in MG_{(i)}} \left\{\alpha'_{(j)} + \beta'_{(j)} P^{MT}_{(j,t)}\right\} + \sum_{w\in MG_{(i)}} C^{WT} P^{WT}_{(w,t)} \\
&+ \sum_{pv\in MG_{(i)}} C^{PV} P^{PV}_{(pv,t)} + \sum_{e\in MG_{(i)}} \left\{C^{BS}_{OM} k_{\underline{SOC}} \left(1-\underline{SOC}\right)\right\} SOC_{(t)} \\
&- \sum_{t\in T}\sum_{i\in MG} \lambda_{(i,t)}\left(k\right)\left(P^{MG\_Ex}_{(i,t)} - P^{MG\_Im}_{(i,t)}\right) \\
&+ \frac{\rho}{2}\sum_{t\in T}\sum_{i\in MG}\left(\left(P^{MG\_Ex}_{(i,t)} - P^{MG\_Im}_{(i,t)}\right) - P^{MG}_{(i,t)}\left(k\right)\right)^2
\end{aligned}
\tag{75}
$$

Subject to: (61), (62), (64), and (66).

The above problem is a mixed integer quadratic programming (MIQP) that can be effectively solved via available commercial software packages. Then, each PMG sends the proposed export (import) power to (from) DN, i.e. $P^{MG\_Ex}_{(i,t)}\left(k+1\right)$ and $P^{MG\_Im}_{(i,t)}\left(k+1\right)$, to the DISCO.

Step3) The DISCO solves the following problem to determine robust solutions, i.e. x, for optimal operation of DN:

$$
\begin{aligned}
x\left(k+1\right) &= \arg\min_{x} \sum_{t\in T} \pi^{WS}_{(t)} P^{WS}_{(t)} + \Gamma_0 \upsilon_0 + \sum_{t\in T} q_{0(t)} + \sum_{t\in T}\sum_{j\in DG} \left\{SUC_{(j)} u^{ON}_{(j,t)}\right. \\
&\left. + \left(\alpha_{(j)} u_{(j,t)} + \beta_{(j)} P^{DG}_{(j,t)} + \gamma_{(j)} P^{DG\,2}_{(j,t)}\right) + SDC_{(j)} u^{OFF}_{(j,t)}\right\} + \sum_{t\in T}\sum_{d\in IL} CIL_{(d,t)} \\
&+ \sum_{t\in T}\sum_{i\in MG} \lambda_{(i,t)}\left(k\right) P^{MG}_{(i,t)} + \frac{\rho}{2}\sum_{t\in T}\sum_{i\in MG}\left(P^{MG}_{(i,t)} - \left(P^{MG\_Ex}_{(i,t)}\left(k+1\right) - P^{MG\_Im}_{(i,t)}\left(k+1\right)\right)\right)^2
\end{aligned}
\tag{76}
$$

Subject to: (60), (63), and (65).

Step 4) The DISCO checks the following criteria. If it is not met, goes to step 5; else, stops the iteration procedure and releases the optimal operating results.

$$
\left[\sum_{t\in T}\sum_{i\in MG}\left(P^{MG}_{(i,t)}\left(k+1\right) - \left(P^{MG\_Ex}_{(i,t)}\left(k+1\right) - P^{MG\_Im}_{(i,t)}\left(k+1\right)\right)\right)^2\right]^{0.5} \le \varepsilon_{thr} \tag{77}
$$

Step 5) The DISCO updates the prices of exchanged power with PMGs as follows:

$$
\lambda_{(i,t)}\left(k+1\right) = \lambda_{(i,t)}\left(k\right) + \rho\left(P^{MG}_{(i,t)}\left(k+1\right) - \left(P^{MG\_Ex}_{(i,t)}\left(k+1\right) - P^{MG\_Im}_{(i,t)}\left(k+1\right)\right)\right) \tag{78}
$$

Then, set $k = k + 1$; DISCO sends the offered values of exchanged power and prices, i.e. $P_{(i,t)}^{MG}(k+1)$ and $\lambda_{(i,t)}(k+1)$ to PMGs; Go to Step 2.

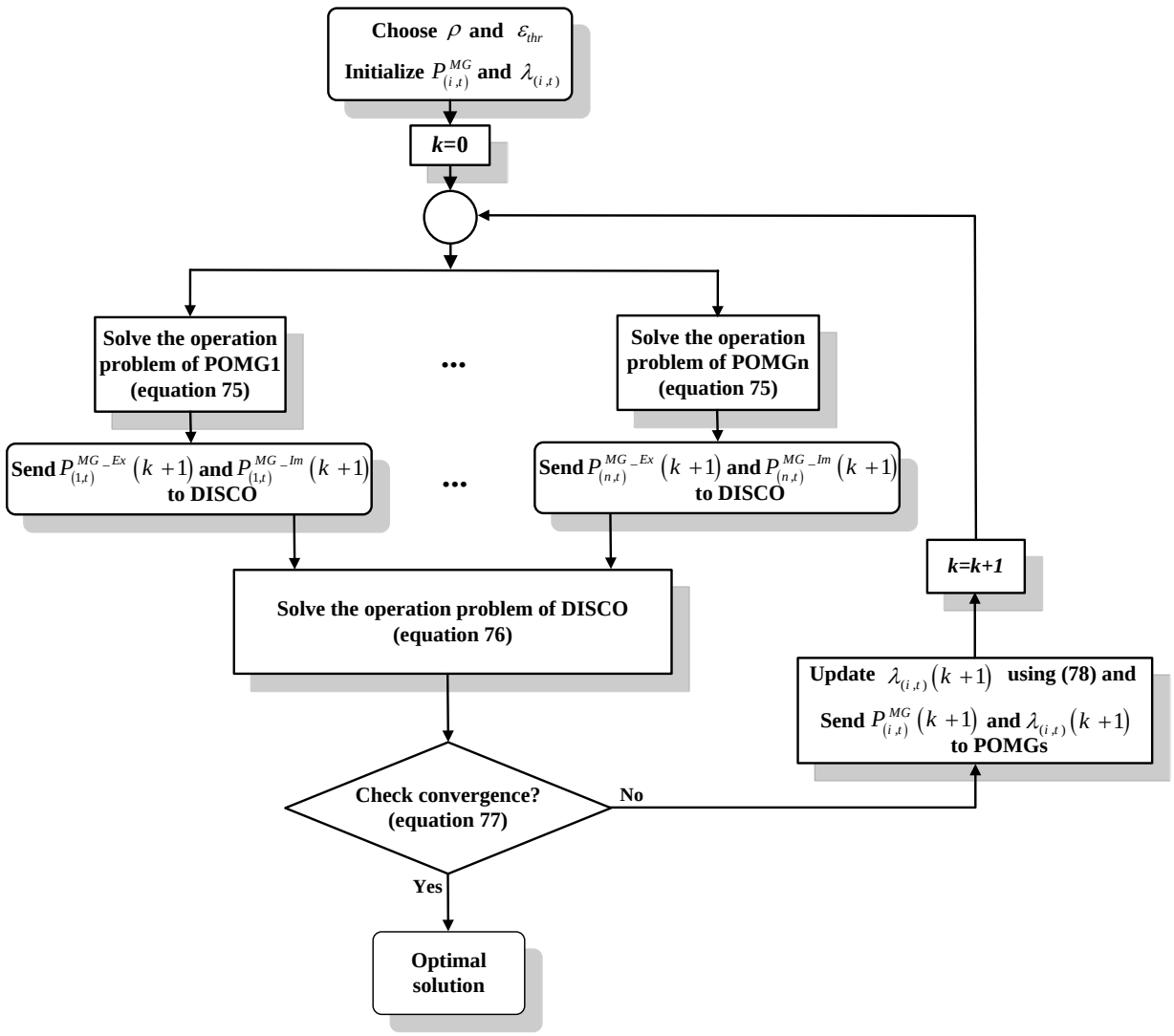

Fig. 2. Flowchart of proposed decentralized robust model

It should be mentioned that the proposed decentralized robust based model is essentially non-convex because of the binary variables; authors of [25] have proved that the ADMM method converges linearly in a finite number of iterations for MILP problems. Meanwhile, some heuristic methods such the alternating optimization procedure [26] and the relax-round-polish procedure [27] can be used if the ADMM based model cannot converge in a suitable number of

iterations. However, this issue is out of scope of this paper and hereby we do not intend to further discuss this topic.

5. Simulation Results

5.1 System data

The proposed model is examined on a modified IEEE 33-bus distribution network with three PMGs which is shown in Fig. 3. The capacity of the substation is limited to 30 MVA. Meanwhile, the exchanged power between DN and PMG1, PMG2, and PMG3 are restricted to ±5, ±4, and ±3 MW, respectively. The thermal limit of feeders and voltage deviation of the buses of DN are considered 10 MVA and ±5%, respectively. The other required data of DN could be retrieved from [28]. The technical and economic information of non-renewable DGs installed in the DN and PMGs are described in Table I [29, 30]. It should be mentioned that all the DGs generate active power at unity power factor. The scaled down wholesale market price and demand on July 18, 2013, recorded at NYISO's PJM are utilized to evaluate optimal operation of DN as shown in Fig. 4 [31]. It should be mentioned that 10% uncertainty is assumed for wholesale market price.

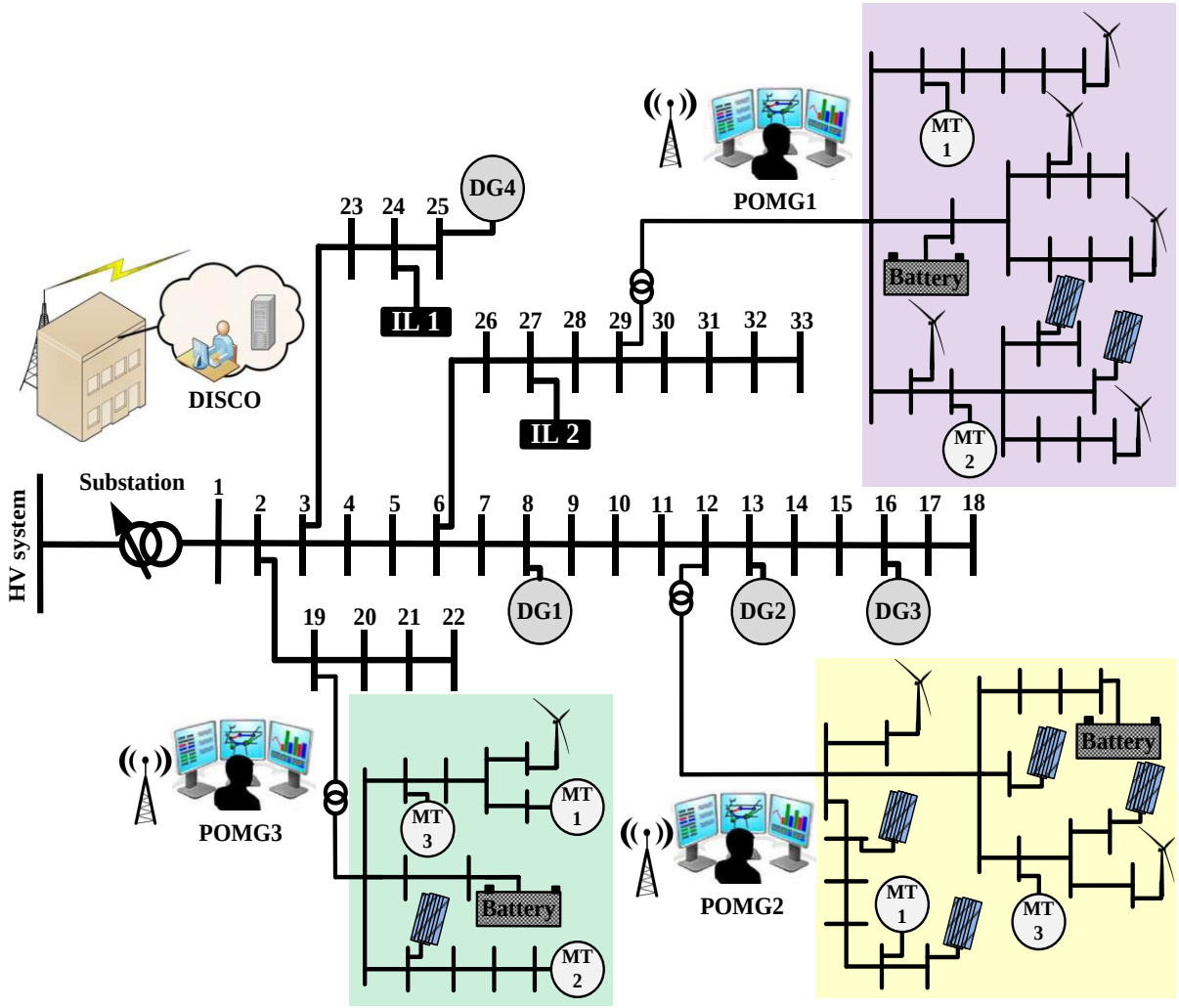

Fig. 3. Single line diagram of test system

Table 1. Parameters of non-renewable DGs

Parameter	Resources of DISCO				Resources of PMGs		
	DG1	DG2	DG3	DG4	MT1	MT2	MT3
$\underline{P}$ (MW)	0.5	0.55	0.55	0.5	0.1	0.1	0.1
$\overline{P}$ (MW)	5	4	3.5	4.5	2	1	1
UR (MW/h)	2.5	2	2	2.5	2	1	1
DR (MW/h)	2.5	2	2	2.5	2	1	1
γ ($/MW^2)	5.7	6.8	6.5	6.2	2.035	0.5768	1.1825

$\beta(\$/\mathrm{MW})$	55.3	53.2	54	53.8	60.28	57.783	61.340
$\alpha(\$)$	34	33.5	34.5	32.8	44	33	44
$UT\ (h)$	2	2.5	2	2.5	2	1	1
$DT\ (h)$	2	2.5	2	2.5	2	1	1
$SUC\ (\$)$	15	13	15	13	150	30	30
$SDC\ (\$)$	10	9	10	9	10	10	10

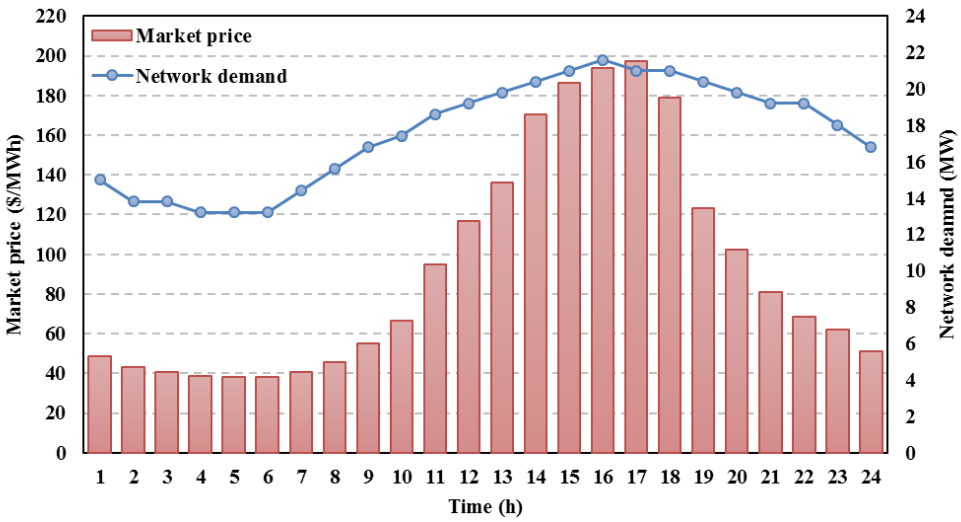

Fig. 4. Hourly network demand and market price

The share of each bus from hourly demand of DN is presented in Fig. 5. It is supposed that all loads of system have a constant power factor of 0.95 lagging. Meanwhile, the step-wise bid-quantity offer package of ILs is given in Table II.

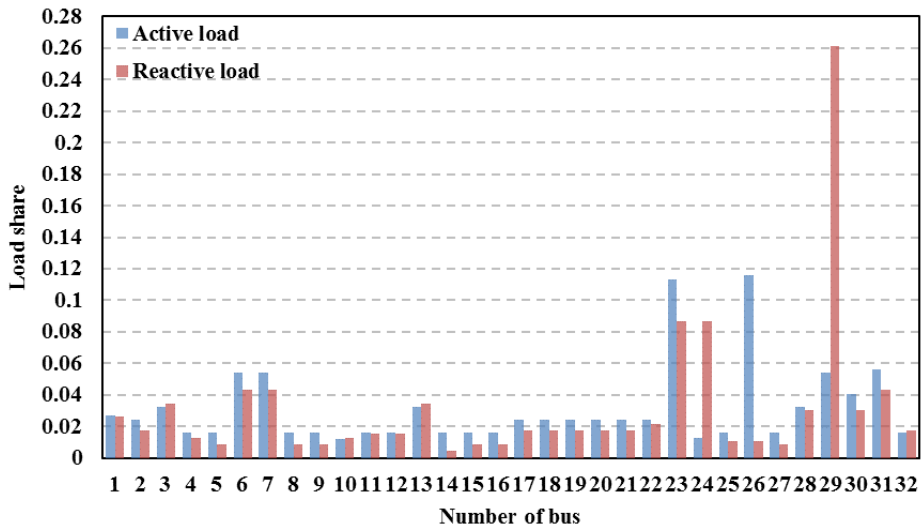

Fig. 5. Load share of each bus from hourly network demand

Table 2. step-wise bid-quantity offer of package of ILs

IL	Quantity (MW)/ Price ($/MWh)			
IL1	0.5/55.20	1.0/110.40	1.5/165.60	2.0/220.80
IL2	0.3/33.12	0.6/66.24	0.9/99.36	1.2/132.48

The hourly demand of PMGs' consumers is shown in Fig. 6 [32]. As mentioned before, the consumers of PMGs are price responsive which their price elasticities are retrieved from [33] and presented in Table 3. It should be mentioned that the average price of wholesale market (i.e. 92 $/MWh) is considered as based energy price (i.e. $\pi_{0(i)}^{MG}$) for responsive consumers of PMGs. Meanwhile, PMGs propose $\lambda_{(i,t)}$ as time-varying prices (i.e. $\pi_{(i,t)}^{MG}$) to their responsive consumers.

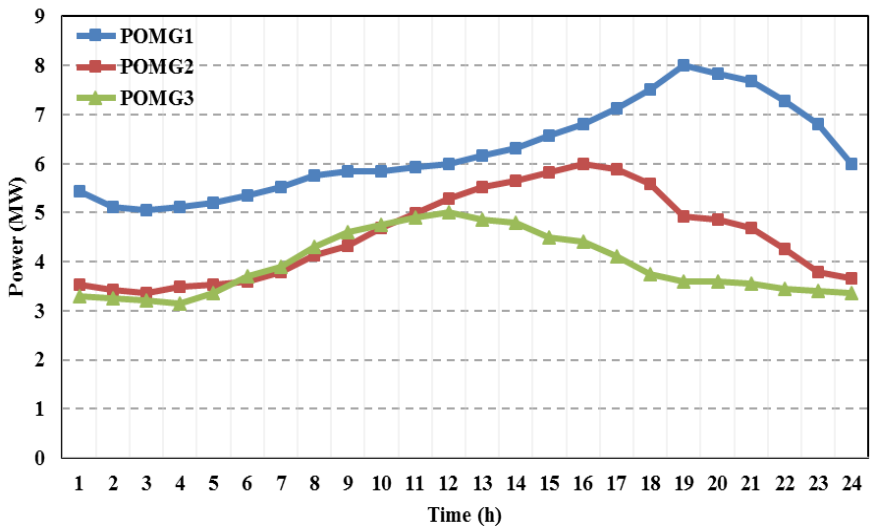

Fig. 6. Hourly demand of PMGs' consumers

Table 3. Price elasticity of consumers

Load type	Time period	On-peak (7-11,17-19)	Mid-peak (11-17)	Off-peak (19-7)
Residential	On-peak	-0.18	0.07	0.06
	Mid-peak	0.07	-0.24	0.1
	Off-peak	0.06	0.1	-0.15
Commercial	On-peak	-0.22	0.08	0.08
	Mid-peak	0.08	-0.26	0.06
	Off-peak	0.08	0.06	-0.1
Industrial	On-peak	-0.24	0.12	0.08
	Mid-peak	0.12	-0.28	0.14
	Off-peak	0.08	0.14	-0.12

The hourly generation of WT and PV are shown in Fig. 7 [34]. It should be mentioned that the installed WTs and PVs in PMGs are the same type. Also, the maintenance cost of WT and PVs

are set 10 and 8 $/MWh, respectively [35]. It is assumed 20% uncertainty for power generation of WTs and PVs.

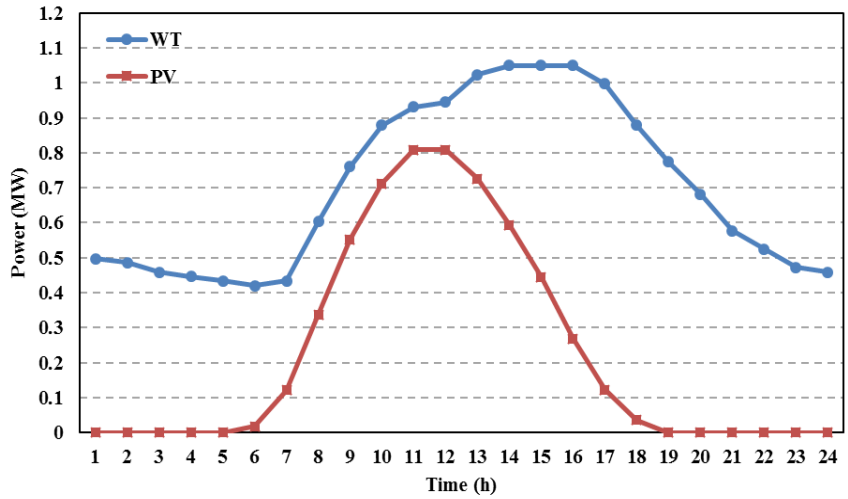

Fig. 7. Hourly generation of WT and PV

The rated capacity of BES in PMG1, PMG2, and PMG3 are 2, 2, and 1 MWh, respectively. The maximum, minimum, and initial SOC of BES are 90%, 10%, and 90% of its capacity. The charging and discharging power of BESs are limited to 0.5 MW. The values of C_{OM}^{BS} and $k_{\underline{SOC}}$ are considered 106.5 $/MWh and 0.15, respectively [36]. Without loss of generality, the value of uncertainty budgets related to wholesale market price and renewable generation, i.e. Γ_0 and $\Gamma_{(t)}$, are assumed 12 and 0.5, respectively. Meanwhile, the penalty factor and threshold of convergence criteria in ADMM based solution algorithm are set to 200 and 0.01, respectively. All case studies are conducted using CPLEX 12.5.1 under GAMS on a 2.50-GHz inter Core i2 CPU personal computer with 4 GB of RAM memory.

5.3 Study results

Fig. 8 shows the total cost of the system and primal residual at each iteration of ADMM based solution algorithm. As can be seen, both total cost of system and primal residual converge rapidly within 4 iterations.

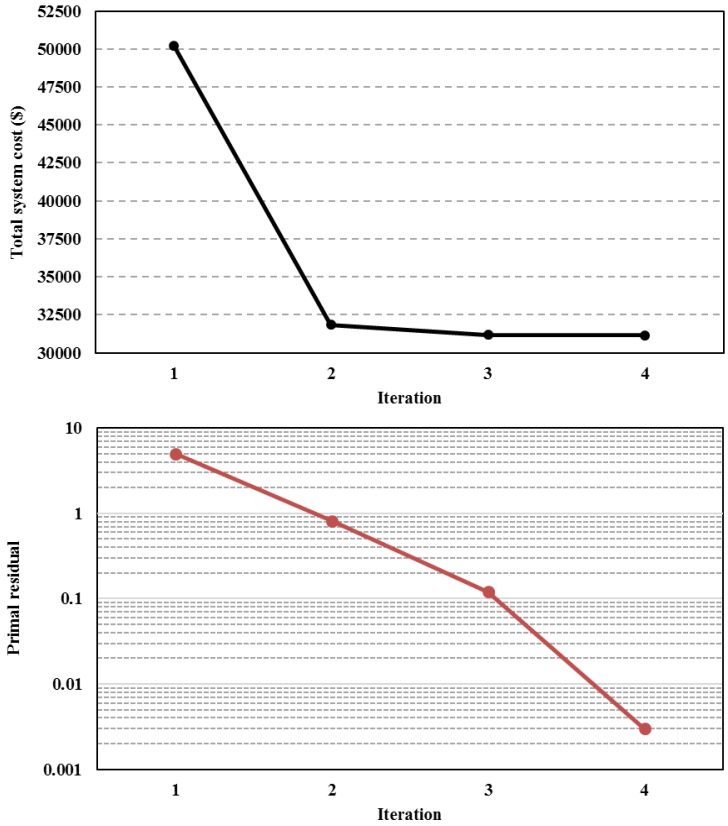

Fig. 8. Total cost of system and primal residual at each iteration

The operation cost of PMGs at each iteration of ADMM based solution algorithm is summarized in Fig. 9. As can be seen, the operation costs of PMGs at first iteration are higher than those at final iteration. This is due to the fact that coordinated dispatch strategies are not announced to PMGs by DISCO at first iteration. Under such circumstances, PMGs could not exchange power with DN and therefore, they should supply their demand with internal resources. After proceeding the iterations, DISCO coordinates the exchanged power of PMGs with DN, therefore, the operation cost of PMGs decrease until the optimal results are obtained. Meanwhile, with coordination of PMGs the total cost of the system is also reduced as illustrated in Fig. 8.

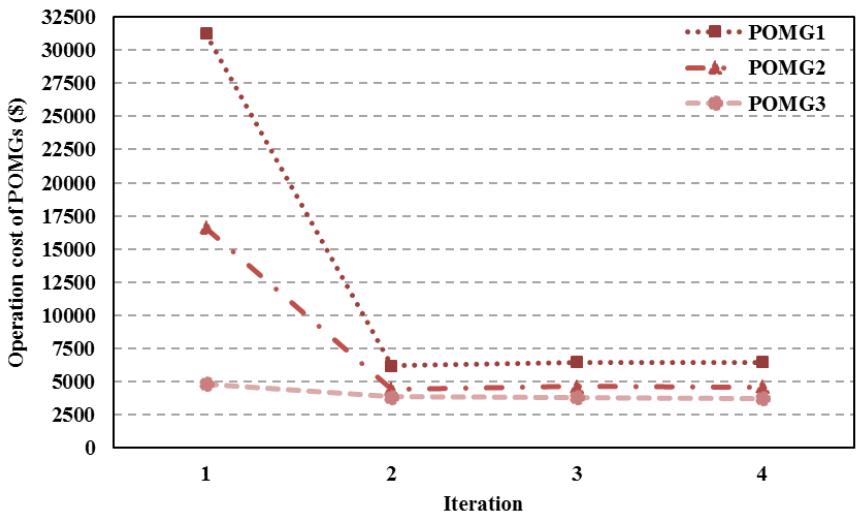

Fig. 9. Operation cost of PMGs at each iteration

The optimized hourly prices of exchanged power between PMGs and DISCO, i.e. $\lambda_{(i,t)}$, are shown in Fig. 10. As it is evident, DISCO offers low prices during off-peak period and high prices during on-peak period. This strategy motivates PMGs to import more power from DN during off-peak period and export more power to DN during on-peak hours. Meanwhile, hourly price of exchanged power for each PMG are different during operating horizon. It is due to the fact that response of PMGs to hourly price depends on their characteristics (i.e. loads, DGs, and etc.) which are different for each PMG.

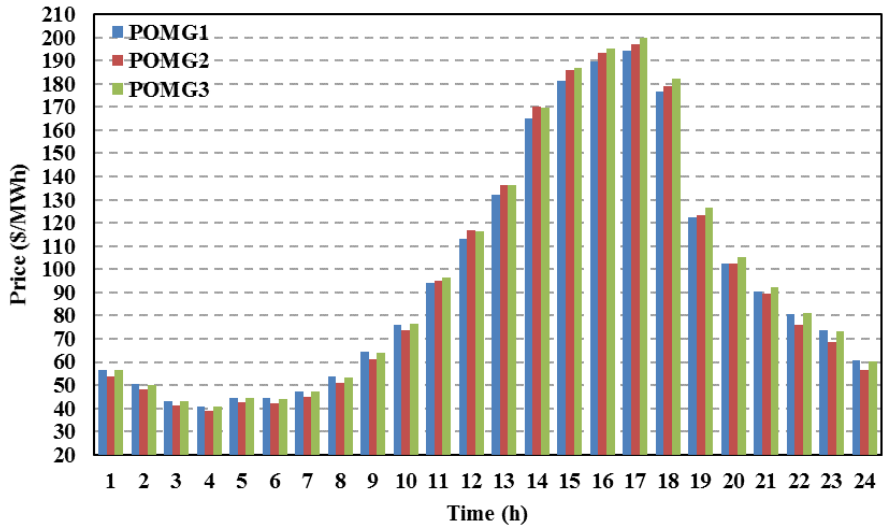

Fig. 10. Optimized hourly prices of exchanged power

The optimized hourly demand of PMGs is shown in Fig. 11. As can be seen, the demand of PMGs is reduced during the hours 11-21, and increased during the hours 1-10 and 22-24. This is due to the fact that responsive consumers of PMGs shift their consumption from relatively high price hours to low price hours. This strategy allows PMGs to sell more power during high price hours to DISCO. Accordingly, DISCO purchases less power from the wholesale market at high price hours to meet demand of DN. Therefore, operating costs of PMGs and DISCO both are reduced.

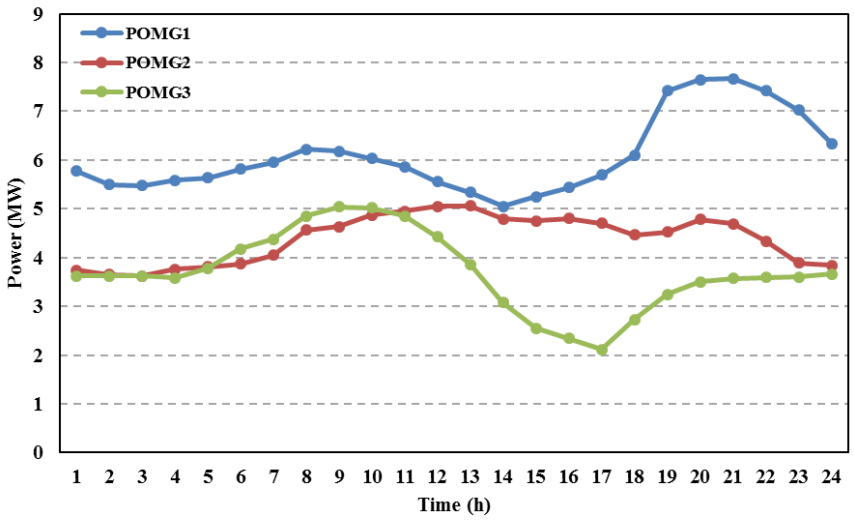

Fig. 11. Optimized hourly demand of PMGs' consumers

The operation results of PMGs are given in Figs. 12, 13, and 14. According to these Figures, WTs and PVs generate their maximum available power. However, MTs are scheduled regarding to prices of exchanged power. As can be seen, MTs are scheduled more during relatively high price hours 11-21. Also, the BESs are discharged during relatively high-price hours (i.e. 14-16) and charged during relatively low-price hours (i.e. 9-11 and 22-24). PMG1 purchases energy at hours 1-10 and 18-24, due to lower prices of exchanged power. With increasing the prices of

exchanged power and share of renewables generation, PMG1 sells power at hours 11-17. Note that the exchanged power is limited to 5 MW. The same argument can be made for other PMGs.

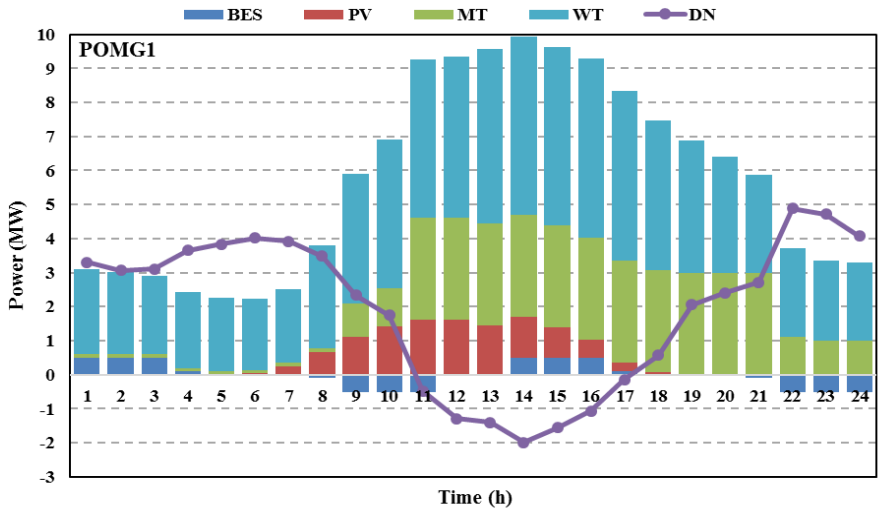

Fig. 12. Operation results of PMG1

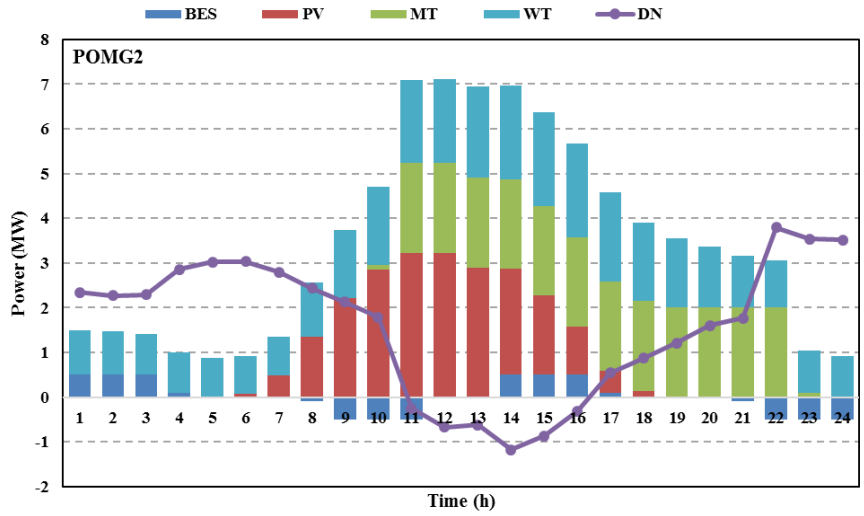

Fig. 13. Operation results of PMG2

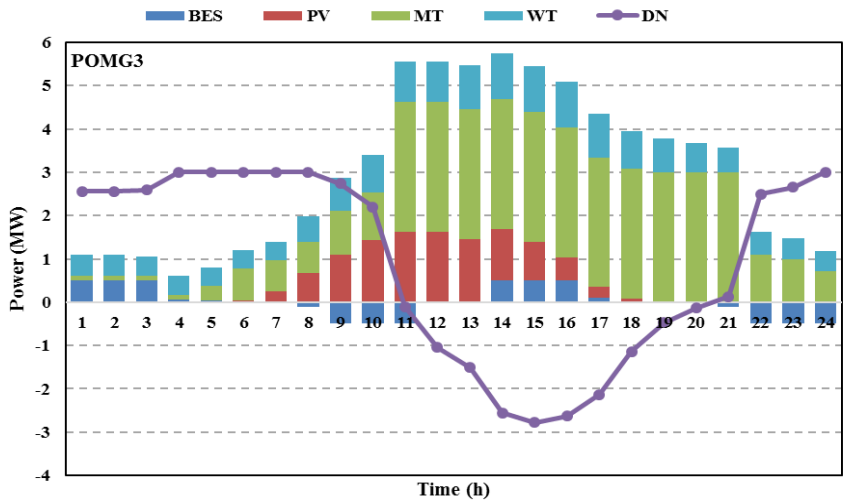

Fig. 14. Operation results of PMG3

The operation results of DISCO are shown in Fig. 15. As can be seen, DISCO purchases more power from the market during low price hours, namely hours 1-11 and 18-24, to supply demands of DN and PMGs. However, as the market prices and demand of DN increase, namely hours 11-17, the DISCO schedules more capacity of DGs and invokes more ILs and purchases power from PMGs. These strategies decreases purchased power from the market during high price periods and therefore, reduce the total cost of the system. Note that during 11-17, due to high market prices, DISCO sells power to the market to make more profit.

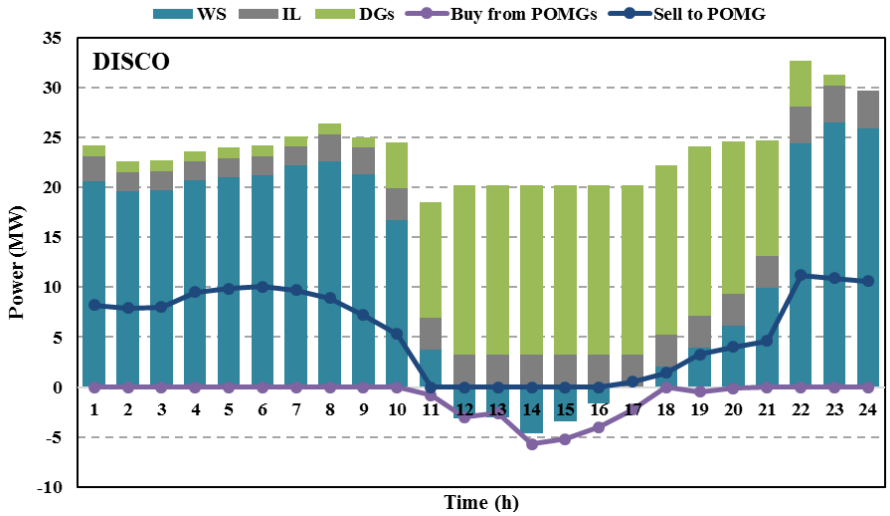

Fig. 15. Operation results of DISCO

To further analyze the optimality and performance of the proposed model, the results are compared with the centralized and uncoordinated models. It should be mentioned that in centralized model, DISCO solves the problem (48)-(54) as the main entity responsible for optimal operating of DN and PMGs. In uncoordinated model, each PMG schedules its resources and determines exchanged power with DN, considering the wholesale prices. The obtained results are summarized in Table 4. For both proposed and centralized models, the results are almost the same as provided in Table 4. It's obvious that the ADMM based solution algorithm can converge to the optimal solution, since the operating cost of entities are only slightly higher than those of the centralized one (by 0.08%). Note that the centralized model is rarely applicable in real practices due to technical and ownership concerns, as explained in Section 5, and the aim of this comparison is to confirm the optimality of the solution obtained from the proposed decentralized method.

To check the economic benefits of the proposed model, it is compared with the uncoordinated model. It is obvious that the operation costs of all entities in the uncoordinated model are clearly higher than the proposed decentralized model by 5.21%. It is due to the fact that in the proposed decentralized model, PMGs and DISCO are coordinated and benefit from power exchanging. PMGs with selling more expensive power and DISCO with purchasing cheaper power gain benefit.

Table 4. Comparison of proposed, centralized, and uncoordinated models

Model	Entity	Operation Cost ($)	Exchanged power cost of entity ($)			
			With DISCO		With Market	
			Buy	Sell	Buy	Sell

Proposed	PMG1	6536	3479	1250	-	-
	PMG2	4727	2778	591	-	-
	PMG3	3768	1906	2492	-	-
	DISCO	31184	-	-	9699	2544
Centralized	PMG1	6526	3482	1247	-	-
	PMG2	4723	2785	590	-	-
	PMG3	3758	1907	2486	-	-
	DISCO	31158	-	-	9691	2547
Un-coordinated	PMG1	7050	-	-	3744	1286
	PMG2	488	-	-	2984	604
	PMG3	3988	-	-	2068	2564
	DISCO	32811	-	-	10015	2584

5.3 Impacts of uncertainties

The impact of uncertain variables, i.e. renewable generation and market price, on the results of the proposed model are investigated in this section. To this end, for different degrees of uncertainties, e.g. $0.1\xi_{(\bullet)}$, $0.5\xi_{(\bullet)}$ and $0.9\xi_{(\bullet)}$, the value of uncertainty budgets, i.e. $\Gamma_{(\bullet)}$, are increased and then, the ADMM based solution algorithm is performed for each case. The variation in the operation cost of DISCO and total operation cost of PMGs are shown in the following figures. As can be seen in Figs. 16 and 17, with increasing the values of ξ^{WS} and Γ_0, operation cost of DISCO and total operation cost of PMGs both grow. The reason is that with increasing the values of ξ^{WS} and Γ_0, the robustness of DISCO operation problem against the uncertainty of wholesale price is increased and therefore, DISCO prefers to provide more power

from reliable resources, e.g. diesel generators, instead of wholesale market with higher cost to meet demand of DN and PMGs. Accordingly, the prices of exchanged power between DISCO and PMGs are increased which increases the operation cost of PMGs.

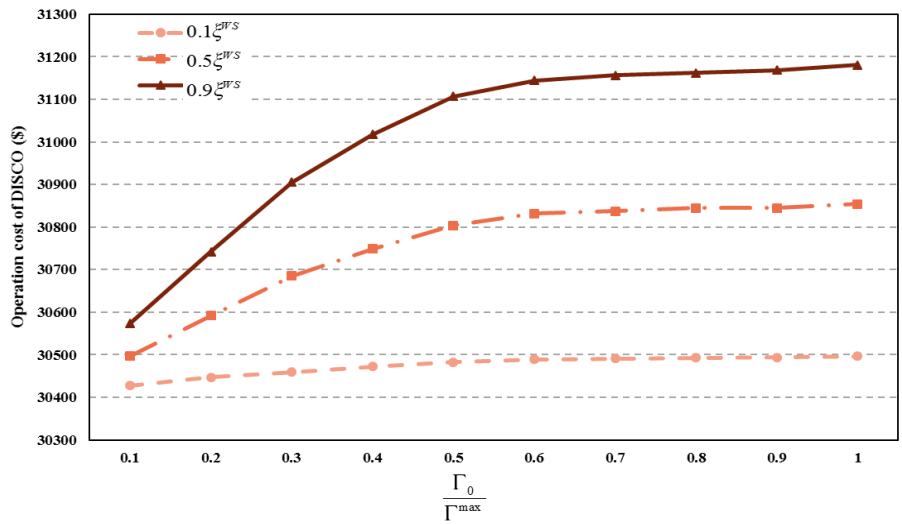

Fig. 16. Impact of ξ^{WS} and Γ_0 on operation cost of DISCO

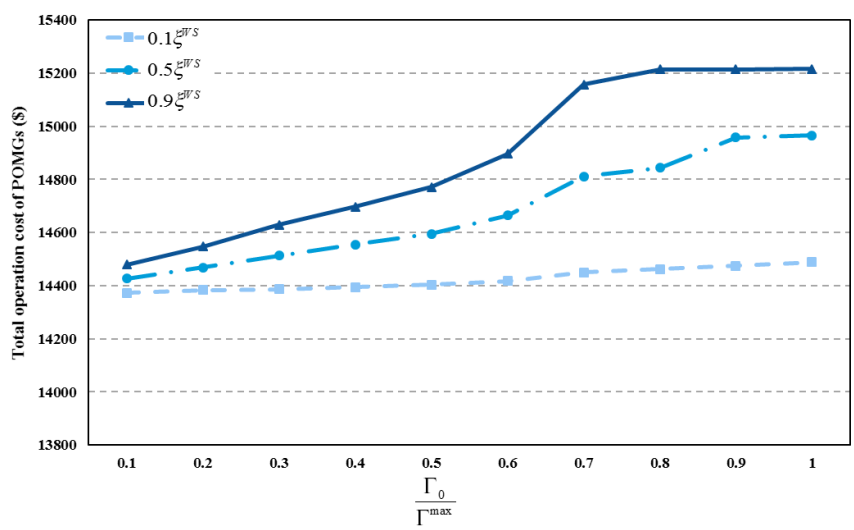

Fig. 17. Impact of ξ^{WS} and Γ_0 on total operation cost of PMGs

As can be seen in Figs. 18 and 19, with increasing the values of ξ^{REN} and $\Gamma_{(t)}$, operation cost of DISCO and total operation cost of PMGs both grow. This is due to the fact that with increasing the values of ξ^{REN} and $\Gamma_{(t)}$, the robustness of PMG operation problem against the

uncertainty of renewable generation is increased and therefore, PMGs are forced to schedule less power of renewable generation during operating horizon which increases their operation cost, significantly. Accordingly, the exported power of PMGs to DN is also reduced which leads to higher operation cost for DISCO.

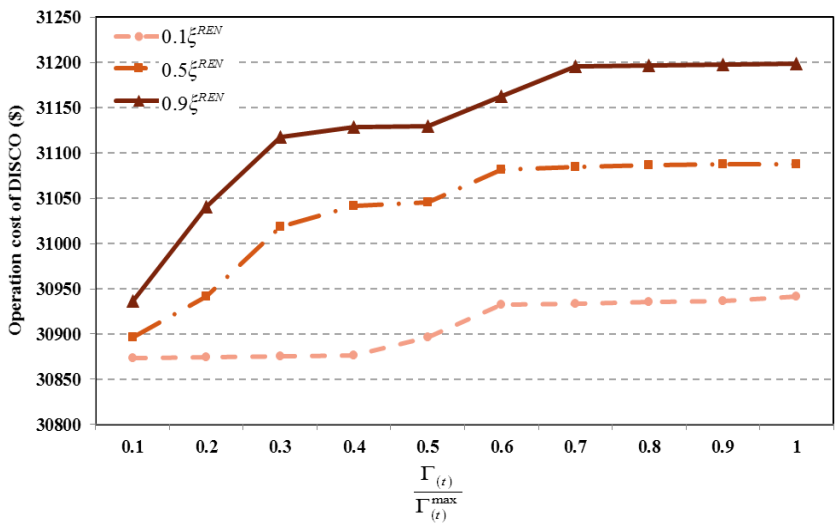

Fig. 18. Impact of ξ^{REN} and $\Gamma_{(t)}$ on operation cost of DISCO

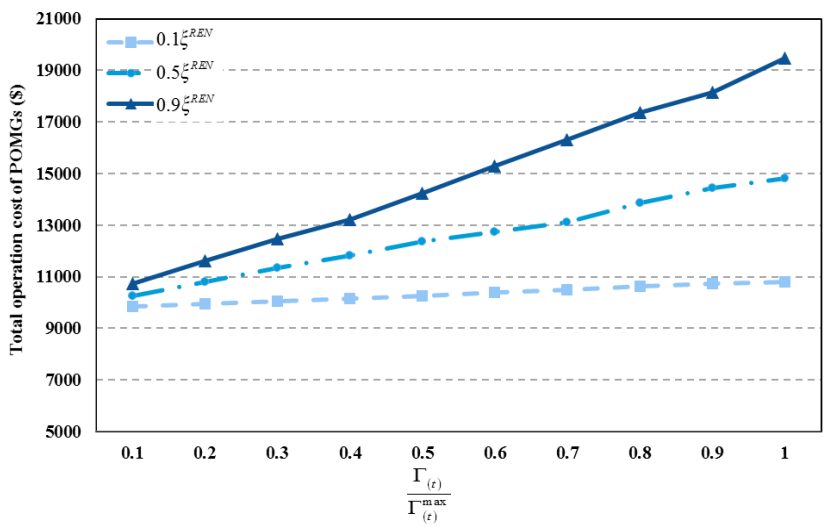

Fig. 19. Impact of ξ^{REN} and $\Gamma_{(t)}$ on total operation cost of PMGs

It could be concluded that as $\xi_{(\bullet)}$ and $\Gamma_{(\bullet)}$ increase, the operation cost of DISCO and PMGs also increase which is consistent with the tradeoff between optimality and robustness of solutions.

5.4 After-the-fact analysis

To justify the robustness of the proposed model, an after-the-fact analysis is presented. In this analysis, 1500 scenarios of uncertain variables, normally distributed, are randomly generated using Monte Carlo (MC) sampling method in a way that (50) and (51) are satisfied. Then, the operation costs of DISCO and PMGs are calculated by solving (1) to (30) for each scenario of wind power generation and market price. The results are shown in the Fig. 20. As can be seen, the operation costs of DISCO and PMGs are lower than those which are obtained by the proposed model as indicated in Table 4 (the optimal operation costs of DISCO, PMG1, PMG2, PMG3, and PMG4 are 31184, 6536, 4727, and 3768, respectively). Therefore, it is confirmed that if uncertain variables fall inside their uncertainty sets, the operation costs of DISCO and PMGs are always lower than the costs obtained by the proposed decentralized robust model.

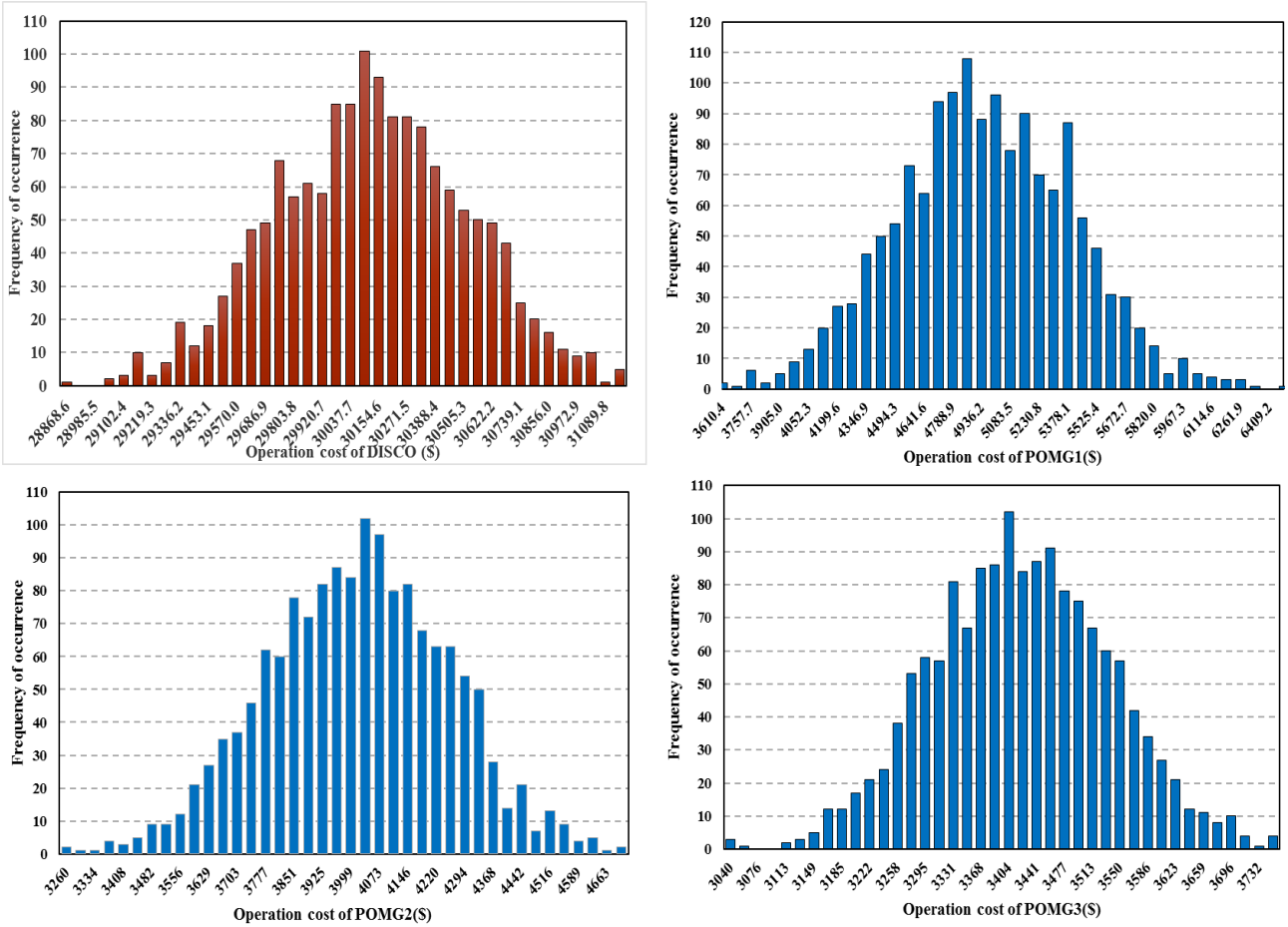

Fig. 20. results of after-the-fact analysis

6. Conclusion

This paper proposed a decentralized robust model for optimal operation of DISCOs with private microgrids. To this end, robust optimization approach was utilized with objective of minimizing total cost of the system under the worst case uncertainties associated with wholesale market price and renewable generation. Then, a solution algorithm based on ADMM method was developed in which DISCO and PMGs were considered as independent entities and minimize their own operation costs, individually. The obtained results proved that the proposed model can successfully converge in finite number of iterations. Likewise, the obtained results confirmed the effectiveness of the proposed model in operational coordination between DISCO and PMGs

while the ownership of each entity was preserved. The performance and optimality of the proposed model were evaluated by presenting a comparison between these results and those obtained from centralized and uncoordinated models. It was shown that the accuracy of results of the proposed model is satisfactory and coordination between DISCO and PMGs bring economic benefit for both. Impact of uncertainties was also studied and revealed that increasing the uncertainties would lead to more operation costs of DISCO and PMGs which can be decreased by properly tuning the uncertainty budgets. Moreover, the robustness of the proposed model was justified using the after-the-fact analysis.

References:

[1] A. A. S. Algarni and K. Bhattacharya, "A Generic Operations Framework for Discos in Retail Electricity Markets," *IEEE Transactions on Power Systems*, vol. 24, pp. 356-367, 2009.

[2] A. Safdarian, M. Fotuhi-Firuzabad, and M. Lehtonen, "A Stochastic Framework for Short-Term Operation of a Distribution Company," *IEEE Transactions on Power Systems*, vol. 28, pp. 4712-4721, 2013.

[3] A. Safdarian, M. Fotuhi-Firuzabad, and M. Lehtonen, "Integration of Price-Based Demand Response in DisCos' Short-Term Decision Model," *IEEE Transactions on Smart Grid*, vol. 5, pp. 2235-2245, 2014.

[4] Q. Wang, C. Zhang, J. Wang, P. Pinson, and J. Ostergaard, "Real-Time Trading Strategies of Proactive DISCO with Heterogeneous DG Owners," *IEEE Transactions on Smart Grid*, vol. PP, pp. 1-1, 2016.

[5] H. Khazaei, B. Vahidi, S. H. Hosseinian, and H. Rastegar, "Two-level decision-making model for a distribution company in day-ahead market," *IET Generation, Transmission & Distribution*, vol. 9, pp. 1308-1315, 2015.

[6] W. Shi, X. Xie, C. C. Chu, and R. Gadh, "Distributed Optimal Energy Management in Microgrids," *IEEE Transactions on Smart Grid*, vol. 6, pp. 1137-1146, 2015.

[7] G. Liu, Y. Xu, and K. Tomsovic, "Bidding Strategy for Microgrid in Day-Ahead Market Based on Hybrid Stochastic/Robust Optimization," *IEEE Transactions on Smart Grid*, vol. 7, pp. 227-237, 2016.

[8] E. Kuznetsova, C. Ruiz, Y.-F. Li, and E. Zio, "Analysis of robust optimization for decentralized microgrid energy management under uncertainty," *International Journal of Electrical Power & Energy Systems*, vol. 64, pp. 815-832, 2015/01/01/ 2015.

[9] G. Liu, M. Starke, B. Xiao, X. Zhang, and K. Tomsovic, "Microgrid optimal scheduling with chance-constrained islanding capability," *Electric Power Systems Research*, vol. 145, pp. 197-206, 2017/04/01/ 2017.

[10] N. Nikmehr, S. Najafi-Ravadanegh, and A. Khodaei, "Probabilistic optimal scheduling of networked microgrids considering time-based demand response programs under uncertainty," *Applied Energy*, vol. 198, pp. 267-279, 2017/07/15/ 2017.

[11] Y. Liu, Y. Li, H. B. Gooi, J. Ye, H. Xin, X. Jiang, *et al.*, "Distributed Robust Energy Management of a Multi-Microgrid System in the Real-Time Energy Market," *IEEE Transactions on Sustainable Energy,* vol. PP, pp. 1-1, 2017.

[12] Y. Z. Li, T. Zhao, P. Wang, H. B. Gooi, L. Wu, Y. Liu, *et al.*, "Optimal Operation of Multi-Microgrids via Cooperative Energy and Reserve Scheduling," *IEEE Transactions on Industrial Informatics,* vol. PP, pp. 1-1, 2018.

[13] N. Nikmehr and S. Najafi Ravadanegh, "Reliability evaluation of multi-microgrids considering optimal operation of small scale energy zones under load-generation uncertainties," *International Journal of Electrical Power & Energy Systems,* vol. 78, pp. 80-87, 2016/06/01/ 2016.

[14] N. Nikmehr and S. N. Ravadanegh, "Optimal Power Dispatch of Multi-Microgrids at Future Smart Distribution Grids," *IEEE Transactions on Smart Grid,* vol. 6, pp. 1648-1657, 2015.

[15] T. Lv, Q. Ai, and Y. Zhao, "A bi-level multi-objective optimal operation of grid-connected microgrids," *Electric Power Systems Research,* vol. 131, pp. 60-70, 2016/02/01/ 2016.

[16] H. S. V. S. K. Nunna and D. Srinivasan, "Multi-Agent based Transactive Energy Framework for Distribution Systems with Smart Microgrids," *IEEE Transactions on Industrial Informatics,* vol. PP, pp. 1-1, 2017.

[17] A. K. Marvasti, Y. Fu, S. DorMohammadi, and M. Rais-Rohani, "Optimal Operation of Active Distribution Grids: A System of Systems Framework," *IEEE Transactions on Smart Grid,* vol. 5, pp. 1228-1237, 2014.

[18] M. Mazidi, H. Monsef, and P. Siano, "Robust day-ahead scheduling of smart distribution networks considering demand response programs," *Applied Energy,* vol. 178, pp. 929-942, 2016.

[19] A. Hussain, V.-H. Bui, and H.-M. Kim, "Robust optimization-based scheduling of multi-microgrids considering uncertainties," *Energies,* vol. 9, p. 278, 2016.

[20] H. Qiu, B. Zhao, W. Gu, and R. Bo, "Bi-level Two-stage Robust Optimal Scheduling for AC/DC Hybrid Multi-microgrids," *IEEE Transactions on Smart Grid,* vol. PP, pp. 1-1, 2018.

[21] M. Mazidi, H. Monsef, and P. Siano, "Incorporating price-responsive customers in day-ahead scheduling of smart distribution networks," *Energy Conversion and Management,* vol. 115, pp. 103-116, 2016/05/01/ 2016.

[22] M. Mazidi, H. Monsef, and P. Siano, "Design of a risk-averse decision making tool for smart distribution network operators under severe uncertainties: An IGDT-inspired augment ε-constraint based multi-objective approach," *Energy,* vol. 116, pp. 214-235, 2016/12/01/ 2016.

[23] D. Bertsimas and M. Sim, "The price of robustness," *Operations research,* vol. 52, pp. 35-53, 2004.

[24] S. Boyd, N. Parikh, E. Chu, B. Peleato, and J. Eckstein, "Distributed optimization and statistical learning via the alternating direction method of multipliers," *Foundations and Trends® in Machine Learning,* vol. 3, pp. 1-122, 2011.

[25] Z. Li, W. Wu, B. Zeng, M. Shahidehpour, and B. Zhang, "Decentralized Contingency-Constrained Tie-Line Scheduling for Multi-Area Power Grids," *IEEE Transactions on Power Systems,* vol. 32, pp. 354-367, 2017.

[26] Z. Li, M. Shahidehpour, W. Wu, B. Zeng, B. Zhang, and W. Zheng, "Decentralized Multiarea Robust Generation Unit and Tie-Line Scheduling Under Wind Power Uncertainty," *IEEE Transactions on Sustainable Energy,* vol. 6, pp. 1377-1388, 2015.

[27] C. He, L. Wu, T. Liu, and M. Shahidehpour, "Robust Co-Optimization Scheduling of Electricity and Natural Gas Systems via ADMM," *IEEE Transactions on Sustainable Energy,* vol. 8, pp. 658-670, 2017.

[28] M. E. Baran and F. F. Wu, "Optimal capacitor placement on radial distribution systems," *IEEE Transactions on Power Delivery,* vol. 4, pp. 725-734, 1989.

[29] M. Khojasteh and S. Jadid, "Decision-making framework for supplying electricity from distributed generation-owning retailers to price-sensitive customers," *Utilities Policy,* vol. 37, pp. 1-12, 2015/12/01/ 2015.

[30] S. Chen, H. B. Gooi, and M. Wang, "Sizing of energy storage for microgrids," *IEEE Transactions on Smart Grid*, vol. 3, pp. 142-151, 2012.

[31] J. New York Independent System Operator, 2013, online available at: and <http://www.nyiso.com/public/markets_operations/index.jsp>.

[32] M. Mazidi, A. Zakariazadeh, S. Jadid, and P. Siano, "Integrated scheduling of renewable generation and demand response programs in a microgrid," *Energy Conversion and Management*, vol. 86, pp. 1118-1127, 2014.

[33] M. Doostizadeh and H. Ghasemi, "A day-ahead electricity pricing model based on smart metering and demand-side management," *Energy*, vol. 46, pp. 221-230, 2012.

[34] M. Q. Wang and H. B. Gooi, "Spinning Reserve Estimation in Microgrids," *IEEE Transactions on Power Systems*, vol. 26, pp. 1164-1174, 2011.

[35] C. Zhang, Y. Xu, Z. Y. Dong, and J. Ma, "Robust Operation of Microgrids via Two-Stage Coordinated Energy Storage and Direct Load Control," *IEEE Transactions on Power Systems*, vol. 32, pp. 2858-2868, 2017.

[36] C. Battistelli, Y. P. Agalgaonkar, and B. C. Pal, "Probabilistic Dispatch of Remote Hybrid Microgrids Including Battery Storage and Load Management," *IEEE Transactions on Smart Grid*, vol. 8, pp. 1305-1317, 2017.