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Published in:
23rd Nordic Seminar on Computational Mechanics (NSCM23)

Publication date:
2010

Document Version
Accepted author manuscript, peer reviewed version

23rd Nordic Seminar on Computational Mechanics
NSCM-23
A. Eriksson and G. Tibert (Eds)

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Key words: Wave guide, Elastodynamics, Asymptotics, Dispersion curves

Summary. This paper is concerned with wave propagation in a curved elastic layer with rectangular cross section. By applying the classical Lamé formulation of displacements, the exact solutions of equations of elastodynamics for such a layer in the plane and antiplane formulations are compared with their counterparts for a straight layer. Simultaneously, the classical Bernoulli-Euler theory for a curved beam is also used in the comparison and the static limit case of the Lamé decomposition is also investigated.

1 INTRODUCTION

In various industrial applications it is necessary to investigate the wave guide properties of circular elastic layer with rectangular cross section. To carry through this task it is reasonable to develop an idealized method where the wave propagation is considered as uncoupled in terms of in-plane and anti-plane[2] motion. This simplification allows for hierarchy of models to be developed and analyzed whilst aiming at a model to describe out-of-plane wave motion.

2 THEORY

The displacement field in an elastic medium can be expressed in terms of longitudinal and transverse motion[1]:

$$\boldsymbol{u} = \boldsymbol{u_l} + \boldsymbol{u_t} \tag{1}$$

Eq. (1) can also be expressed in terms of potentials:

$$\boldsymbol{u} = grad\varphi + curl\boldsymbol{\psi} \tag{2}$$

Wherein φ and $\boldsymbol{\psi}$ are scalar and vector potentials, respectively.

3 THE ANALYSIS

3.1 Exact solution for in-plane wave motion for straight and curved layer

By applying Eq. (2) to derive an expression for the displacement and stress field in cylindrical coordinates[4], it is possible formulate the dispersion equation for the straight and curved elastic layer, See Figure 1, with the application of different boundary conditions.

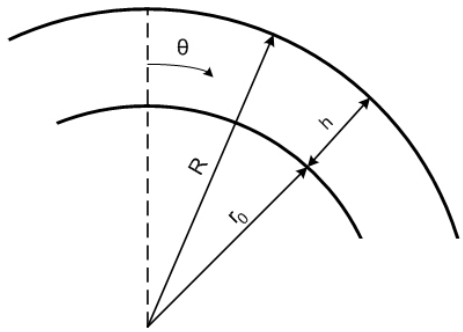

Figure 1: Curved layer

Figure 2 and Figure 3 are showing the dispersion curves for in-plane wave motion for the straight and curved elastic layer, respectively, in plane strain formulation.

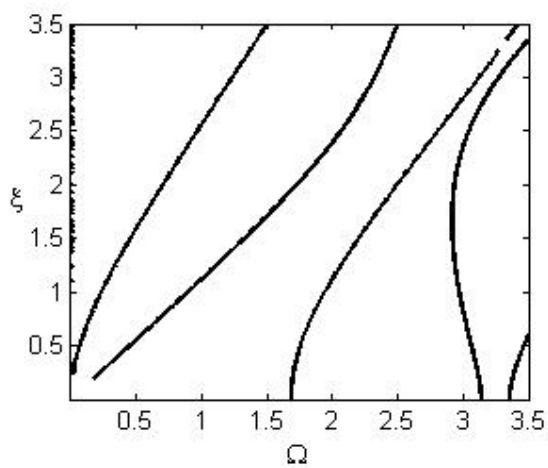

Figure 2: Dispersion curve for a straight elastic layer with free boundaries

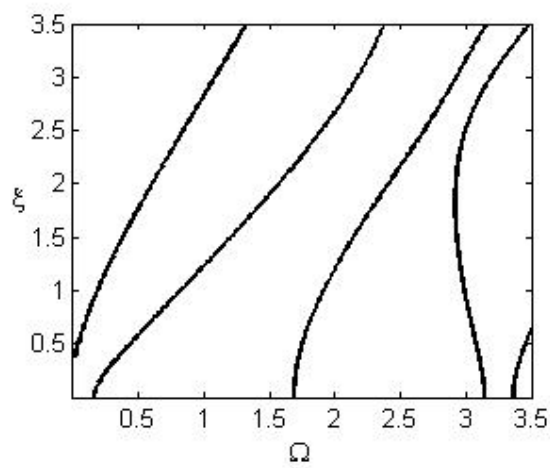

Figure 3: Dispersion curve for a curved elastic layer with free boundaries and a ratio r_0/h=5

As it is observed, there is a good agreement between the dispersion curves for the two different cases even though the ratio r_0/h is small.

3.2 Exact solution for curved layer and curved beam theory

To further analyze the exact solution of the dispersion equation for the curved layer the classical Bernoulli-Euler beam theory is applied[3]. Figure 4 and Figure 5 illustrate the dispersion curves for low and high frequency range, respectively, for r_0/h=5 where Ω is the non-dimensional frequency and ξ is the non-dimensional wave number.

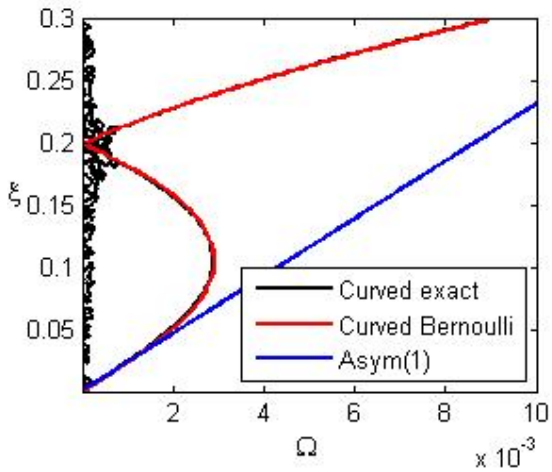

Figure 4: Dispersion curves, low frequency range

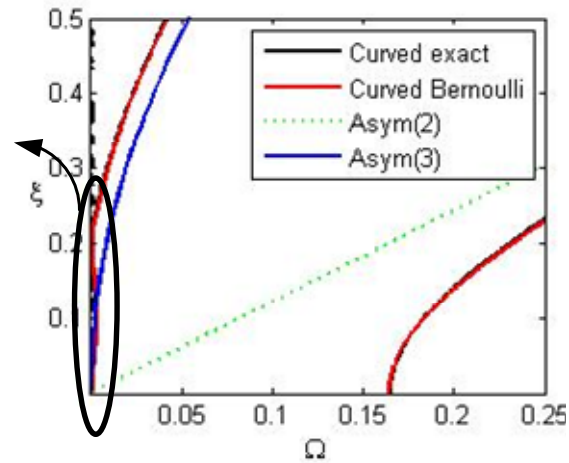

Figure 5: Dispersion curves, low frequency range

Both figures show good agreement between the exact and curved beam solution. The blue line on Figure 4 is the low frequency asymptotic expansion of the form $\xi = 2\sqrt{3}\kappa\Omega$, where $\kappa = \frac{c_l}{c_0}$, c_l is the longitudinal wave speed and $c_0 = \sqrt{\frac{E}{1-\nu^2}}$. On Figure 5 the dottet line represents the high frequency asymptotic for the longitudinal wave which has the form $\xi = \kappa \times \Omega$ and the blue line represents the high frequency asymptotic for the flexural wave in the form $\xi = \sqrt{2\sqrt{3}\kappa\Omega}$. The imperfections on Figure 4 at the low frequencies appear due to numerical instability. It indicates that the Lamé decomposition fails at the static limit.

3.3 Anti-plane

So far in-plane waves have been investigated in plane strain formulation. As the aim of the research project is to formulate the out-of plane waves, the first step is to consider the anti-plane problem formulation. Firstly, the waves are considered as uncoupled, which allows to reduce the expression for the displacement field to only depend upon one scalar potential instead of four as shown in Eq. (1).

The wave motion in the anti-plane problem formulation for the curved layer is a shear wave propagating along the curvature, see Figure 6. Displacements occur in the out-of-plane x-direction, but these are independent on the same coordinate and instead dependent upon the coordinates perpendicular to the x-direction(in-plane coordinates). The latter description is exactly the definition of shear wave motion in anti-plane problem formulation.

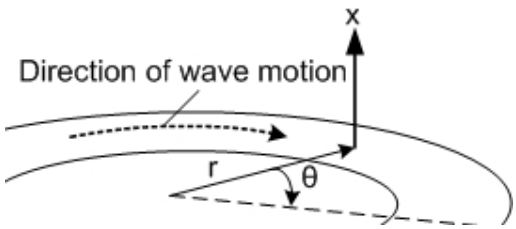

Figure 6: Wave direction

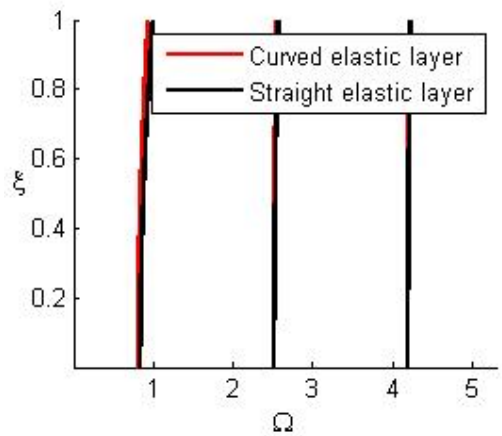

Figure 7: Dispersion curves, anti-plane

Figure 7 illustrates a comparison of the dispersion curves for shear waves in a straight and a curved layer with r_0/h=5, where mixed boundary conditions are introduces. Thereby, it is stated that there is a good agreement between the two cases also with anti-plane problem formulation.

8 CONCLUSION

Based on the comparison of dispersion curves for the straight and curved elastic layer it is concluded the difference is negligible even for relatively large curvatures both in plane and anti-plane formulation. The classical curved beam theory is validated by comparison with the exact solution.

Further work on the low frequency limit for the exact solution should be carried through by applying asymptotic analysis. This is to avoid the numerical instability which occurs when frequency tends to 0.

As an extension to this work, it should be aimed to obtain the dispersion curves with the Wave Finite Element Method(WFE) and compare these with the exact solution. Thereby, a WFE model can be tuned and applied for situations where analytical solutions would not be possible to carry through.

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