Experimental verification of an empirical model for time variation of overtopping discharge

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ABSTRACT

An empirical model for time variation of overtopping discharge is verified through a comparison to experimental data. The verification is performed for a parameter range covering overtopping ramps typically used in wave energy converters utilizing overtopping.

1 INTRODUCTION

In the present paper an empirical model for time variation of overtopping discharge is verified through a comparison to experimental data. The motivation for this is that little or no knowledge is presently available regarding the time variation of overtopping discharge for ramp layouts typical for wave energy converters of the overtopping type. When utilizing wave overtopping for power production the overtopping waves typically enters a reservoir before the water is lead through a turbine back to the sea. In order to optimize the reservoir size and the control strategy for the turbines, so the loss of energy in reservoir and turbines are minimized, it is important to know how the irregular nature of ocean waves influence the variation of the overtopping discharge.

In literature the main focus has so far been on mean overtopping discharge for sea defense structures like seawalls, breakwaters and dikes, see e.g. Burcharth and Hughes (2000) or Van der Meer (1998) for an overview. In some cases also the probability of an overtopping event, as well as the distribution of the largest overtopping volumes (e.g. the mean overtopping volume from the 1/250 largest overtopping events) have been investigated. However, as the objective of these studies mainly have been to investigate extreme overtopping events for sea defence structures, designed to avoid or at least limit the amount of overtopping, they cannot in general be expected to cover the parameter ranges that are of interest for wave energy devices, where generally maximum potential energy of overtopping volumes is wanted. Thus, in the present study the attention is specially directed to situations with small values of the relative crest freeboard \( R = R_c/H_s \) (smaller than, say, 0.75). The equations given by Van der Meer & Janssen (1995), on which the model for the time variation of overtopping is based, have been developed for breakwaters and dikes that typically have larger values of \( R \) and for this reason a verification of the model is necessary. As stated by Kofoed & Frigaard (2000) the overtopping formula given by Van der Meer & Janssen (1995) over-predicts the mean overtopping discharge for low \( R \) values.

The experimental results used in this paper for the verification of the model for time variation of overtopping discharge is based on laboratory measurements of overtopping discharge for ramps suitable for use in wave energy devices utilizing overtopping, see Kofoed (2000) or Kofoed and Frigaard (2000).

2 EXISTING EMPIRICAL MODEL

In this investigation of the overtopping discharge time dependency experimental
results are used to compare with the method used by Jakobsen & Frigaard (1999) for simulating the time variation of overtopping discharge in the power simulation software for the wave energy converter Wave Dragon (WDpower). In this case a time series of the overtopping discharge is necessary to know for a given wave situation and overtopping ramp geometry in order to enable calculation of power output from the turbines in the wave energy converter. In figure 1 an example of the results of a simulation performed using WDpower is shown. Based on such simulations turbine configurations and control strategies can be tested, see Madsen & Frigaard (2000).

If this comparison shows that the method for simulating the time variation of overtopping discharge is valid for the parameter ranges typical for wave energy converters of the overtopping type then also wanted time dependent overtopping discharge parameters can be calculated using this method.

As described in Jakobsen & Frigaard (1999) the overtopping discharge used in the software WDpower are calculated using the expression for probability of overtopping \( P_{ot} \) given by Van der Meer & Janssen (1995):

\[
P_{ot} = e^{-\left(\frac{H_s}{R_c}\right)^2} \quad (1)
\]

where \( H_s \) is the significant wave height, \( R_c \) is the crest freeboard and \( c \) is a constant set to 1.21.

Furthermore, the following expression (also given by Van der Meer & Janssen (1995)) for the probability \( P_{V_w} \) of a certain overtopping volume in a wave \( V_w \) given that overtopping occurs, is used to calculate the volume of an overtopping wave:

\[
P_{V_w} = 1 - e^{-\left(\frac{V_w}{a}\right)^2} \quad , \quad a = 0.84 \frac{q T_m}{P_{ot}} \quad \Leftrightarrow \quad V_w = 0.84 \frac{q T_m}{P_{ot}} \left(- \ln\left(1 - P_{V_w}\right)\right)^{1/2} \quad (2)
\]

where \( q \) is the mean overtopping discharge and \( T_m \) is the mean wave period.
2.1 Simulation procedure

In order to calculate a time series of overtopping volumes the following recipe is used:

- $P_{ot}$ is calculated using eqn. 1.
- $q$ is calculated using some overtopping formula or as in this investigation simply taken from a model test.
- For a chosen number of waves $N$ (each assumed to be $T_m$ long) the following is done:
  - A random number $p$ between 0 and 1 is drawn
  - If $p > P_{ot}$ then $V_w^i$ is set to 0, else $V_w^i$ is calculated using eqn. 2.
- The obtained series of $V_w^i$'s ($V_w^1$ to $V_w^{N}$) is then converted into a discharge time series $q_{sim}(t)$ in order to enable a comparison with a measured discharge time series from the model tests $q_{meas}(t)$.

The idea is then to compare $q_{sim}(t)$ and $q_{meas}(t)$ in order to see whether the used method for simulating the time variation of overtopping discharge is applicable for situations typical for wave energy converters of the overtopping type and not only breakwaters, dikes etc. for which the used equations have been developed.

2.2 Comparison of simulated and experimental data

The comparison of $q_{sim}(t)$ and $q_{meas}(t)$ is done by comparing the results of an analysis done in the following way for each of the discharge time series:

- The discharge time series is divided into $N_{window}$ sub-series each $T_{window}$ long.
- For each of the sub-series the average discharge is calculated so $N_{window}$ average discharge values $q_{window}^i$ (for $i = 1 .. N_{window}$) are obtained.
- Each of the values $q_{window}^i$ are normalized by the average discharge of the whole time series $q$ ($q_{window}^i/q$) and average (which should be 1) and the standard deviation of these values are calculated.

If the results from the analyses of the two time series are the same it can be concluded that the simulation method models what is measured.

Figure 2: Photos from the model tests. (A curved, not a linear, ramp profile is shown.)
In the study on optimisation of overtopping ramps described in Kofoed (2000) a very large amount of model tests have been carried out. A large variety of geometric setups and wave conditions have been tested. However in this investigation one series of tests have been selected for the analysis.

### 3.1 Experimental setup

In the tests used in the present study a ramp with limited draught (modelling a floating overtopping device) has been used. The geometry of the ramp is as follows:

- Linear ramp profile.
- Relative draught \( d_r/d = 0.4 \) (\( d_r \), draught, water depth \( d = 0.50 \) m).
- Crest freeboard normalized by water depth \( R_c/d = 0.1 \).
- Ramp angle \( \alpha = 30^\circ \).

See figure 2 for photos from the model test setup, and figure 3 for a sketch of the model test setup.

### 3.2 Wave conditions

It has been chosen to look at 2 wave situations both with a peak period \( T_p = 1.13 \) s and significant wave height \( H_s = 0.08 \) and 0.16 m, respectively. In both tests irregular 2-D waves are used. The irregular waves are generated using the parameterised JONSWAP-spectrum with a spectral enhancement factor \( \gamma = 3.3 \), corresponding to a location in the Danish part of the North Sea.

These wave conditions results in the following characteristic parameters:

- Relative crest freeboard \( R = 0.61 \) and 0.37.
- Wave steepness \( s_p = 4.3 \% \) and 8.6 %.

The reason for choosing these combinations of geometry and wave conditions for the analysis is that the resulting relative crest freeboard \( R \) is typical for what is reasonable to use in a wave energy converter utilizing wave overtopping.
3.3 Overtopping measuring technique

In the model tests the range of the overtopping discharge have been very large due to the large range of wave conditions and geometries tested. Therefore, the design of the overtopping measuring system is a compromise between being able to measure very large and very small amounts of overtopping.

The system that has been chosen is shown in figure 3. The system consists of a reservoir, a pump and a wave level gauge. The reservoir is placed beside the overtopping ramp in order to allow waves to pass under the ramp, as this is not extending to the bottom. Between the ramp and the reservoir a perforated damping wall is placed in order to decrease the amount of disturbance in the water level measurements done by the water level gauge. The water level gauge and the pump are connected to a PC that monitor and record the water level in reservoir. Once a preset maximum water level is reached the pump is activated for a fixed time period (3 s in the used setup) and the pump volume of water is then known from a calibration of the pump (approx. 100 l in the used setup).

Based on the measured water level in the reservoir the overtopping volume, and thereby also the discharge, during a test can be found. Furthermore, as the water level in the reservoir is measured continuously, the overtopping discharge time series \( q_{\text{meas}}(t) \) during each test can be calculated by differentiation, see figure 4. When calculating \( q_{\text{meas}}(t) \) the signal from the water level gauge is corrected by adding a piece of water level time series measured during the calibration of the pump at the time where the pump is emptying the reservoir. This is done in order to ensure that a continuous overtopping discharge time series \( q_{\text{meas}}(t) \) is obtained. Though, in spite all efforts it has not been possible to make a perfect correction, which means that \( q_{\text{meas}}(t) \) is not completely correct at the time of the pumping. This can

![Figure 4: Example of water level time series measured in the overtopping reservoir (top) and the corresponding calculated overtopping discharge time series (bottom).](image)
also be seen from figure 4 where it is seen that the overtopping discharge sometimes is negative. A negative discharge can of course not occur, but is an effect of the problems at the time of pumping (the large negative peaks) and the fact that disturbances in the water level measurements occurs due to small waves in the reservoir. However, if the average overtopping discharge is calculated even for very small windows \( T_{\text{window}} \) (down to the order of 10 s) these will be correct also although pumping should occur within \( T_{\text{window}} \). Another reason for not using window sizes smaller than approx. 10 s is the fact that the measured water level in the reservoir is delayed and smoothed by distance from the ramp and the basin, and the perforated damping wall.

Thus, it is actually these difficulties given by the measuring technique that is the reason for using the indirect method for verification of the model for overtopping discharge time variation applied in this paper.

### 4 TEST RESULTS AND COMPARISON WITH EMPIRICAL MODEL

For each of the 2 tests chosen for this analysis the comparison is done using a window size of 60 s in model scale, corresponding to approx. 60 waves. The results of this are shown in figure 5.

Furthermore, the analysis have been done using different values for \( T_{\text{window}} \) for the test with \( R = 0.61 \). The results of this are given in figure 6.

In table 1 the standard deviations of \( q_{\text{window}}/q \) \((i = 1 \ldots N_{\text{window}})\) for \( q_{\text{meas}}(t) \) and \( q_{\text{sim}}(t) \) is given along with the ratio between these.

<table>
<thead>
<tr>
<th>( R = 0.61, T_{\text{window}} = 300 \text{ s (5 sub-series)} )</th>
<th>( T_{\text{window}} = 300 \text{ s (5 sub-series)} )</th>
<th>( T_{\text{window}} = 120 \text{ s (15 sub-series)} )</th>
<th>( T_{\text{window}} = 60 \text{ s (30 sub-series)} )</th>
<th>( T_{\text{window}} = 30 \text{ s (60 sub-series)} )</th>
<th>( T_{\text{window}} = 10 \text{ s (80 sub-series)} )</th>
<th>( T_{\text{window}} = 60 \text{ s (30 sub-series)} )</th>
<th>( R = 0.37, T_{\text{window}} = 60 \text{ s (30 sub-series)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>St. dev. (( q_{\text{window}}/q )) for ( q_{\text{meas}}(t) )</td>
<td>0.12</td>
<td>0.17</td>
<td>0.26</td>
<td>0.39</td>
<td>0.57</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>St. dev. (( q_{\text{window}}/q )) for ( q_{\text{sim}}(t) )</td>
<td>0.10</td>
<td>0.16</td>
<td>0.20</td>
<td>0.28</td>
<td>0.50</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>Ratio</td>
<td>1.20</td>
<td>1.06</td>
<td>1.30</td>
<td>1.39</td>
<td>1.14</td>
<td>0.89</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Standard deviations of \( q_{\text{window}}/q \) \((i = 1 \ldots N_{\text{window}})\) for \( q_{\text{meas}}(t) \) and \( q_{\text{sim}}(t) \), and the ratios between these.

From the presented results the following can be observed:

- For the tests with \( R = 0.61 \) and 0.37 with \( T_{\text{window}} = 60 \text{ s} \) (figure 5) it is seen that good agreement is found between the analysis of \( q_{\text{sim}}(t) \) and \( q_{\text{meas}}(t) \). However, from table 1 it is see that for the test with \( R = 0.61 \) the standard deviation for \( q_{\text{meas}}(t) \) is 30 % larger than for \( q_{\text{sim}}(t) \), while for the test with \( R = 0.37 \) the standard deviation for \( q_{\text{meas}}(t) \) is 11 % smaller than for \( q_{\text{sim}}(t) \). For the simulation of overtopping for the evaluation of turbine configuration etc. in a wave energy converter these deviations are considered acceptable.

- From results for the test with \( R = 0.61 \) and varying \( T_{\text{window}} \) (figure 6) it is seen that the standard deviation for \( q_{\text{meas}}(t) \) is larger (6 – 39 %) than for \( q_{\text{sim}}(t) \) for all values of \( T_{\text{window}} \). Thus the tendency is in general the same as seen for \( T_{\text{window}} = 60 \text{ s} \).

- From results for the test with \( R = 0.61 \) and \( T_{\text{window}} = 30 \text{ and 10 s (60 sub-series)} \) (figure 6) is is seen that \( q_{\text{window}}/q \) for a few subseries is negative. This supports that the limit of how small a value of \( T_{\text{window}} \) for which the analysis is reasonable is approx. 10 s corresponding to the order of 10 waves.

- For both \( q_{\text{meas}}(t) \) and \( q_{\text{sim}}(t) \) it is seen from table 1 that the standard deviation of \( q_{\text{window}}/q \) decrease for increasing \( T_{\text{window}} \).
Figure 5: Results for the 2 tests with $R = 0.61$ (left) and 0.37 right. The accumulated probability density for $\frac{q_{\text{window}}}{q}$ is plotted for $q_{\text{sim}}(t)$ and $q_{\text{meas}}(t)$, respectively.

Figure 6: Results for test with $R = 0.61$ where different $T_{\text{window}}$ have been applied ($T_{\text{window}} = 300$ s (top), 120 s (middle left), 60 s (middle right), 30 s (bottom left) and 10 s (bottom right), respectively). The accumulated probability density for $\frac{q_{\text{window}}}{q}$ is plotted for $q_{\text{sim}}(t)$ and $q_{\text{meas}}(t)$, respectively.
5 CONCLUSION

The objective of the investigation has been to verify an existing empirical model for the time variation of overtopping discharge in order to justify the use of the model in a parameter range outside the range for which the used equations (eqn. 1 and 2) has originally been established. This has been done by comparing experimental data with data simulated by the method used by Jakobsen & Frigaard (1999). This comparison showed a reasonable agreement between the measured and simulated data and it is therefore concluded that the method is applicable also for low values of relative crest freeboards that are typical for ramps used in wave energy converters utilizing the overtopping principle.

Due limitations in the applied measuring technique utilized in the model tests used for the comparison, it has not been possible to verify the simulation method on a wave to wave basis, but only the time variation of the overtopping discharge down to a resolution of approx. 10 waves. This is, however, considered satisfactory for evaluation of turbine configuration etc. in a wave energy converter as it is not realistic to control and adjust turbines etc. more often than for each 10 waves.

However, the fact that the simulated overtopping discharge time series results in standard deviations of \(q_{\text{window}}/q\) \((i = 1 \ldots N_{\text{window}})\) that for one case is smaller than the measured, and in another case it is larger indicates that the simulation method probably can be improved by modifying eqn. 1 and/or 2. It is intended to do a more detailed analysis in the future in order to investigate the need for a modification more thoroughly.

6 ACKNOWLEDGEMENT

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REFERENCES


