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An Improved Virtual Inertia Control for Three-Phase Voltage Source Converters Connected to a Weak Grid

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Abstract—The still increasing share of power converter-based renewable energies weakens the power system inertia. The lack of inertia becomes a main challenge to small-scale modern power systems in terms of control and stability. To alleviate such adverse effects from inertia reductions, e.g., undesirable load shedding and cascading failures, three-phase grid-connected power converters should provide virtual inertia upon system demands. This can be achieved through directly linking the grid frequency and voltage references of DC-link capacitors/ultracapacitors. This paper reveals that the virtual inertia control may possibly induce instabilities to the power converters under weak grid conditions, which is caused by the coupling between the d- and q-axis as well as the inherent differential operator introduced by the virtual inertia control. To tackle this instability issue, this paper proposes a modified virtual inertia control to mitigate the differential effect, and thus alleviating the coupling effect to a great extent. Experimental verifications are provided, which demonstrate the effectiveness of the proposed control in stabilizing three-phase grid-connected power converters for inertia emulation even when connected to weak grids.

Index Terms—Frequency regulation, renewable energy, stability, virtual inertia, weak grid, power converter.

I. INTRODUCTION

The main driving forces to deploy and develop a large amount of renewable energies integrated into modern power systems include the reduction of carbon footprint and increment of clean energy production [1]. Despite being pursued worldwide, the employment of renewable energies even partly to replace fossil fuels (expected to completely phase out in the future) may retrofit the entire power grid and challenge the stability of modern power systems [2]. One of the major concerns, which has already been acknowledged in small-scale power systems, e.g., in Ireland and Great Britain, refers to the frequency instability due to the lack of inertia in the system with a high penetration level of power converter-interfaced renewable energies [3].

In conventional power systems, synchronous generators operating at a speed in synchronism with the grid frequency act as the major grid interfaces [4]. When a frequency change due to the power imbalance between generation and demand occurs, synchronous generators autonomously slow down or speed up in accordance with the grid frequency. In this way, the synchronous generators release/absorb the energy to/from the power grid so that the power mismatch can partially be compensated. This effect is quantitatively evaluated by the per unit kinetic energy, which is defined as power system inertia [4], [5]. However, this phenomenon changes in modern power systems, because most of renewable energy sources (RESs), such as wind energies and solar photovoltaics (PV), are coupled to the power grid through power electronic converters [1], [6]. Different from synchronous generators, grid-connected power converters normally operate in the maximum power point tracking (MPPT) mode to optimize the energy yield without any inertia contribution [7]. This is because there is no kinetic energy in such power conversion systems that can be used as rotational inertia [8]. As more synchronous generators are phasing out and being replaced by power converters, the entire power system becomes more inertia-less, being a major concern for stability and control. More specifically, without sufficient inertia, the grid frequency and/or the rate-of-change-of-frequency (RoCoF) may be apt to go beyond the acceptable range under severe frequency events, leading to generation tripping, undesirable load shedding, or even system collapses [9].

Various solutions to tackle the lack of inertia issue have already been proposed in the literature. One straightforward approach is to change the requirements of RoCoF withstand capabilities of generators [3]. Although this solution has been accepted as an efficient and suitable solution by the system operators in Ireland/North Ireland [10], the high costs associated with generator testing mainly hinder its wide applications. In addition, since this solution involves no effort
for inertia enhancement, the inertia issue cannot be completely resolved. Another possibility for inertia enhancement is the use of synchronous condensers, i.e., synchronous generators without prime movers or loads [11]. However, high capital and operating costs have deterred the widespread adoption of synchronous condensers.

Similar to synchronous generators, wind turbines also feature rotating masses and the associated kinetic energy. However, their rotating speeds are usually decoupled from the grid frequency for a better speed control to optimize the energy harvesting [12], [13]. In this regard, variable-speed wind generation systems normally contribute zero synchronous inertia to the power grid. To exploit the stored kinetic energy in wind turbines, the electromagnetic torque (or grid-injected power) can be changed in response to the grid frequency during frequency events [13]. Through this approach, the emulated inertia or virtual inertia will be synthesized by wind turbines. In [14], the electromagnetic torque is proportionally linked to the RoCoF (i.e., df / dt) for inertia emulation. References [15–17] propose several modified virtual inertia controls without considering the aerodynamics and speed recovery processes of wind turbines. As analyzed by [18] and [19], speed recovery processes will greatly change the inertia response of wind turbines and may cause a recurring frequency dip or even rotor stall. As such, the emulated inertia from wind turbines and synchronous inertia are still not identical, as proven by the wind inertial response in Hydro-Quebec [3].

Another emerging approach aims to generate virtual inertia through battery storage systems. This can be achieved by introducing a proportional link between RoCoF (i.e., df / dt) and battery power references [20–22]. As mentioned, similar inertia emulation schemes have already been applied to wind turbines [23], [24]. The formulation of battery power references necessitates the fast and accurate detection of RoCoF signals, which is considered as a significant challenge by the power system operators in Ireland and Great Britain [3]. To tackle this issue, the possibility of using a frequency-locked-loop (PLL) for RoCoF detection is discussed in [21]. In addition, it is revealed that power converters may have instability concerns when adopting the df / dt-based inertia emulation scheme [25]. [26]. Furthermore, slow inertia emulation (which can be achieved through the use of a first-order lag of a time constant around 1 s) can address the instability concerns [26]. The analysis and conclusion provided in [26] are of great importance and can provide useful guidelines for virtual inertia design.

As compared to batteries, ultracapacitors feature a higher power density and are therefore suitable for inertia emulation [27]. In addition to ultracapacitors, capacitors are normally necessary in the DC-links of grid-connected power converters for voltage support and harmonic filtering [28]. These capacitors may also be exploited to reap the benefit of their capabilities for inertia emulation with a small or even no change of hardware [29], [30]. Regarding control implementations, the inertia emulated by capacitors and ultracapacitors are very similar. Because of adjustable capacitor and ultracapacitor voltages, RoCoF detection is no longer necessary. By proportionally linking the capacitor voltage and the grid frequency, virtual inertia can be expected from the capacitors/ultracapacitors [29], [31], [32]. This approach can be very effective and simple in terms of control designs. Therefore, it has been regarded as a promising solution and extended to high-voltage direct current (HVDC) applications such as modular multilevel converters (MMCs) [33–35].

It is known that weak grids featuring large grid impedances may make three-phase grid-connected power converters unstable due to the resonance caused by high-order passive filters [36], coupling among multiple power converters [37], and influence of the phase-locked-loop (PLL) on the current control [38]. In addition to the above-mentioned causes, this paper reveals that the strong coupling between the control loops (i.e., the dq-reference control systems) and the differential operator are responsible for the instability of grid-connected power converters with virtual inertia from capacitors/ultracapacitors.

The rest of this paper is organized as follows. Section II presents the concept of virtual inertia and the fundamentals of three-phase power converters with the virtual inertia control. With the system models, the instability issue is further explored in Section III. Section IV introduces the proposed modified virtual inertia control for stability enhancement. The effectiveness of the proposed control to enhance the stability under weak grids is experimentally verified in Section V. Finally, Section VI provides the concluding remarks.

II. THREE-PHASE GRID-CONNECTED POWER CONVERTERS WITH VIRTUAL INERTIA CONTROL

Power electronics is the key to renewable energy integration, and power converters as grid interfaces are replacing conventional synchronous generators. However, presently, the power system inertia is solely contributed by the rotating masses of synchronous generators. As a result, the lack of inertia may challenge the operation and control of modern power systems [3], [9].

A. Concept of Virtual Inertia

To address the issue of less inertia in more-electronics power systems, capacitors and/or ultracapacitors have been increasingly exploited to absorb/release power in a similar way as conventional synchronous generators do in the case of frequency events for inertia emulation [29–32]. Fig. 1 illustrates the mapping between the synchronous generator and the capacitor, where the inertia constant of the synchronous generator \( H \) is defined as the ratio of the kinetic
energy stored in the rotating masses of the synchronous generator \((J\omega_0^2/2)\) to its rated power \(S_{\text{base}}\), which can be expressed as [4]

\[
H = \frac{J\omega_0^2}{2S_{\text{base}}},
\]

where \(J\) denotes the combined moment of inertia of the synchronous generator and turbine. \(\omega_0\) stands for the rated rotor mechanical speed. Similarly, the inertia constant of the capacitor \(H_{\text{cap}}\) can be defined as the ratio of the electrical energy stored in the capacitor \((C_{\text{dc}}V_{\text{dc,ref}}^2/2)\) to its rated power \(S_{\text{base}}\), which can be expressed as [29]

\[
H_{\text{cap}} = \frac{C_{\text{dc}}V_{\text{dc,ref}}^2}{2S_{\text{base}}},
\]

in which \(C_{\text{dc}}\) and \(V_{\text{dc,ref}}\) represent the capacitance and rated capacitor voltage, respectively. The rotor speed \(\omega_r\) (note that the electrical speed \(\omega_e\) equals the mechanical speed \(\omega_m\) for synchronous generators with one pair of poles) and capacitor voltage \(v_{\text{dc}}\) have the same role in determining the corresponding inertia constant, \(H\) and \(H_{\text{cap}}\), as noticed in (1) and (2). Therefore, the virtual inertia with an equivalent inertia coefficient of \(H_{\text{cap}}K_{\text{ov,pu}}\) can be expected from the capacitor after the per unit capacitor voltage and the per unit rotor speed or the grid frequency are linked through the proportional gain \(K_{\text{ov,pu}}\), where [29]

\[
\Delta v_{\text{dc,pu}} = K_{\text{ov,pu}}\Delta \omega_{\text{pu}}.
\]

**B. Power Converters with Virtual Inertia Control**

Fig. 2 shows the schematic diagram of a three-phase grid-connected power converter with the virtual inertia control, where the power grid is modelled as a series connection of an ideal voltage source \(v_{\text{abc}}\) and a grid inductor \(L_g\). In the case of weak grids (large \(L_g\)), the voltage drop across \(L_g\) can be considerable, thereby making the measured voltages at the point of common coupling (PCC) \(v_{\text{gabc}}\) significantly different from the ideal grid voltages \(v_{\text{abc}}\). A two-level three-phase power converter with an output inductor filter \(L_e\) is employed to investigate the potential instability issue due to inertia emulation. As shown in Fig. 2, the control structure of the system consists of two parts—a virtual inertia controller and a conventional cascaded voltage/current controller implemented in the synchronous \(dq\)-frame [29], [30]. The virtual inertia controller is expected to change the DC-link voltage reference by providing a voltage difference \(\Delta v_{\text{dc,ref}}\).

The voltage adjustment by the virtual inertia controller should be proportional to the change of the grid angular frequency \(\Delta \omega\) in order to generate an appropriate amount of virtual inertia, while the voltage/current controller simply regulates the DC-link voltage \(v_{\text{dc}}\) to follow the voltage reference adjusted by the virtual inertia controller.

Detailed control schemes will be elaborated in the following sections based on the system and control parameters in Table I. It should be highlighted that the adopted virtual inertia control regulates the DC-link voltage in proportional to the grid frequency for inertia emulation. Therefore, it only needs the frequency signal rather than the \(df/dt\) signal, and hence it is different from the \(df/dt\)-based scheme. In addition, the adopted virtual inertia control is applied to “grid-following” power converters, which are controlled as AC current sources and cannot operate alone in the islanded mode.

**III. INSTABILITY UNDER WEAK GRID CONDITIONS**

This section will detail the control philosophy and explore the instability for the three-phase grid-connected power converters with the virtual inertia control under weak grid conditions.

**A. Control without Virtual Inertia**

The relationship between the converter voltages \(v_{\text{abc}}\) and grid voltages \(v_{\text{gabc}}\) can mathematically be described by the following equation in the synchronous \(dq\)-frame [39]:

\[
\begin{align*}
\dot{v}_{d} &= v_{d} + L_{\text{g}} \frac{di_{d}}{dt} - \omega_{\text{pu}}i_{q} \\
\dot{v}_{q} &= v_{q} + L_{\text{g}} \frac{di_{q}}{dt} + \omega_{\text{pu}}i_{d},
\end{align*}
\]

where \(v_{d}(t)\) and \(v_{q}(t)\) denote the \(d\)- and \(q\)-axis components of converter voltages, \(v_{d}(t)\) and \(v_{q}(t)\) represent the \(d\)- and \(q\)-axis components of grid voltages, \(i_{d}(t)\) and \(i_{q}(t)\) designate the \(d\)- and \(q\)-axis components of converter currents, respectively, \(L_{\text{g}}\) is the total inductance (i.e., \(L_{\text{g}} = L_{\text{g}} + L_{e}\)), and \(\omega_{\text{pu}}\) stands for the fundamental angular frequency. The terms \(\omega_{\text{pu}}L_{\text{g}}i_{d}(t)\) and \(\omega_{\text{pu}}L_{\text{g}}i_{q}(t)\) are caused by the cross-coupling effect between the
d- and q-axis. Although the cross-coupling effect due to the filter inductance $L_c$ may be suppressed by the current feedforward control, the effect of the grid inductance $L_g$ can be dominant in a weak grid condition, and this effect is difficult to be predicted and eliminated [1]. To facilitate the analysis, the current feedforward control is not included here. The system in (4) can be expressed in the complex frequency domain as

$$
\begin{align*}
\left[ V_{vd}(s) - V_d(s) + \alpha_0 L_i s G_{plant}(s) = I_{cd}(s) \right] & \\
\left[ V_{iq}(s) - V_q(s) - \alpha_0 L_i s G_{plant}(s) = I_{cq}(s) \right],
\end{align*}
$$

where $G_{plant}(s)$ can be represented by

$$
G_{plant}(s) = \frac{1}{L_s}.
$$

Proportional integral (PI) controllers are employed as the current controller $G_i(s)$ and voltage controller $G_v(s)$, expressed as

$$
G_i(s) = K_{ip} + \frac{K_i}{s},
$$

$$
G_v(s) = -(K_{ip} + \frac{K_i}{s}),
$$

where the minus sign is caused by the definition of converter current directions in Fig. 2. Considering the time-delay introduced by reference computations and pulse updates, whose effect can be simplified as a first-order lag to approximately model its low-frequency characteristics for simplicity [38]:

$$
G_d(s) = \frac{1}{T_d s + 1},
$$

in which $T_d = 1.5 / f_s$ with $f_s$ being the sampling frequency, the control architecture of the cascaded voltage/current controller is shown in Fig. 3. It is clear from Fig. 3(a) that the DC-link voltage $V_{dc}$ is regulated to its reference $V_{dc, ref}$ (normally it is a constant $V_{dc, ref}$) through the control of the $d$-axis current $i_{cd}$. Variations in the $d$-axis current $i_{cd}$ will directly affect $V_{dc}$ through a transfer function $G_d(s)$, which is caused by the real-time power balance between the AC-side and DC-side of the three-phase power converter. Under the assumption of an ideal lossless conversion, it can be derived as

$$
G_v(s) = \frac{-3V_d}{2V_{dc, ref}C_{dc} s},
$$

where $V_d$ denotes the rated value of $V_d$ and $V_{dc, ref}$ stands for the reference value of $V_{dc}$. The grid synchronization can be achieved using a PLL. It enables the detection of the phase-angle from the PCC voltages $V_{abc}$. According to [38] and [39], the PLL will influence both the $d$- and $q$-axis current control due to the transformations between natural frame and synchronous frame. When the grid-connected power converter is operated with a unity power factor, the influence of the PLL on the converter control is reflected in the $q$-axis. As shown in Fig. 3(b), the extra terms $I_{cd}\Delta \theta_{pll}$ and $V_d\Delta \theta_{pll}$ are introduced by the PLL. $I_{cd}$ denotes the rated value of $i_{cd}$, and $\Delta \theta_{pll}$ represents the difference between the phase-angle locked by the PLL and that of PCC voltages. Although the
PLL is assumed to only affect the $q$-axis current control, it should be noted that the $d$-axis current $i_{d}$ will influence the PLL (e.g., its phase angle $\Delta \theta_{\text{PLL}}$) through the coupling between $d$-axis and $q$-axis, and this effect will be analyzed in the following sections.

A small-signal model of the PLL is given in Fig. 4, from which the relationship between $\Delta \theta_{\text{PLL}}$ and $v_{iq}$ can be derived and represented as [38]

$$G_{\text{PLL}}(s) = \frac{\Delta \theta_{\text{PLL}}(s)}{v_{iq}(s)} = \frac{K_{\text{PLL},p} + s + K_{\text{PLL},i}}{s^2 + s + V_d K_{\text{PLL},p} + s + V_d K_{\text{PLL},i}}.$$  \hspace{1cm} (11)

where $K_{\text{PLL},p}$ and $K_{\text{PLL},i}$ denote the proportional and integral gains of the PLL loop filter (i.e., a PI controller), respectively. Detailed derivations of the PLL model and its influence on the current control are elaborated in [38] and [39]. It is revealed that the PLL proportional gain $K_p$ rather than the PLL integral gain $K_i$ will play a dominant role in determining the system stability [39]. An excessively large $K_p$ will destabilize the power converter control under weak grids. In contrast, an excessively small $K_p$ will deteriorate the PLL dynamics. In this paper, $K_{\text{PLL},p}$ and $K_{\text{PLL},i}$ are designed to yield a crossover frequency of 75 Hz and a phase margin of 75° for the PLL control. This design guarantees the stable operation of power converters without the virtual inertia control as well as fair PLL dynamics.

The closed-loop transfer functions of the $d$- and $q$-axis current control $G_{cd,cl}(s)$ and $G_{cq,cl}(s)$ can be derived from Fig. 3 as

$$G_{cd,cl}(s) = \frac{i_{cd}(s)}{i_{cd,ref}(s)} = \frac{G_i(s)G_q(s)G_{\text{plant}}(s)}{1 + G_i(s)G_q(s)G_{\text{plant}}(s)}.$$  \hspace{1cm} (12)

$$G_{cq,cl}(s) = \frac{i_{cq}(s)}{i_{cq,ref}(s)} = \frac{L_i G_i(s)G_q(s)G_{\text{plant}}(s)}{L - L_q G_q(s)G_{\text{PLL}}(s)[1 + G_i(s) + V_d G_i(s) + L_i G_i(s)G_q(s)G_{\text{plant}}(s)]}.$$  \hspace{1cm} (13)

Furthermore, the voltage-loop gain without the virtual inertia control $G_{v,cl,wo}(s)$ is derived as

$$G_{v,cl,wo}(s) = \frac{v_{dc}(s)}{\Delta v_{dc}(s)} = G_i(s)G_{cd,cl}(s)G_v(s).$$  \hspace{1cm} (14)

Moreover, the closed-loop transfer function of the voltage control without the virtual inertia control $G_{v,cl,wo}(s)$ can be represented as a function of $G_{v,cl,wo}(s)$ as

$$G_{v,cl,wo}(s) = \frac{v_{dc}(s)}{v_{dc,ref}(s)} = \frac{G_{v,cl,wo}(s)}{1 + G_{v,cl,wo}(s)}.$$  \hspace{1cm} (15)

**Fig. 5** illustrates the Bode diagram of the voltage-loop gain without the virtual inertia control $G_{v,cl,wo}(s)$, where a positive gain margin of 44 dB can readily be obtained, indicating a large stability margin. **Fig. 6** illustrates the pole-zero map of the closed-loop transfer function of the voltage control without the virtual inertia control $G_{v,cl,wo}(s)$, and the corresponding zoom-in inset is also provided. Since all the closed-loop poles are in the left-half-plane, the voltage and current control under weak grid conditions can always be stable with the parameter values listed in Table I.

### B. Control with Virtual Inertia

Referring back to Fig. 3(b), because of the coupling effect between the $d$- and $q$-axis, variations of $i_{d}$ will change $\Delta \theta_{\text{PLL}}$. Considering the coupling effect, it is possible to derive the transfer function from $i_{cd}$ to $\Delta \theta_{\text{PLL}}$, i.e., $G_{id,\Delta \theta}(s)$, as

$$G_{id,\Delta \theta}(s) = \frac{\Delta \theta_{\text{PLL}}(s)}{i_{cd}(s)} = \omega L_\phi G_{\text{PLL}}(s)G_{\text{plant}}(s).$$  \hspace{1cm} (16)

The Bode diagram of $G_{id,\Delta \theta}(s)$ is illustrated in Fig. 7. It is obvious that $G_{id,\Delta \theta}(s)$ exhibits a low-pass filter characteristic. As compared to that of $i_{cd}$, the magnitude of $\Delta \theta_{\text{PLL}}$ has been...
attenuated more than −40 dB. This is because the PLL transfer function \( G_{\text{pll}}(s) \) can be approximated to a gain \( (1 / V_d) \) in the low frequency band according to (11), and \( V_d \) is greater than 100 V here. As a result, the \( d \)-axis current control has almost no effect on the PLL, and thus the coupling effect between the two axes are often ignored when analysing the instability issue due to PLLs for simplicity [39].

By directly linking the change of the capacitor voltage reference \( \Delta V_{dc,\text{ref}} \) and the change of the grid angular frequency \( \Delta \omega \) through a proportional gain \( K_{\text{env}} \), virtual inertia can be provided by the three-phase grid-connected converter [29]. It should be emphasized that \( \Delta \omega \) is normally obtained from the PLL in practice. Mathematically, \( \Delta \omega \) is a time-derivative of the phase-angle \( \Delta \theta_{\text{PLL}} \), i.e., \( \Delta \omega = \Delta \theta_{\text{PLL}} \). As can be observed from Fig. 4. Therefore, the virtual inertia control inherently introduces a differential operator between \( \Delta \theta_{\text{PLL}} \) and \( \Delta V_{dc,\text{ref}} \), as shown in Fig. 8, where the entire control structure for the three-phase power converter with the virtual inertia control is shown. Note that \( \Delta \theta_{\text{PLL}} \) is expressed as the time derivative of \( \Delta \theta_{\text{PLL}} \) in Fig. 8, while \( \Delta \theta_{\text{PLL}} \) is represented as the integral of \( \Delta \omega \), in Fig. 4. In fact, these two representations are equivalent.

According to Fig. 8, the transfer function from \( i_{cd} \) to \( \Delta V_{dc,\text{ref}} \), denoted as \( G_{\text{id},\Delta V_{dc,\text{ref}}}(s) \), is given by
\[
G_{\text{id},\Delta V_{dc,\text{ref}}}(s) = \frac{\Delta V_{dc,\text{ref}}(s)}{i_{cd}(s)} = G_{\text{id},s}(s)K_{\text{env}}s.
\]
(17)

Although the differential operator in (17) inserted between \( \Delta \omega \), and \( \Delta \theta_{\text{PLL}} \) is quite necessary for inertia emulation, it also makes \( \Delta V_{dc,\text{ref}} \) very sensitive to \( i_{cd} \), particularly for a large value of \( K_{\text{env}} \) as evidenced by the magnitude diagram of \( G_{\text{id},\Delta V_{dc,\text{ref}}}(s) \) shown in Fig. 9, which serves to illustrate the effect of the virtual inertia control. The mechanism for magnitude amplification can be explained by the transfer function \( G_{\text{id},\Delta V_{dc,\text{ref}}}(s) \), namely \( \Delta \omega_{\text{PLL}}(s) / V_p(s) \), which is essentially the multiplication of the PLL transfer function in (11) and the differential operator \( s \):

\[
G_{\text{id},\Delta V_{dc,\text{ref}}}(s) = G_{\text{id},s}(s)K_{\text{env}}s.
\]

As compared to (11), the differential operator reshapes the PLL transfer function from a low-pass filter \( G_{\text{PLL}}(s) \) to a high-pass filter \( G_{\text{PLL,\text{HP}}}(s) \). In addition, \( G_{\text{PLL,\text{HP}}}(s) \) can be approximated as a gain of \( K_{\text{PLL,\text{HP}}} \) in the high-frequency band.

Since \( K_{\text{PLL,\text{HP}}} > 1 \text{ (rad/s)/V} \), the magnitude will be amplified by the differential operator. The effect of magnitude amplification, together with the considerable phase-shift of \( G_{\text{id},\Delta V_{dc,\text{ref}}}(s) \), tends to destabilize the power conversion system, and the instability issue will be disclosed as follows.

It can be obtained from Fig. 8 that the branch from \( i_{cd} \) to \( \Delta V_{dc,\text{ref}} \) in parallel with the branch from \( i_{cd} \) to \( V_d \). First, note that one branch links \( i_{cd} \) to \( V_d \) through \( G_{\text{dc},s} \). Additionally, the other branch starts from the term \( i_{cd} \Delta \omega_{\text{PLL}} \) (i.e., \( i_{cd} \) multiplied by \( \omega_0 L_d \)), goes through the \( q \)-axis current control.

---

**Fig. 7.** Bode diagram of \( G_{\text{id},\Delta \theta_{PLL}}(s) \), i.e., the transfer function from \( i_{cd} \) to \( \Delta \theta_{PLL} \).

**Fig. 8.** Entire control structure for the three-phase power converter with the virtual inertia control.

**Fig. 9.** Bode diagram of \( G_{\text{id},\Delta V_{dc,\text{ref}}}(s) \), i.e., the transfer function from \( i_{cd} \) to \( \Delta V_{dc,\text{ref}} \) with the conventional virtual inertia control. 

\[
G_{\text{PLL,\text{HP}}}(s) = \frac{K_{\text{PLL,\text{HP}}}s^2}{s^2 + V_p K_{\text{PLL,\text{HP}}}s + V_d K_{\text{PLL,\text{HP}}}^2}.
\]

(18)
to $\Delta i_d$, and finally reaches $\Delta V_{dc,ref}$ through the multiplication of $K_{av}$ and $s$. Since the inputs of the two branches (namely $i_d$) are the same, and the corresponding outputs are operated in the same summing node, the two branches are in parallel. The additional branch from $i_d$ to $\Delta V_{dc,ref}$ essentially models the effect of the virtual inertia control, which may cause the instability issue, as will be detailed. Substitution of (16) into (17), it is noted that the gain of this additional branch is determined by the grid inductance $L_g$. If $L_g = 0$ mH, the additional branch will be disabled. Therefore, the instability issue will not occur under stiff grids where $L_g = 0$ mH. In addition, the increase of $L_g$ will intensify the influence of the virtual inertia control on the system stability. Since the two branches are in parallel, they can be combined with $\Delta V_{dc,ref}$ being regarded as a part of $V_{dc}$. In this sense, $(V_{dc} - \Delta V_{dc,ref})$ will be the output of the modified voltage-loop in replacement of $V_{dc}$. The loop-gain of the modified voltage-loop with the conventional virtual inertia control $G_{v,cl,wc}(s)$ can be derived as

$$G_{v,cl,wc}(s) = \frac{V_{dc}(s) - \Delta V_{dc,ref}(s)}{\Delta V_{dc}(s)} = G_v(s)[G_{id,cl}(s) - G_{id,ref}(s)].$$

(19)

The Bode diagram of $G_{v,cl,wc}(s)$ is shown in Fig. 10, where the negative phase margins and gain margins clearly demonstrate the instability of the voltage control. Moreover, the similarities between the magnitude curves of Fig. 9 and Fig. 10 indicate that $G_{id,ref}(s)$ may greatly influence the voltage control because of its magnitude amplification effect. To further verify the instability of the voltage control, the closed-loop transfer function of the voltage control with the conventional virtual inertia control $G_{v,cl,wc}(s)$ is derived as

$$G_{v,cl,wc}(s) = \frac{G_v(s)[G_{id,cl}(s) - G_{id,ref}(s)]}{1 - G_v(s)G_{id,cl}(s)G_{id,ref}(s) + G_v(s)G_{id,cl}(s)G_{id,ref}(s)}.$$

(20)

Substituting of (14) into (15) and then comparing (15) with (20), it is noted that an additional term $-G_v(s)G_{id,cl}(s)G_{id,ref}(s)$ is added to the denominator of $G_{v,cl,wc}(s)$, and this term causes the instability of the voltage control.

Fig. 11 presents the pole-zero map of the voltage control with the conventional virtual inertia control, where a gain $K_{av}$ of 14.32 is used and henceforth, as given in Table I. As observed, a pair of conjugate poles appearing in the right-half plane (RHP) will make the voltage control unstable, which is consistent with Fig. 10. Although the presented stability analysis is on the basis of a single closed-loop transfer function of the voltage control, it essentially describes the relationship between the most concerned input and output of practical multiple input multiple output power conversion systems.

IV. PROPOSED MODIFIED VIRTUAL INERTIA CONTROL FOR STABILITY ENHANCEMENT

According to the previous analysis, the magnitude amplification of the differential operator should be responsible for the system instability. In this section, the differential operator is intentionally modified in order to reduce its high-frequency amplification effect, and thus to improve the system stability.

As mentioned before, $G_{pll,orf}(s)$ can be simplified into $K_{pll,p}$ in the high-frequency band. This is because of the second-order term ($K_{pll,p}s^2$) in the numerator of $G_{pll,orf}(s)$ in (18). The proposed control scheme aims to reduce the coefficient of this second-order term. Its fundamental principle is shown in Fig. 12, where the change of the modified angular frequency $\Delta \omega_m$ will be used instead of $\Delta \omega_r$ to generate $\Delta V_{dc,ref}$ through the proportional gain $K_{av}$. The transfer function $G_{pll,sum}(s)$ from $V_{eq}$ to $\Delta \omega_m$ can be derived from Fig. 12(a) as

$$G_{pll,sum}(s) = \frac{(K_{pll,p} - K_p)s^2 + K_{pll,pr}q + V_d}{s^2 + V_dK_{pll,q}s + V_dK_{pll,i}},$$

(21)
Regardless of phase

Referring to (22), this additional phase lag is caused by the achieved at the expense of an additional 90°
dramatic attenuation of the magnitude of

is more of concern. When

magnitude of

The Bode diagram of

transfer function from

where

Km denotes the proportional gain of the proposed modified virtual inertia control. Notice that this control scheme will not affect the frequency detection when the PLL locks the grid voltage, since

= 0 in steady state. By comparing

in (18) and

s, it is noted that the proposed control scheme essentially attaches the following transfer function to the differential operator:

\[ G_m(s) = \frac{G_{pll,om}(s)}{G_{pll,ov}(s)} = \frac{(K_{pll,p} - K_m)s + K_{pll,p}}{K_{pll,p}s + K_{pll,p}}, \]  

(22)

where

G_m(s) would be a lead-lag compensator when

Km ≠ K_{pll,p} or a low-pass filter if

Km = K_{pll,p}. Considering (22), the transfer function from

\[ i_{cd} \]

to

\[ \Delta V_{dc,ref} \]

should be reorganized as

\[ G_{id,\Delta ref,wp}(s) = \frac{\Delta V_{dc,ref}(s)}{i_{cd}(s)} = G_{id,Ap}(s)G_m(s)K_m s. \]  

(23)

The Bode diagram of

is plotted in Fig. 13. As

Km increases and approaches

(Km ≠ K_{pll,p}), the magnitude of

drops without any compromise on the phase-shift, particularly in the high-frequency band which is more of concern. When

Km reaches

(Km = K_{pll,p}), the dramatic attenuation of the magnitude of

can be achieved at the expense of an additional 90-degree phase lag. Referring to (22), this additional phase lag is caused by the degradation of the numerator order of

when

Km = K_{pll,p}. Regardless of phase-shifts, the proposed control scheme manages to attenuate the magnitude amplification, and this result can be beneficial in terms of system stability.

Furthermore, the loop-gain of the modified voltage-loop with the proposed virtual inertia control

is written as

\[ G_{r_{-},wp}(s) = \frac{V_{dc}(s) - \Delta V_{dc,ref}(s)}{\Delta V_{dc}(s)} = G_i(s)G_{id,Ap}(s)\left[G_m(s) - G_{id,\Delta ref,wp}(s)\right]. \]  

(24)

Fig. 14 illustrates the Bode diagram of the voltage-loop gain with the proposed virtual inertia control

, where the stability enhancement can clearly be observed. When

K_m = 0.5\times K_{pll,p}, although the system featuring a negative phase margin and a negative gain margin is unstable, both margins are improved. The case of

results in a system with a gain margin of 4.35 dB and a phase margin of 34.1 degrees, and these positive stability indices indicate the system is stable.

The closed-loop transfer function of the voltage control with the proposed virtual inertia control

is derived as
\[
G_{v_{cl,wp}}(s) = \frac{v_{dc}(s)}{v_{dc,ref}(s)} = \frac{G_v(s)G_{id,cl}(s)G_{dv}(s)}{1-G_v(s)G_{id,cl}(s)G_{dv}(s) + G_v(s)G_{id,cl}(s)G_{dv}(s)}.
\]

The Pole-zero map of \(G_{v_{cl,wp}}(s)\) is further provided in Fig. 15. Note that all the closed-loop poles of the voltage control are located in the left-half-plane, thereby demonstrating a stable system. It should be commented that as long as the pole of \(G_m(s)\) locates leftwards to its zero, i.e., \(|K_{pil,p} - K_m| < K_{pil,p}\), the proposed control scheme will enable magnitude attenuation and consequently, the stability is improved.

V. EXPERIMENTAL VERIFICATIONS

The instability issue of the three-phase grid-connected converters with the virtual inertia control will further be experimentally investigated in this section. Moreover, the effectiveness of the proposed modified virtual inertia control in addressing the instability issue will also be verified. In the experiments, the grid voltages \(v_{abc}\) were formed by a virtual synchronous generator (VSG), which is essentially a three-phase power converter exhibiting the same electrical terminal characteristics as conventional synchronous generators, providing the base power system inertia and forming the grid voltages [40]. The reason for the employment of the VSG lies in the replacement of conventional synchronous generators and proper regulation of the grid frequency so as to demonstrate the benefits of virtual inertia. A step-by-step design of the VSG control, together with its design parameters, can be found in [29] and [41]. In addition to the VSG, other parts of the system with the parameters listed in Table I are shown schematically in Fig. 2. Control systems were implemented in a dSPACE control platform (Microlabbox). An eight-channel oscilloscope (TELEDYNE LECROY: HDO8038) was used to capture all the experimental waveforms.

Fig. 16 presents the steady-state experimental results of the three-phase power converter with the conventional virtual inertia control, where \(\Delta f_i (\Delta \omega_r = 2\pi \Delta f_i)\) denotes the grid frequency change measured by the PLL, and the other notations can be found in Fig. 2. In this case, the VSG is employed solely to provide the grid voltages with a fixed frequency as an AC voltage source. As observed, the current waveforms \(i_{abc}\) are seriously distorted. It should be mentioned that the saturation units are necessary in practice to limit the variation ranges of the DC-link voltage \(v_{dc}\) and frequency \(f_r\) to avoid over voltages. Due to the saturation units, \(v_{dc}\) and \(f_r\) can only oscillate within certain ranges rather than being completely unstable. Therefore, the oscillations in the grid frequency change \(\Delta f_i\) imply that the virtual inertia control through the direct link between \(f_r\) and \(v_{dc}\) should be responsible for the instability. Fortunately, the instability issue can successfully be addressed once the proposed modified virtual inertia control is enabled, as demonstrated in Fig. 17. It is clear that the oscillations in the grid frequency change \(\Delta f_i\) and the DC-link voltage \(v_{dc}\) disappear. Due to the stable DC-link voltage \(v_{dc}\), the converter currents become almost distortion-free with negligible variations (induced by power converter losses). These experimental results agree well with the pole-zero maps shown in Fig. 11 and Fig. 15. It should be noted that the instability due to the virtual inertia control remains even if the PLL is replaced by other more...
robust synchronization units, such as the frequency-locked-loop (FLL) [21], or with modified PLL designs.

Through the proposed virtual inertia control, the three-phase grid-connected power converter may effectively contribute virtual inertia to weak power grids. For illustration, Fig. 18 shows the grid frequency and the DC-link voltage responses of the power converters with and without the virtual inertia control when they experienced a 5% step-up load change. Such a step-up load change causes a relatively serious frequency event, which is sometimes used to emulate generation or load tripping in real power systems [5], [42]. It should be mentioned that the step-up load change causes the demanded power to be greater than the generated power. As a consequence, the grid frequency drops, which is seen by all the synchronous generators as a common signal to increase their respective power outputs for balancing the power mismatch between generation and demand. In the case of such a frequency event, it is important to keep the frequency drop, as well as its changing rate, i.e., the rate of change of frequency (RoCoF), below the limits defined by grid codes [3]. Otherwise, excessive frequency drops may further cause generation and load tripping, thus leading to cascaded failures. In extreme cases, system blackouts may even occur [4].

In Fig. 18, the VSG is operated to regulate the grid frequency in a similar way as conventional synchronous generators do. As compared to the grid-connected power converter without the virtual inertia control, the power converter with the virtual inertia control registers a change of its DC-link voltage that is proportional to the grid frequency for inertia emulation during the frequency event. As a result, the inertia constant is improved from 5 s to 7.5 s, indicating a 50% enhancement of the power system inertia. The increased inertia can contribute to an 11% increment of the frequency nadir (i.e., the lowest frequency point) and a 33% improvement of the RoCoF, as demonstrated in Fig. 18. Note that the extent of performance improvements depends on the emulated virtual inertia, which is further determined by the DC-link voltage, voltage variation range, and DC-link capacitance, as detailed in [29].

VI. CONCLUSION

This paper has identified the potential instability issue for three-phase grid-connected power converters with the virtual inertia control when they are connected to weak power grids. The exploration indicates that the virtual inertia control introduces an extra link between the $d$-axis current and the DC-link voltage reference through the $d$- and $q$-axis coupling, $q$-axis current control, phase-locked-loop, and virtual inertia controller. Specifically, there is an inherent differential operator inside the link, which is responsible for the instability, when the converter is connected to weak grids. Accordingly, the virtual inertia control has been intentionally modified in order to mitigate the amplification effect introduced by the differential operator. As a result, three-phase grid-connected power converters can effectively contribute virtual inertia to weak power grids, which have been verified by the experimental results. The major contributions of this paper can be summarized as follows:

1. Identify the instability issue faced by three-phase grid-connected power converters with the virtual inertia control under weak grids;
2. Explain the mechanism for the instability issue;
3. Propose a modified virtual inertia control scheme to resolve the instability issue.

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