



**AALBORG UNIVERSITY**  
DENMARK

**Aalborg Universitet**

Efficient Finite Element Calculation of Ny

Clausen, Johan; Damkilde, Lars; Krabbenhøft, K.

*Published in:*

Proceedings of the Eleventh International Conference on Civil, Structural and Environmental Engineering Computing

*Publication date:*

2007

*Document Version*

Publisher's PDF, also known as Version of record

[Link to publication from Aalborg University](#)

*Citation for published version (APA):*

Clausen, J., Damkilde, L., & Krabbenhøft, K. (2007). Efficient Finite Element Calculation of Ny. In B. H. V. Topping (Ed.), *Proceedings of the Eleventh International Conference on Civil, Structural and Environmental Engineering Computing* Civil-Comp Press.

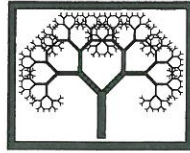
#### **General rights**

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal -

#### **Take down policy**

If you believe that this document breaches copyright please contact us at [vbn@aub.aau.dk](mailto:vbn@aub.aau.dk) providing details, and we will remove access to the work immediately and investigate your claim.



©Civil-Comp Press, 2007

Proceedings of the Eleventh International Conference on  
Civil, Structural and Environmental Engineering Computing,  
B.H.V. Topping, (Editor),  
Civil-Comp Press, Stirlingshire, Scotland, paper 20, 2007.

## Paper 20

# Efficient Finite Element Calculation of $N_\gamma$

J. Clausen<sup>1</sup>, L. Damkilde<sup>2</sup> and K. Krabbenhøft<sup>3</sup>

<sup>1</sup>*Department of Civil Engineering, Aalborg University, Denmark*

<sup>2</sup>*Esbjerg Institute of Technology, Aalborg University, Esbjerg, Denmark*

<sup>3</sup>*Centre for Geotechnical and Materials Modelling, University of Newcastle, New South Wales, Australia*

**Keywords:** Mohr-Coulomb, bearing capacity, implicit integration, stress update, elasto-plastic constitutive matrix, non-linear FEM.

This paper deals with the computational aspects of the Mohr-Coulomb material model, in particular the calculation of the bearing capacity factor  $N_\gamma$  for a strip and circular footing. The main focus is on the stress update algorithm for the elasto-plastic finite element computations. The merits of a return mapping algorithm which carries out all the manipulations in principal stress space will be examined and exemplified for a Mohr-Coulomb material. The method was first presented in ref. [1].

By simple geometric means methods of determining which type of stress return should be invoked are presented. For each type of return, i.e. return to a plane, return to a corner and return to the apex, explicit formulae for the updated stress and the corresponding constitutive matrix is presented. When expressed in the principal stress space these formulae turn out to be extremely simple, when compared to the return mapping implementation in the general stress space of for example Crisfield [2].

The Mohr-Coulomb material model is known to cause problems in numerical calculations, due to the singularities present at the corners and at the apex. In some applications these problems are overcome by slightly modifying the criterion by rounding of the corners and thereby eliminate the singularities. The drawback of this modification is that calculated results, in general, will not converge towards the exact solutions. With the presented method the singularities are handled in a simple and robust manner and therefore no problems are experienced, and it is shown that the obtained results converge toward the exact values with great precision. Also the efficiency of different methods of handling the corner and apex singularities is evaluated.

Some comments are also given as how to mend some of the problems of the general return mapping implementation of Crisfield [2].

## References

- [1] J. Clausen, L. Damkilde and L. Andersen. Efficient return algorithms for associated plasticity with multiple yield planes, *International Journal for Numerical Methods in Engineering*, 66:1036–1059, 2006.
- [2] M.A. Crisfield. *Non-Linear Finite Element Analysis of Solids and Structures. Vol. 2: Advanced topics*, John Wiley & Sons, 1997.