Analysis of Oscillation Frequency Deviation in Elastic Coupling Digital Drive System and Robust Notch Filter Strategy

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Abstract—Mechanical resonance is a common problem in drive systems with elastic coupling. On-line adaptive notch filter is widely used to make system stable and the key of this method is to identify natural torsional frequency from speed feedback signal. However, because of common adoption of digital control and expansion of system bandwidth, oscillation frequency of the system is more likely to deviate from natural torsional frequency to a higher one. When oscillation frequency is shifted, the enabled notch filter with erroneous notch frequency causes an oscillation with a lower frequency and even makes resonance severer. In order to explain this phenomenon, the classical two-mass model based classification of resonances is checked at first. Then, by taking digital control, current loop delay and saturation nonlinearity into consideration, an improved digital mechanical resonance model is proposed and a criterion for oscillation frequency deviation is finally obtained. Furthermore, a more widely applicable and robust notch filter tuning strategy with no oscillation rebound is presented. In the end, the validity of aforementioned analysis and strategy is verified by experimental results.

Index Terms—Two-mass system, mechanical resonance, digital control, notch filter.

I. INTRODUCTION

Elastic joint like shaft, gear, ball-screw, timing-belt and coupling, is an essential part in industrial applications such as factory automation, computer numerical control machine tools, and industrial manipulators. Although elastic joint is cost-effective and flexible, it causes mechanical resonance and even leads to equipment damage [1]-[3]. Due to the market competition, companies prefer the controller with high gains parameters to improve production efficiency. However, with higher gains controllers are more likely to cause system instability and excite resonance, especially in flexible transmission system. Hence, guaranteeing the stability of the system is the precondition for improving the performance.

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Fig. 1. Phenomenon of mechanical oscillation frequency deviation.

Hardware solutions such as increasing the stiffness coefficient of elastic joint, adding passive damping into system and choosing actuator with suitable inertia are most direct methods, but not economical. Thus, companies tend to modify controller and algorithm to make up for mechanical deficiencies and optimize the performance of in-use equipments. Suppression methods for mechanical resonance can be mainly divided into four groups:

1) Filter strategies: adding a low-pass or notch filter to conventional speed control structure to damp resonance-frequency component and compensate for the phase lag. Then, modified frequency-domain characteristics make system stable [1],[4]-[8].

2) Input shaping methods: correcting input reference and using iterative learning control in industrial assembly line with repeated tasks, which can suppress terminal vibration and make trajectory smooth [1],[9].

3) State variable measurement and estimation: using sensors or observers to achieve additional feedback or feedforward from one or more selected state variables to improve system characteristics and performances [10]-[13].

4) Modern control theories: utilizing predictive, adaptive, fuzzy, neural, and other modern control theories in two-mass system to cover the shortage of classical control strategies and enhance robustness of systems [14]-[22].

Among those methods, the filter strategies are more convenient for operation and easier to be achieved in digital controller, and notch filter based resonance damping is one of the most widely used methods. In conventional on-line adaptive notch filter (OANF), Fast Fourier Transformation is used to identify the resonance frequency \( \omega_{\text{NTF}} \) (Natural Torsional Frequency) of the system from speed error signal in order to set the notch frequency. This method can complete the tuning of parameters and make system return to stability in a short time [4]-[7]. However, OANF still has some limitations in some cases that can be shown in Fig. 1. When speed feedback of the system is close to reference \( \omega_r \), an oscillation (frequency is \( \omega_2 \),
\( \omega_2 > \omega_{\text{NTF}} \) appears. After adding a notch filter with notch frequency as \( \omega_2 \), the resonance frequency shifts to \( \omega_3 \) \((\omega_{\text{NTF}} < \omega_3 < \omega_2) \) and the resonance amplitude may become even larger, which does not satisfy the expectation. Although iterative tuning of notch frequency may possibly stabilize the system, this process is slow and not smooth enough. Hence, a more widely applicable and effective robust notch filter strategy with no oscillation rebound is needed.

In this paper, motivated by the above observations, a classification, based on traditional model, of mechanical resonances in transient and steady state has been done at first. Due to the limitation of traditional model, the self-sustained oscillation cannot be described clearly. Thus, an improved digital drive mechanical resonance model is built up to explain this type of oscillation. In the modeling process, some theories in LCL filter resonance are adopted [23]-[25]. Based on this improved model, a criterion for oscillation frequency deviation is obtained and a conclusion is drawn that choosing \( \omega_{\text{NTF}} \) as notch frequency is more effective. In order to expand the applicable range of notch filter, a robust on-line adaptive notch filter (ROANF) is proposed. Adjustments of sampling period and output limit of speed loop are used to guarantee robust and smooth resonance suppression performance of ROANF. The effectiveness of the proposed model and strategy is verified on a 750W PMSM drive platform with elastic couplings.

II. CLASSICAL MECHANICAL RESONANCE MODEL AND RESONANCE FREQUENCY CLASSIFICATION

A. Two-Mass Model

Fig. 2 is a classical schematic for two-mass model. An elastic coupling is used to connect the motor and the actuator, whose stiffness is \( K \), and damping is \( C_w \). \( J_m \) and \( J \) represent the inertia of motor and load respectively. Speed of motor \( \omega_m \) is dependent on electromagnetic torque \( T_e \) and shaft torque \( T \). Load torque \( T_1 \) and \( T_2 \) act together on the actuator, deciding its speed \( \omega_l \). Due to the fact that \( \omega_m \) may be inconsistent with \( \omega_l \), there is an angle difference \( \theta \) between motor angle \( \theta_m \) and load angle \( \theta_l \). This deformation \( \theta \) of the shaft is the cause of \( T \).

Kinematics equations of the system are illustrated as follows

\[
\begin{align*}
J_m \omega_m' &= T_e - T_1 \\
J \omega_l' &= T - T_1 \\
T &= C_w (\theta_m - \theta_l) + K (\theta_m - \theta_l) \\
\omega_m &= \dot{\theta}_m \\
\omega_l &= \dot{\theta}_l
\end{align*}
\]

The torque generated by the SPMMS is given by

\[
T_e = 1.5 \rho \left[ \psi_r i_q + \left( L_a - L_q \right) i_r i_q \right] = 1.5 \rho \psi_r i_q = K \dot{i}_q
\]

in which \( \rho \) is the pole pair number, \( \psi_r \) is the flux produced by the rotor magnet, \( L_{a,q} \) and \( i_{r,q} \) are the equivalent inductances and currents in the direction of the magnet flux and the quadrature direction, and \( K \) is the torque coefficient. Considering \( i_q \) is easier to detect than \( \dot{T}_e \) in experiment and there is a proportional relationship between these two values, \( i_q \) is chosen as a representation of torque in the follow-up analysis. The nominal current \( i_{qN} \) corresponds to the nominal torque \( T_{eN} \) of the motor.

Then, the control diagram of two-mass system can be obtained as Fig. 3. Because \( C_w \) is small in most industrial systems, it is neglected in the following analysis. Hence, the transfer function of \( \omega_m \) and \( T_e \) can be deduced as

\[
G_s(s) = \frac{\omega_m}{T_e} = \frac{1}{(J_m + J)s^{2} + J \omega_l i_q + K_s}
\]

Furthermore, natural torsional frequency \( \omega_{\text{NTF}} \) and anti-resonance frequency \( \omega_{\text{ARF}} \) of system are defined as

\[
\omega_{\text{NTF}} = \sqrt{(J_m + J)K_s / (J_m J)} \quad \omega_{\text{ARF}} = \sqrt{K_s / J}
\]

B. Classification of Mechanical Resonance Frequency

In order to analyze the frequency of mechanical resonance, the model of the controller is also necessary besides the kinematics equation (3). Considering the bandwidth of current loop is much higher than that of speed loop, the torque coefficient \( K \) is preferably used to represent the current loop \( G_{\text{ARC}}(s) \) in classical mechanical resonance model. On the other hand, influence of the integral term in speed loop controller \( G_{\text{ASR}}(s) \) on the system concentrates on low frequency band far below \( \omega_{\text{NTF}} \) in industrial applications. Considering this paper focuses on phase characteristics near \( \omega_{\text{NTF}} \), hence, only the proportional gain \( K_p \) is used to replace \( G_{\text{ARC}}(s) \) and the integral part is ignored. To sum up, the control diagram of classical two-mass drive system can be shown as Fig. 4 in which \( i_{qN} \) is the output of the speed loop controller. Next, the following discusses the mechanical resonance frequency under transient and steady state of the classical mechanical resonance model.

1) Transient State

From Fig. 3, transfer function from \( T_1 \) to \( \omega_m \) can be given as

\[
G_s(s) = \frac{\omega_m}{T_1} = \frac{-K_s}{(J_m + J_s) \omega_l i_q + (J_m - J_s) K_s}
\]
By combining (3) and (5), the unit step response of \( \omega_m \) in the \( s \)-domain can be expressed as

\[
\omega_m(s) = G_t(s)T_e(s) - G_z(s)T_i(s)
\]

\[
\omega_m(s) = \frac{1}{J_m + J_L} \left( \frac{T_e(s) - T_i(s)}{s} + \frac{(J_i T_e(s) - J_o T_i(s))s}{J_m (s^2 + \omega_{ntf}^2)} \right)
\]

(6)

Using inverse Laplace transform to rewrite (6), the unit step response expression of \( \omega_m \) in the time domain can be written as

\[
\omega_m(t) = \frac{T_e - T_i}{J_m + J_L} + \frac{J_i T_e - J_o T_i}{J_m \omega_{ntf}} \sin(\omega_{ntf}t)
\]

(7)

According to (7), in transient state when tracking the step reference, the ramp response of the drive system with elastic coupling contains a resonance component whose frequency is \( \omega_{ntf} \). The magnitude of this resonance component is inversely proportional to frequency and independent with \( T_i \).

2) Steady State

In dual closed loop drive system, once the speed feedback reaches the vicinity of the reference, speed loop recovers from saturation and continues to function. According to Fig. 4, the closed loop transfer function \( G_c(s) \) can be expressed as

\[
G_c(s) = G_{asr}(s)G_{acm}(s)G_i(s)/[1 + G_{asr}(s)G_{acm}(s)G_i(s)]
\]

\[
= \frac{K_m K_i (J_i s^2 + K_i)}{J_m J_i s^3 + K_i J_i s^2 + (J_m + J_L) K_i s + K_e K_i}
\]

(8)

The characteristic equation of (8) can be given as

\[
s^3 + \frac{K_m K_i J}{J_m} s^2 + \frac{K_i}{J_m} \omega_{ntf}^2 s + \frac{K_e K_i}{J_m} \omega_{ntf} = \left(s^2 + 2\zeta_s \omega_s s + \omega^2_s\right)(s + K_i) = 0
\]

(9)

in which \( \zeta_s \) and \( \omega_s \) represent the damping and frequency coefficients of oscillating element respectively and the \( K_i \) is a coefficient that depends on the parameters of the system.

According to (3), the phase lag of the system in classical model is less than \( \pi/4 \). Thus, the drive system is bound to be stable if it is a minimum phase system. Based on (9), when \( \zeta_s > 0 \) and \( K_i > 0 \), the frequency of damped oscillation \( \omega_s \) after speed loop returning to work can be given as

\[
\omega_s = \omega_s \sqrt{1 - \zeta^2}
\]

(10)

In summary, the mechanical resonance analyzed in classical model can be shown in Fig. 5. Because the oscillation in steady state decays gradually, the steady state oscillation is relatively small. Hence, the speed signal is given in the form of error in Fig. 5. In transient state, speed error is the difference between the ramp component in (7) and speed feedback.

Besides the phenomena in Fig. 5, there is another type of mechanical resonance in modern digital drive system, which can be shown in Fig. 6. In the process of speed regulation, when speed reference is a step signal, speed loop saturates rapidly and current loop is able to track the constant reference from speed loop quickly in transient state. Thus, the influence of the digital control and current loop delay can both be ignored. The conventional model can still apply in transient state of Fig. 6. However, in steady state, the resonance in high gain system is a severe self-sustained oscillation with frequency is \( \omega_{osc} \) and sometimes \( \omega_{osc} \) is larger than \( \omega_{ntf} \). This phenomenon is different from the analysis of the conventional model. Hence, a more comprehensive model is needed.

III. PROPOSED IMPROVED MECHANICAL RESONANCE MODEL IN ELASTIC COUPLING DIGITAL DRIVE SYSTEM

In mechanical resonance model for digital drive system, more factors need to be taken into account. In order to refine the conventional model, this part is focused on the modeling of three key factors including equivalence of current loop, impact of digital control and saturation nonlinearity of speed controller’s output.

A. Equivalence of Current Loop

In traditional model, the current loop is usually regarded as torque coefficient \( K_t \) or first-order low pass filter because of its higher bandwidth than that of speed loop. However, due to the maximum phase lag of a first-order low pass filter is \( \pi/2 \) and the torque coefficient \( K_t \) does not provide any phase lag, the total phase lag is no more than \( \pi \) and the system is always stable under these two kinds of approximation methods. This conclusion conflicts with Fig. 6. In addition, the phase function of low pass filter includes inverse trigonometric function, which is not convenient for applied calculation.

Since the gain near \( \omega_{ntf} \) is large, the amplitude attenuation from current loop can be ignored. Hence, this paper chooses delay model to simulate the current loop. In order to use delay model to reflect the bandwidth of current loop, the phase of the cutoff frequency \( \omega_c \) is set as \( -\pi/4 \). The relationship between time constants, \( T_i \) in delay model and \( T_i \) (\( 2\pi/\omega_c \)) in current loop, can be given as

\[
T_i = \frac{\pi}{4\omega_c} = T_i/8
\]

(11)

The transfer function of the delay-model-based current loop can be given as...
\[ G_{ACR}(s) = K_e e^{-st} \left| _{t=\tau_R} = K_e e^{-st/\tau_R} \right. \] (12)

Taking (12) into consideration, the simplified open loop transfer function can be expressed as

\[ G_{ACR}(s) \cdot G_z(s) = \frac{J_0^2 + K_m}{J_n^2 + K_m} \cdot \frac{1}{(J_n + J_s) s} \cdot K_e e^{-st/\tau_R} \] (13)

The phase of (13) can be expressed as

\[ \angle G_{ACR}(s) \cdot G_z(s) \bigg| _{s=j\omega} = -\frac{\omega T_s}{8} \left ( \theta < \omega_{ARF} \right ) \]

\[ \angle G_{ACR}(s) \cdot G_z(s) \bigg| _{s=j\omega} = -\frac{\omega T_s}{8} \left ( \omega_{ARF} < \omega < \omega_{NTF} \right ) \]

\[ \angle G_{ACR}(s) \cdot G_z(s) \bigg| _{s=j\omega} = -\frac{\omega T_s}{8} \left ( \omega > \omega_{NTF} \right ) \] (14)

**B. Impact of Digital Control**

Although digital control is flexible and widely used, it brings phase lag to system inevitably and affects the stability of the system. Hence, a ZOH with time constant \( T_{ASR} \) is added to the model to simulate the impact from digital control. \( T_{ASR} \) is also the sampling period of speed loop. The transfer function of ZOH \( G_0(s) \) can be given as

\[ G_0(s) = \left ( 1 - e^{-T_{ASR}s} \right ) / s \] (15)

Substituting \( s=j\omega \) into (15) yields

\[ G_0(j\omega) = T_{ASR} \sin(\omega T_{ASR} / 2) e^{-\omega^2 T_{ASR} / 2} \] (16)

According to (16), the phase of \( G_0(j\omega) \) can be given as

\[ \angle G_0(j\omega) = e^{-\omega^2 T_{ASR} / 2} = -\omega T_{ASR} / 2 \] (17)

In order to unify those time constants, assuming \( T_{ASR} \) is the sampling period of current loop and \( p_1, p_2 \) are proportionality coefficients, which are related to the control strategy [26],[27]. In most cases, the description equations are as follows

\[ p_1 = T_s / T_{ASR} \geq 6 \]

\[ p_2 = T_{ASR} / T_{ACR} \geq 1 \] (18)

Adding (15) and \( G_{ASR}(s) \) into (13), using (18) to unify those time constants as multiples of \( T_{ASR} \) and extracting the phase component of ZOH only. The open loop transfer function of digital system control model can be expressed as

\[ T(s) = G_n(s) \cdot G_{ASR}(s) \cdot G_{ACR}(s) \cdot G_z(s) \]

\[ = \frac{J_0^2 + K_m}{J_n^2 + K_m} \cdot \frac{1}{(J_n + J_s) s} \cdot K_e e^{-\omega T_s / 2} \] (19)

The phase of (19) can be expressed as

\[ \angle T(s) \bigg| _{s=j\omega} = \theta_n + \theta_{ACR} + \theta_1 \]

\[ = \begin{cases} (p_1 + 4p_2) / 8 \cdot T_{ASR} \cdot \omega \cdot \frac{\pi}{2} & (\omega < \omega_{ARF}) \\ -\frac{\pi}{2} & (\omega_{ARF} < \omega < \omega_{NTF}) \\ 0 & (\omega > \omega_{NTF}) \end{cases} \] (20)

According to (20), there is enough phase lag in the modified transfer function \( T(s) \) rather than the conventional model. It means that if the gain of system is large and the phase margin is negative, the system becomes unstable and the subsequent oscillation inferred from the above analysis is more realistic.

**C. Saturation Nonlinearity of Speed Controller’s Output**

In linear control theory, the prerequisite for undamped self-sustaining oscillation is rigorous. Thus, damped or divergent oscillation is more common. However, there is an undamped self-sustaining oscillation in steady state of Fig. 6. In order to explain this phenomenon, the nonlinear analysis method is used. The describing function of saturation nonlinearity of speed controller’s output (output limit is \( \pm A \)) can be given as

\[ N_0(X/A) = 2 \pi \left( \frac{\omega}{\alpha} \right) \frac{1}{\pi} \left( \frac{A}{X} \right)^2 \] (21)

The control diagram of improved mechanical resonance model can be shown in Fig. 7. In order to simplify the operation, this part reunifies those time constants as multiples of \( T_{ASR} \). Next, assuming \( \lambda=p_1 / 8p_2 \) and substituting the whole ZOH into the system transfer function. The open loop transfer function of system in the \( s \)-domain \( G(z) \) can be expressed as [23],[24]

\[ G(z) = Z \left( K_e e^{-\omega T_{ASR}} G_0(s) G_z(s) \right) = K_F K_p Z \frac{\omega - 1/\ell}{\omega + 1/\ell} + Z \left( G(s) e^{-\omega T_{ASR}} \right) \] (22)

in which \( \ell \) is an integer, \( 0 \leq m < 1 \), and \( \ell = \lambda - m \).

According to the method of residues, (22) can be rewritten as

\[ Z \left( \frac{G(s)}{s} \right) e^{-\omega T_{ASR}} = \sum_{i=1}^{\infty} \text{Res} \left( \frac{z \cdot G_z(s) e^{-\omega T_{ASR}}}{s} \right) \] (23)

Substituting (3) into (23) yields
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\[ G(z) = K_p K_c z^{-1} \left( \frac{T_{ASR}}{z} + m z + 1 - m \right) \left( J_m + J_f \right) (z) \left( J_m \right)^{-2} \left( \sin m \omega_{NTF} T_{ASR} + \sin \omega_{NTF} T_{ASR} \right) \]

(24)

The \( w \)-transform \( z=(w+1)/(w-1) \) is used to transform (24) from \( z \)-transform to \( w \)-transform. By separating the linear part and nonlinear part, Fig. 7 can be reconfigured to Fig. 8. The negative inverse describing function curve \(-1/N_0(X/A)\) and the linear transfer function curve \(G(w)\) are both plotted in Fig. 9 (the crossover point is \( \alpha \)). According to the describing function method [28], if the amplitude of oscillation becomes larger, the action point departs from the surroundings of \( G(w) \) and the system restores stability. Finally, the action point returns to \( \alpha \) again. In contrast, if the amplitude of oscillation becomes smaller, the action point moves to the right of the \( \alpha \). Right part of \(-1/N_0(X/A)\) is surrounded by \( G(w) \), which means the system becomes unstable and the oscillation amplitude becomes larger. Then, the action point moves back to \( \alpha \) as well. Hence, the crossover point \( \alpha \) corresponds with undamped self-sustaining oscillation and the frequency of \( \alpha \) is the oscillation frequency \( \omega_{OSC} \). Because \(-1/N_0(X/A)\) must be a negative real number less than \(-1\), if \( \alpha \) exists, it must locate on the negative real axis. Thus, there is a phase relationship of \( \omega_{OSC} \) as follows

\[ \angle G(j \omega_{OSC}) = -\pi \]

(25)

In order to simplify the operation, (20) is used to replace \( G(w) \) in (25). Then, the expression of \( \omega_{OSC} \) can be given as (because for the sake of work efficiency the phase lag of real system cannot be too large, the possibility of \(-\pi\) crossover point takes place near or before \( \omega_{ARF} \) is small, so it is ignored)

\[ \omega_{OSC} = \frac{\omega_{NTF}}{4 \pi \pm 4 \pi \tan \left( \omega_{ARF} \right)} \]

(26)

In this paper, the frequency deviation means \( \omega_{OSC} > \omega_{NTF} \). According to (20) and (26), the calculation formula of shifted frequency \( \omega_{OSC} \) of the system and its deviation criterion can be given as (transforming the frequency unit to Hertz (\( \omega=2\pi f \))

\[ f_{OSC} = \frac{1}{2} \left( T_c + 4T_{ASR} \right), \left( T_c + 4T_{ASR} < 2/T_{NTF} \right) \]

(27)

According to (27), the impact of \( T_c \) and \( T_{ASR} \) on \( f_{OSC} \) can be shown as Fig. 10. With the decrease of the delay from current and speed loop, \( f_{OSC} \) gradually increases. In the previous industrial drive system, \( T_c \) and \( T_{ASR} \) are relatively large. Hence, almost all \( f_{OSC} \) in system are equal to \( f_{NTF} \). However, high performance and bandwidth become the characteristics of modern digital control systems. The decrease of the system delay means \( \omega_{OSC} \) would be more likely to deviate from \( \omega_{NTF} \).

To sum up, the decrease of \( T_c \) and \( T_{ASR} \) are key factors for oscillation frequency deviation; the saturation nonlinearity coming from the limitation of speed loop output is the cause of the undamped self-sustaining oscillation of systems in steady state when the phase margin of the system is negative. The saturation avoids the occurrence of severer diverging oscillation and protects the systems (tuning \( A \) can constrain the maximum oscillation amplitude). Based on the improved model, the expression of \( \omega_{OSC} \) and a deviation criterion of oscillation frequency (27) are concluded. Because (27) is

concise and intuitive, it can be applied in reality to identify whether the resonance deviation happens or not.

IV. PROPOSED ROBUST NOTCH FILTER STRATEGY

A. Traditional On-Line Adaptive Notch Filter Theory

The transfer function of classical notch filter can be given as

\[ G_{n}(s) = \frac{s^2 + 2\omega_n s + \omega_n^2}{s^2 + b s + \omega_n^2} \]

(28)

where \( \omega_n \) is the notch frequency and \( b \) is -3dB rejection bandwidth factor. The phase of (28) around \( \omega_n \) can be given as

\[ \angle G_{n}(j \omega_n) = \arctan \frac{\omega_n}{\omega_n^2 - \omega_n^2} = -\pi/2 \]

(29)

Assuming \( k = 2/T_{ASR} \), \( \Omega \) is the rejection bandwidth and the magnitude of notch filter at \( \omega_r = \pm \Omega/2 = -\pi \). Applying Tustin transformation to discretize the notch filter (28) and DFFlt structure [8],[23] to modify the notch factor yields

\[ G_n(z) = \frac{1}{2} \left( 2 a_2 z^{-1} - a_2 z^{-2} + 2 a_1 z^{-2} \right) \]

(30)

in which \( a_1 \) and \( a_2 \) can be expressed as

\[ \begin{align*}
a_1 &= \frac{2 \cos \omega_n T_{ASR}}{1 + \sqrt{10^{60} - 1 \tan \left( \omega_n T_{ASR}/2 \right)}} \\
a_2 &= \frac{1 - \sqrt{10^{60} - 1 \tan \left( \omega_n T_{ASR}/2 \right)}}{1 + \sqrt{10^{60} - 1 \tan \left( \omega_n T_{ASR}/2 \right)}}
\end{align*} \]

(31)

One of the conventional OANF parameter tuning methods can be shown in Fig. 11. A predetermined error boundary is set to get the rejection bandwidth \( \Omega \), and the on-line FFT result of speed feedback error signal is used to find the notch frequency \( \omega_n \) in this method [6]. After parameter determination, according to (24) and (30), the open loop transfer function of the system with notch filter \( G'(z) \) can be given as
By substituting \( z=(w+1)/(w-1) \) into (32), when \( \omega_{\text{OSC}}=\omega_{\text{NTF}} \), the bode diagram of \( G'(w) \) can be shown in Fig. 12. According to Fig. 12 and (29), OANF can not only decrease the gain around \( \omega_{\text{NTF}} \) but also reduce the phase lag after \( \omega_{\text{NTF}} \) at the cost of increase of the phase lag before \( \omega_{\text{NTF}} \). The improvement from both magnitude and phase characteristics make system restore stability effectively. However, in Fig. 13, different with Fig. 12, the criterion in (27) is valid and \( \omega_{\text{OSC}} \) is larger than \( \omega_{\text{NTF}} \). The phase delay coming from OANF causes the new \( \omega_{\text{OSC}} \) to move closer to \( \omega_{\text{NTF}} \) and even result in a server oscillation. Thus, OANF is not suitable in the condition of \( \omega_{\text{OSC}}<\omega_{\text{NTF}} \).

### B. Process of Robust On-line Adaptive Notch Filter

Also based on Fig. 13, if \( \omega_{\text{NTF}} \) is set as \( \omega_m \), the notch filter can still make up for the phase lag around the \( \omega_{\text{OSC}} \) to guarantee the stability of system. Hence, no matter whether the criterion in (27) is valid or not, \( \omega_m=\omega_{\text{NTF}} \) is an effective choice. Considering the time of transient-oscillation stability with frequency is \( \omega_{\text{NTF}} \) is relatively short and the FFT analysis is not reliable, thus, the steady state on-line identification of \( \omega_{\text{NTF}} \) is the focus of proposed ROANF strategy.

According to (27) and Fig. 10, \( T_{\text{ASR}} \) and \( T_f \) are influencing factors for \( \omega_{\text{OSC}} \) in drive system. An appropriate increase for \( T_{\text{ASR}} \) or \( T_f \) can make criterion in (27) invalid again and \( \omega_{\text{OSC}} \) return to \( \omega_{\text{NTF}} \). Because the parameters of the current loop controller need to be conservative enough to ensure the security of system, the adjustment of \( T_{\text{ASR}} \) is finally adapted in ROANF to extract \( \omega_{\text{NTF}} \) of system. In addition, because larger \( T_{\text{ASR}} \) means larger oscillation amplitude, the output limit of speed loop \( A \) is reduced moderately before ROANF parameters identification is finished. Overall, the control diagram and schematic diagram (\( T_{\text{ASR}} \) is the initial sampling period of speed loop and it is assumed to be no longer than 2ms) of ROANF can be shown as Fig. 14 and Fig. 15 respectively, and the detailed process of ROANF is listed as follows.

### V. Experimental Results

The experiment setup is shown in Fig. 16, consisting of a Zynq-based driver and a DSP-based driver, two SPMSMs and two elastic couplings. Two SPMSMs are both equipped with 2500 impulses/revolution incremental optical encoders. The
The frequency domain analysis of current loop identifies the characteristic frequencies of the system. Combining DSP-Based Motor Driver and Zynq-Based Motor Driver, the elastic couplings are investigated. One kind of sweep signal—Chirp Signal (the frequency of which increases with time) is used to describe the influence of delays in current control loop and transmiss analysis. The speed loop control and current loop control (a classical PI controller) of the platform is finished in FPGA, an external interrupt is sent to ARM and a register is used to record the times of interrupts to judge the frequency (corresponding to sampling period) of speed loop control. Hence, \( T_{\text{ASR}} \) can be adjusted easily. In order to verify the parameter of the elastic coupling is unknown or not and transmiss analysis. The relationship between \( i_{\text{ref}} \) and \( i_{c} \) feedback to identify the frequency domain characteristics of the current loop and get the value of \( T_{c} \). The analysis results of current loop with different parameters are shown in Fig. 17.

Apart from the modification of current limitation and speed loop sampling period, only the parameters of speed controller’s output filter are changeable. Especially, in order to reflect the importance of the choice for notch frequency \( f_{\text{osc}} \) and to enhance the contrast effects with the performance of OANF, both \( \Omega \) and \( \gamma \) of OANF and ROANF are set as 100π and 3 instead of self-tuning. Thus, only \( f_{\text{osc}} \) is the changeable in OANF and ROANF settings for the follow-up experiments.

### Oscillation Frequency Deviation Phenomenon

At first, off-line two-mass mechanical systems identification [29],[30] is used to determine the standard value of oscillation frequency \( f_{\text{osc}} \) of OANF and ROANF. Considering the results of these two kinds of couplings are similar, only the data of coupling 1 is given out. In order to verify the validity of the proposed model, oscillation frequencies \( f_{\text{osc}} \) of the platform with different \( T_{\text{ASR}} \) (from 1ms to 2.5ms) and \( T_{c} \) (from 2.25ms to 3.60ms) are recorded in Fig. 18.

According to Fig. 18, when \( T_{\text{ASR}}>2\text{ms} \), \( f_{\text{osc}} \) is close to \( f_{\text{NTF}} \) (159Hz). However, when \( T_{\text{ASR}}<2\text{ms} \), \( f_{\text{osc}} \) shifts from \( f_{\text{NTF}} \) to a higher frequency. \( f_{\text{osc}} \) tends to increase with the decrease of \( T_{\text{ASR}} \) and \( T_{c} \). Submitting the groups of data \( T_{\text{ASR}}<2\text{ms} \) from Fig. 18 into (27) indicates error remains several tens of Hertz from expected \( f_{\text{osc}} \). Thus, assuming \( G_{\text{val}}(s)=e^{-\beta s} \) as the modification factor to reflect the delay coming from damping, signal filtering and transmission. Combining \( G_{\text{val}}(s) \) with (27), the fixed expression and criterion can be given as

\[
    f_{\text{osc}} = 2\left(\frac{1}{T_{c} + 4T_{\text{ASR}} + 8T_{m}^2}\right) \cdot \left(\frac{1}{T_{c} + 4T_{\text{ASR}} + 8T_{m}^2} < 2\left| f_{\text{NTF}} \right| \right) \tag{34}
\]

Fig. 17. Analysis results of current loop with different parameters.

Fig. 18. Oscillation frequency of systems with different \( T_{\text{ASR}} \) and \( T_{c} \).
Fig. 19. Effect of OANF.

in which $T_m$ can be calculated by taking data from Fig. 18 to (34) and calculating the arithmetic mean.

By setting $T_m$ as 0.4531ms, estimation error has been narrowed down to several Hertz, which is within acceptable range. In addition, the parameter conservation from (27) does not cause the failure of ROANF because the determined $\Omega$ is larger than the required minimum rejection bandwidth.

New oscillation frequency $f_{OSC}$ and oscillation amplitude variation of the system after OANF is enabled are shown in Fig. 19 (a) and Fig. 19 (b) respectively. In case of $f_{OSC}>f_{NTF}$, OANF can suppress resonance successfully. However, when $f_{OSC}>f_{NTF}$, OANF causes a lower frequency oscillation with an amplitude even hundreds rpm larger (in the case of $T_{ASR}=1$ms), which is consistent with previous derivations.

B. Resonance Damping Test without Load Torque

For verifying the effectiveness of ROANF, two sets of comparative experiments in the case of $f_{OSC}>f_{NTF}$ and $f_{OSC}=f_{NTF}$ are carried out and the results are shown in Fig. 20. The Fig. 20 (a) and (c) are the speed and current responses with OANF and ROANF. In addition, the Fig. 20 (b) and (d) are FFT results of speed error signal during 5~6s (before notch filter is enabled) and 7~8s (after notch filter is enabled) respectively for Fig. 20 (a) and (c). $A$ of OANF and ROANF is both set as 3A. However, $T_{ASR}$ and $A$ of ROANF are adjusted to 2ms and 0.9A at motor start-up stage. In the experiment, there is an oscillation frequency identifying process about 2s in OANF and ROANF.

In Fig. 20 (a) and (b), when $f_{OSC}>f_{NTF}$, because oscillation frequency has deviated, the enable of OANF lowers the frequency (159.5Hz) and makes its amplitude larger (+178rpm). In contrast, $T_{ASR}$ adjustment of ROANF make $f_{OSC}$ return to $f_{NTF}$ effectively. Then, the correct notch frequency identification achieved a good resonance damping performance.

Fig. 20. Speed and current responses with OANF and ROANF.

Besides, in Fig. 20 (c) and (d), when $f_{OSC}=f_{NTF}$, $T_{ASR}$ is 2ms at first and no more adjustment of $T_{ASR}$ is needed for ROANF. Although both OANF and ROANF make system stable, the adjustment of ROANF’s speed output limit decreases the oscillation amplitude and ensures the safety of the equipment.
In conclusion, experiment results verify the effectiveness of oscillation frequency deviation criterion and the robustness and disturbances rejection ability of ROANF.

VI. CONCLUSION

This paper has classified several kinds of mechanical resonance in elastic coupling drive system. In order to identify the severe self-sustained oscillation in steady state of high-performance digital drive system, an improved digital drive mechanical resonance model has been built up by taking delay of current loop, influence of digital control and saturation nonlinearity in speed loop into consideration. Based on this model, the phenomenon that system’s oscillation frequency deviated from the natural torsional frequency is discussed and an oscillation frequency deviation criterion is obtained. Finally, a robust on-line adaptive notch filter is proposed. Appropriate adjustments of sampling period and output limit of speed loop are used to identify the correct notch frequency and achieve robust and effective resonance damping in different systems. Both the oscillation frequency deviation analysis and the performance of robust on-line adaptive notch filter are verified on an elastic coupling PMSM platform. The future work will focus on analyzing the influence of damping in mechanical resonance frequency and testing other types of suppression methods in systems with oscillation frequency deviation.

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