Abstract—In this paper, a conceptual multi-zone model for climate control of a live stock building is elaborated. The main challenge of this research is to estimate the parameters of a nonlinear hybrid model. A recursive estimation algorithm, the Extended Kalman Filter (EKF) is implemented for estimation. Since the EKF is sensitive to the initial guess, in the following the estimation process is split up into simple parts and approximate parameters are found with a non recursive least squares method in order to provide good initial values. Results based on experiments from a real life stable facility are presented at the end.

I. INTRODUCTION

In order to improve live-stock production performance, modern stables are equipped with advanced controllers and equipment for providing a convenient indoor climate. Consequently, the failure detection of components and controllers is of crucial importance, as component failures may lead to unacceptable loss of animal productions. Besides, replacing the failed components is time consuming and costly for the farmer. The majority of failure detection methods are model-based, because detection of a fault or failure is easy and reliant on fault free in comparison with faulty model. Overall, there are two methods for modeling, the first one relies on analyzing input and output data and the second one is mathematical modeling which uses physical laws for the system. In [1] it is discussed how to perform a dynamic temperature modeling based on input and output data. In [2], a steady state indoor climate model for pig stable is presented. However; it must be noted that [3, 4] shows a third method which is a combination of the two main ideas such that at first physical laws is utilized to derive a model and thereafter its parameters are estimated by analyzing the input and output data. This is known as grey box modeling in the literature[13].

In reality the airspace inside a large livestock building is incompletely mixed, and this concept has fostered the idea of multi zone climate modeling. Where models separate into non-interacting [5] or interacting zone models [6].

The aim of the work presented here is to derive a model for active fault detection and isolation of the pig stable ventilation system which is validated by a laboratory as a typical equipped stable. The model is an extension of previous research in this laboratory [3, 4] aiming at a more representative model of the real systems. In fact, both previous works were conducted with control objective in mind, where robust control designs allow for less accurate modeling. In addition, standard control design tools restricts the model domain, while performance of fault detection mechanism depends on model accuracy and small improvement on an absolute linear scale may reduce the detection error rate by orders of magnitude. In both [3, 4], the experiment data for estimation of inlets and outlets is provided from manufacturer data sheets, therefore the simulated model do not fit well with the stable measurements. During the research presented here, it is tried to define the model parameters according to the laboratory experiments and rely on a nonlinear estimation method. In [4], the pressure for the entire stable is assumed constant and consequently the stationary flow between zones is considered insignificant in comparison with the incoming and outcoming flows and thus neglected. Whereas, the pressures of zones of the stable are allowed to differ in [3], approximations are introduced by linearization, which reducing model accuracy.

In the present work, the pressure is defined by more precise equations and consequently the stationary flows between zonal borders are included. Due to the indoor and outdoor conditions, the airflow direction varies between any adjacent zones. Therefore, the system behavior is represented with different discrete dynamic equations (piecewise equation). In the literature, these kinds of systems with behavior expressed by piecewise equations are classified as hybrid systems [10].

Multi-zone hybrid models are generally not linear in their parameters and their estimation is one of the challenges for this research. The parameters are estimated by a recursive estimation algorithm, the extended kalman filter (EKF), as it is able to converge precisely to the parameters of the nonlinear hybrid models. Furthermore, the EKF is sensitive to the parameter changes which are useful for online or active fault detection. A data set is acquired from a real scale pig stable. The verification of the prediction and measurement output validates the performance of the simulated model.

The paper is organized as follows: in section 2 descriptions of the mathematical modeling are given. Thereafter the suggested estimation algorithm is presented in section 3. Section 4 represents the experiments setup, and the accomplishments of EKF and modeling by presenting
experimental results are described in section 5. Finally the conclusion and remarks are presented in section 6.

II. MODEL DESCRIPTION

The airspace inside the stable is incompletely mixed, so it is divided into three conceptually homogeneous parts which is called multi-zone climate modeling. Due to the indoor and outdoor conditions, the airflow direction varies between adjacent zones. Therefore, the system behavior is represented with different discrete dynamic equations. In more details, each flow direction depends on its relevant condition (invariant condition) and as long as the condition is met by the states, the system behavior is expressed according to the appropriate dynamic equations. Once the states violates the invariant condition and satisfies a new one, the system behavior is defined with a new equation. A schematic diagram of the stable system is illustrated in Fig. 1, and the general information of the facility of laboratory is given in [4]. More details about the relevant condition for the airflow behavior is defined with a new equation. A schematic diagram of the test stable is illustrated in Fig. 1, and their relevant equations are given:

\[
q^{st}_{i-1,i} = m_1(P_{i-1} - P_i) \quad (1) \\
q^{st}_{i,i+1} = m_2(P_i - P_{i+1}) \quad (2)
\]

where \(P_i\) is pressure inside zone \(i\), which is calculated by the mass balance equation of (10) for every zone. \(m_1\) and \(m_2\) are constant coefficients, and \(q^{st}_{i-1,i}\) and \(q^{st}_{i,i+1}\) are stationary flows.

\[
q^{st}_{i-1,i} = [q^{st}_{i-1,i}]^+ - [q^{st}_{i-1,i}]^- \quad (3)
\]

the use of square brackets is defined as:

\[
[q^{st}_{i-1,i}]^+ = \max(0, q^{st}_{i-1,i}) \quad (4) \\
[q^{st}_{i-1,i}]^- = \min(0, q^{st}_{i-1,i}) \quad (5)
\]

A. Mathematical Modeling

The model is intended to be a realistic representation of internal temperatures for all multi-zone types of livestock buildings. It is divided into subsystems as follows: Inlet model for both windward and leeward, outlet model, and stable heating system, and finally the dynamic model of temperature based on the heat balance equation.

Fig. 1. Schematic diagram of the test stable

![Fig. 1. Schematic diagram of the test stable](image)

Fig. 2. Illustration flow for zone \(i\)

![Fig. 2. Illustration flow for zone \(i\)](image)

1. Inlet Model

An inlet is basically built into an opening in the wall and it consists of a hinged flap for adjusting amount and direction of the incoming air. Compared to the results in [3, 4], the following approximated model for airflow \(q^{in}_{i} [m^2/s]\) into the zone is suggested.

\[
q^{in}_{i} = k_{i}(\alpha + \text{leak})\Delta P_{inlet} \quad (6)
\]

\[
\Delta P_{inlet} = 0.5C_pV_{ref}^2 - P_i + \rho \alpha g \left(1 - \frac{T_{ref}}{T_{in}}\right)(H_{NLP} - H_{inlet}) \quad (7)
\]

where \(k_{i}\) and \(\text{leak}\) are constants, \(\alpha\) is the opening angle of the inlets, \(\Delta P_{inlet}(pa)\) is the pressure difference across the opening area and interfered from thermal buoyancy and wind effect, \(\rho\) is the outside air density, \(V_{ref}\) is the wind speed, \(C_p\) stands for the wind pressure coefficient. \(H\) stands for height and \(H_{NLP}\) is the neutral pressure level which is calculated from mass balance equation [12].

2. Outlets Model

The outlet is a chimney with an electrically controlled fan and plate inside. The following simple linear model is presented according to [3, 4]:

\[
q^{fan}_{i} = u \cdot c_{i} - d \cdot \Delta P_{outlet} \quad (8)
\]

with defining \(\Delta P_{outlet}\) as [11]:

\[
\Delta P_{outlet} = \frac{1}{2} \rho C_{p\text{outlet}} V_{ref}^2 - P_i + \rho g \frac{T_{out} - T_i}{T_i} (H_{NLP} - H_{outlet}) \quad (9)
\]

\[
\sum q_{in}\rho + \sum q_{out}\rho = 0 \quad (10)
\]

where \(c\) and \(d\) are constants and \(u\) is fan voltage.

3. Stable Heating Model

The overall stable heating model is taken from [8] and represented by the equations:

\[
\dot{Q}_{heater} = C_1(T_{in} - T_{win})C_2 \quad (11)
\]

\[
C_1 = m_{heater}c_{water} \quad (12)
\]

\[
C_2 = \exp \left[ -\frac{U_{heater}d_{pipe}}{m_{heater}c_{water}} \right] - 1 \quad (13)
\]
where $m_{heater}$ is the mass flow rate of heating system, the heat capacity is presented by $c_p_{water}$, $T_{in}$ and $T_{win}$ are temperature inside and outside of the stable and incoming flow of the heating system, $U_{heater}$ is the overall average heat transfer coefficient and $A_{pipe}$ is the cross area of the pipe. In order to derive a more precise stable heating model, $C_2$ is estimated from the laboratory experiments.

4. Modeling Climate Dynamics

The following formulation for temperature for each zone inside the stable is driven by thermodynamic laws and given by:

$$M_i c_p T_i \frac{dT_i}{dt} = Q_{i-1,i} + Q_{i,i-1}^+ + Q_{i,i+1}^+ + Q_{i+1,i}^+ + Q_{in,i} + Q_{conv,i} + Q_{source,i}$$

$$Q = m_i c_p T_i$$

$$Q_{i-1,i} = \left[ q_{i-1,i}^+ \right] \cdot \rho_i c_p T_{i-1}$$

$$Q_{i,i-1} = \left[ q_{i,i-1}^- \right] \cdot \rho_i c_p T_i$$

(15)

where $Q_{in,i}$ represents the heat transfer by mass flow through inlet and outlet, $Q_{i-1,i}$ denotes heat exchange from zone $i-1$ to zone $i$ and $Q_{i,i-1}$ presents the heat exchange from zone $i$ to $i-1$ and vice versa for $Q_{i,i+1}$ and $Q_{i+1,i}$ which cause by stationary flow between zones. $Q_{conv}$ is the convective heat loss through the building envelope and described as $U_{wall}(T_i - T_o)$, $Q_{source,i}$ is the heat source and consists of animal heat production and heating system, and finally $m_i$ is the mass flow rate.

As seen in Fig. 3, there are four different directions for the stationary flow in the stable based on defined invariant conditions by pressure as (1-4), which yields four piecewise smooth equations for the indoor temperature of each zone.

In the following the model is presented as hybrid state space equations:

$$\begin{align*}
\frac{dT_i}{dt} &= f(T, u, q), \\
q &= h_3(T, P, u) = \begin{bmatrix} q_{in,1-3} \\ q_{out,1-3} \end{bmatrix}, \\
\alpha &= \begin{bmatrix} a_{i=1,...6} \\ u_{i=1,3} \end{bmatrix}, \\
\nu &= Q_{heater, stable}
\end{align*}$$

(16)

(17)

where $f$ represents the hybrid state space equation for dynamics of the temperature, and flow equations are comprised in $h_3$. $u$ is input, $P$ denote the vector of pressures insides each zone and output of the system is given by $z$, and the $h_2$ function comprises the mass balance equation of (10).

III. PARAMETER ESTIMATION

In order to identify the model parameters by EKF, the state space model must be augmented by parameter variation dynamics:

$$\begin{align*}
X &= \begin{bmatrix} f \\ C \end{bmatrix} = \begin{bmatrix} f(T, u, q) + v \\ 0_{1 \times 1} \end{bmatrix}, \\
q &= h_3(X, P, u), \\
h_2(P, X, u) &= 0, \\
z &= h_3X + w,
\end{align*}$$

(18)

where $C$ is the coefficient matrix with zero dynamics, $w$ is the measurement noise and consequently will be defined from sensor errors and is assumed to be zero mean. $v$ is the process noise and can be estimated from variance error of the actuators and other equipments of the ventilation system. A discrete model is given as:

$$\begin{align*}
X(k) &= f(T_{k-1}, u_{k-1}, q_{k-1}) + \begin{bmatrix} v_{k-1} \\ 0_{1 \times 1} \end{bmatrix}, \\
Z_{k-1} &= h_3(X_{k-1}) + w_{k-1}, \\
q_{k-1} &= h_3(x_{k-1}, P_{k-1}, u_{k-1}), \\
h_2(P_{k-1}, X_{k-1}, u_{k-1}) &= 0
\end{align*}$$

(19)

After extension of the space model with parameters, the next step in the EKF for achieving the estimation step is to linearizing the non-linear discrete equation (19) using a first order Taylor series expansion around the estimate

$$X_{k-1}(-), X_k \equiv f(X_{k-1}(-)) + \phi_X(X_{k-1} - X_{k-1}(-)) + \begin{bmatrix} v_{k-1} \end{bmatrix}.$$  

(20)

where $\phi_X$ is the Jacobian matrix of $f$ with respect to $X$. Since $f$ is a function of $X, u, q$ and their relations are implicit, the chain rule for several variable hypothesis is used to find the Jacobian matrix:

$$\phi_X \equiv \frac{\partial f(X, u, q)}{\partial X} = f_X(X, u, q) \frac{\partial X}{\partial X} + f_q(X, u, q) \frac{\partial q(X, u, P)}{\partial X}$$

(21)

(22)
\[ \frac{\partial q(X,u,P)}{\partial x} = h_3(x,u,P) \frac{\partial X}{\partial x} + h_2(x,u,P) \frac{\partial P}{\partial x} \]  

(23)

according to Eq. 19:

\[ \frac{\partial h_2(x,u,P)}{\partial x} = 0 = h_2(x,u,P) \frac{\partial P}{\partial x} + h_2(x,u,P) \frac{\partial X}{\partial x} \]

\[ \Rightarrow h_2(x,u,P) \frac{\partial P}{\partial x} = -h_2(x,u,P) \]

(24)

with respect that \( h_2(x,u,P) \) is a square matrix, and multiplying the both side of equation with \((h_2(x,u,P))^{-1}\) it can be written as:

\[ \frac{\partial P}{\partial x} = (h_2(x,u,P))^{-1} h_2(x,u,P) \]

and finally with substituting the equation of (23) and (24) in (22) and implementing the appropriate invariant condition due to the equations of (1-4), the Jacobian matrix for the hybrid model will be defined.

The discrete extended kalman algorithm which consists of two steps is presented as follows:

1. Prediction stage:

\[ \hat{X}_k(-) = f_{k-1}(\hat{X}_{k-1}(+)) \]  

(25)

\[ P_k(-) = \varphi_{k-1}P_{k-1}(+)\varphi_{k-1}^T + Q_{k-1} \]  

(26)

2. Update stage

\[ \bar{K}_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \]  

(27)

\[ \hat{X}_k(+) = \hat{X}_k(-) + \bar{K}_k(z_k - \hat{z}_k) \]  

(28)

\[ P_k(+) = (1 - \bar{K}_kH_k)P_k(-) \]  

(29)

where \( Q = E \begin{bmatrix} [V_{k-1}] & [V_{k-1}]^T \\ [0_{1x1}] & [0_{1x1}]^T \end{bmatrix} \) is the covariance matrix of the process noise, and \( R = E[w_{k-1}w_{k-1}^T] \) is the covariance matrix of the measurement noise. \( \bar{K}_k \) is the Kalman gain at time \( t_k \) \( \hat{X}_k(+) \) the expected value of \( X_k \) given the \( k \) measurements, \( \hat{X}_k(-) \) is the predicted state estimation and \( H_k \approx \frac{\partial h_1}{\partial x} |_{x=x_k(-)} \). \n
\[ X_k(+) = E(X_k/z_0, i = 1, \ldots, k + 1). \]  

(30)

\( P_k(-) \) is the covariance matrix of the prediction error

\[ P_k(-) = E[(X_k - X_k(-))(X_k - X_k(-))^T/z_i, i = 1, \ldots, k], \]  

(31)

\( P_k(+) \) is the covariance matrix of the estimation error

\[ P_k(+) = E[(X_k - X_k(+))(X_k - X_k(+))^T/z_i, i = 1, \ldots, k], \]  

(32)

IV. EXPERIMENT OUTLINE

The experimental data were collected from a real large scale live-stock building with slow dynamic behavior with time constants around 10 minutes. The actuator settings (control signals) for ventilation systems are a Pseudo-Random Digital Signal (PRDS) with time granularity of 10 minutes and an amplitude variation. In fact, in order to excite the dynamic of the system, the amplitude of the control signals vary with multi-rate, for example voltage of the fan (substitute inside the chimney) changes between 0-2 volt and after 2 hours it turns to 5-8 volt and so on. There is also a similar scenario for the inlets. Temperature of the stable heating systems is held at 40 degrees with small oscillation; while, the flow of the heating system is fixed. For further information about the experiment design; see [4]. The system was running totally around 7 hours with a two minutes sampling period. The two-third of experiments is implemented for parameter estimation or in other word for constructing the appropriate model and the remaining is utilized for model validation. The experiment was conducted during spring when the deviation of wind is large and this additional disturbance cause more model uncertainty.

In the following, the signals for different inputs are illustrated, as it is shown in Fig. 4.a and b, the ventilation systems changed considerably in order to obtain more temperature deviation for precise validation. The stable heating systems are constant; Fig. 4.c and d

V. RESULTS AND DISCUSSION

The results are divided in two parts, the EKF result and model validation.

A. EKF estimation:

In according to the literature [12], the EKF algorithm is highly depended on prior knowledge of the system and tuning factors, such as initial value of the parameters, process and measurements noise and covariance matrix. In order to find rough estimates of the parameters, the modeling task is divided into several parts. At first a model with single input and output (SISO) is defined, then the results are implemented for a modified model as a multi input-output (MIMO) system and finally the multi-zone system is obtained relying on previous results. The preliminary estimation and parameters of inlet and outlets for the simplified SISO model are conducted by standard least square.

The EKF algorithm is used to estimate 14 parameters. The state and measurement for the EKF are:

\[ x = [T_1, T_2, T_3, m_1, m_2, UA_{wall1}, UA_{wall2}, UA_{wall3}, k_{11}, k_{12}, k_{13}, C_{11}, C_{12}, C_{13}, V_1, V_2, V_3] \]

\[ z = [T_1, T_2, T_3, Q_{out1}, Q_{out2}, Q_{out3}, \Delta P_{in1}, \Delta P_{in2}, \Delta P_{in3}] \]

The initial and final values of the parameters are given in Table 1 and the result of EKF are illustrated in Fig. 5. The figure illustrates the results of prediction error by the constant predictor and the EKF according to the following equations:

\[ \varepsilon_{CP} = \sqrt{\sum (y_{k-1} - \hat{y}_k)^2} \]

\[ \varepsilon_{EKF} = \sqrt{\sum (\hat{y}_k - y_k)^2} \]  

(33)

(34)
where $\hat{y}_k$ is estimated state and $y_k$ is the measurement. As it is clearly seen from Fig. 5, the EKF estimation error is less than the constant prediction error. So, it illustrates the benefits of the recursive estimations algorithm for the case of nonlinear parameter estimation.

B. Model validation

As it was mentioned in the previous part, the dynamic model for the indoor temperature of the stable is derived in multi-steps. At first the approximated parameters of the inlet and outlet are derived from SISO modeling, and thereafter the relevant equations for the stationary flows are analyzed and rough approximation of the parameters of the dynamic temperature equation are defined according to MIMO modeling. Finally the entire relevant parameters are estimated by the EKF.

The result not only yields consistent positive estimation of the parameters value, but also confirms the performance of simulated model in comparison with the measurement. However, the model of the inlet is a simple linear model, the Fig. 6 for the prediction and measurement flow, illustrates that the model quite fits the measurements except for the peak of the graph, where there is a small discrepancy. In Fig. 7, the surface demonstrates the characteristic of the fan with pressure-voltage-flow data for the prediction and measurement data. As it is clearly seen, the linear model almost fits the real data except for the 0 and 10 voltages. This discrepancy yields that the linear model cannot represent well the nonlinear characteristic of the fan for those of points.

The validation is carried out for open loop and with the inputs signals which were not used in the estimation process. Then the simulation output was compared with the measurements. Fig. 8 presents the measurement and predicted data of the indoor temperature of the stable for every zone. It illustrates, that there is non neglectable discrepancy attributed to modeling error. The modeling error can be contributed to several factors such as sharp deviation of wind which

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>VALUES OF THE MODEL COEFFICIENTS</th>
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<tbody>
<tr>
<td>Coefficients</td>
<td>Initial values</td>
</tr>
<tr>
<td>$m_1$</td>
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</tr>
<tr>
<td>$m_2$</td>
<td>1</td>
</tr>
<tr>
<td>$U A_{wall1}$</td>
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<td>$C_d$</td>
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<td>100000</td>
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<tr>
<td>$V_3$</td>
<td>10000</td>
</tr>
</tbody>
</table>
mentioned before, heat capacity of the construction material, the latent heat loose through evaporation, the degree of air mixing, building leakage and large scale livestock building which cause high uncertainty.

VI. CONCLUSION

A conceptual multi zone model for the indoor climate of a live-stock building was derived. The model was nonlinear in its parameters. An Extended Kalman Filter (EKF) was used because it is able to converge to the parameters of the nonlinear hybrid models; besides, the next aim of this research is active fault detection, and recursive estimation methods are well suited for such problems. It must be noted that, the EKF depends on the initial values and tuning factors, hereby a prior knowledge of the system is required. An experiment confirmed the performance of the EKF and generally the multi-zone model, which tracks the trace of real data; however, some discrepancy between predicted and measurement values were observed. The model uncertainty is an unavoidable aspect of model identification and here related to undesirable environment disturbances.

Future work will address open questions for using analytical input-output modeling instead of grey box.

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REFERENCES