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Man-Induced Vibrations

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Introduction

Human motion can cause various types of periodic or transient dynamic loads. The periodic loads are mainly due to jumping, running, dancing, walking and body rocking. Transient loads primarily result from single impulse loads, such as jumping and falling from elevated positions. The response to these loads are of primary interest for the structural engineer, whereas the exact load as a function of time generally is of minor importance. This is true when the loading time (contact duration) $t_p$ is small compared to the largest natural periods $T_n = 2\pi/\omega_n$ of the structure. The present study is mainly concerned with spectator-induced vertical vibrations on grandstands. The idea is to use impulse response analysis and base the load description on the load impulse. If the method is feasible, it could be used in connection with the formulation of requirements in building codes.

During the last two decades work has been done on the measurement of the exact load functions and related response analysis. A recent work using a spectral description has been performed by Per-Erik Erikson [9] and includes a good literature survey. Bachmann and Ammann [1] give a good overview of vibrations caused by human activity. Other relevant references have been included in the reference list.

Periodic motion

The forces acting on a human body performing periodic motion can be decomposed in several ways. In this section the vertical motion is considered. A body shown in figure 1 (left) with mass $m$ is acted upon by a gravitation force $F_g = mg$, a constant reaction force $F_c = \gamma mg$ and a dynamic force $F_d$. The constant reaction force $F_c$ exists, if the body is in continuous contact with a structure and it is the
minimum value of the total reaction force \( R = F_c + F_d \). The remaining force is the dynamic force \( F_d \geq 0 \). The motion of the body is determined by

\[ m\ddot{x} = F_d + F_c - F_g \]  

(1)

The momentum of the periodic motion is also periodic. Conservation of momentum over one time period \( T_p \) is found as

\[ \int_0^{T_p} (F_d + F_c - F_g) \, dt = 0 \]  

(2)

Upon inserting the constant forces \( F_d \) and \( F_c \) the equation (2) yields the periodic impulse \( I \) of the dynamic force:

\[ I = \int_0^{T_p} F_d \, dt = (1 - \gamma)mgT_p \]  

(3)

Simple human motions can often be modelled by a few periodic impulses. The load-time function of walking, shown in figure 1 (right) could be modelled by a constant reaction force \( F_c \) and one periodic load function \( F_d \) (impulse). In a simple model \( F_d \) would consist of periodic Dirac impulses corresponding to impacts on the structure. Between impacts the body moves in a conservative force field. The simple model then corresponds to bouncing of a ball. Anticipating that the center of mass of the body moves a distance \( h \) in the vertical direction, the maximum change in potential energy is \( (1 - \gamma)mgh \). Setting the maximum potential energy equal to the kinetic energy before impact \( \frac{1}{2}mu^2 \) yields the impact velocity:

\[ v = \sqrt{2(1 - \gamma)gh} \]  

(4)
The change in momentum $\Delta P$ due to impact is determined by the mass and the change in velocity $2u$:

$$\Delta P = 2mv = 2m\sqrt{2(1 - \gamma)gh}$$

Setting the momentum change equal to the impulse of the dynamic force $I = \Delta P$ yields

$$\gamma = 1 - \frac{8h}{gT_p^2}$$

For discontinous contact such as jumping ($\gamma = 0$) equation (6) gives a direct mechanical link between the motion height and the time period, eg. $h = \frac{1}{8}gT_p^2$. For continous contact it can be used for estimation of the static reaction force. Using a load time function from brisk walk from Baumann & Bachmann [2] with $T_p = 0.5s$ and $\gamma = 0.53$, shown in figure 1 (right), the vertical movement of the center of mass becomes $h = 0.14m$.

In the case of human motion we can estimate the movement of the center of mass, whereby it is possible using equation and (3) and (5) to find the impulse needed for the periodic motion. If the human motion is known as a function of time, the time derivative of the momentum gives the total force on the body $F = \frac{d}{dt}(mv)$. This would enable determination of the load function.

**Impulse response**

The structural response to the impulsive loads from for example human motion is considered in the next two sections, firstly for a single impulse load and secondly for one periodic impulse load.

For a single degree of freedom system with (structural) mass $M$, undamped eigenfrequency $\omega_n$ and damping ratio $\xi$ the displacement response to a Dirac impulse $I$ is given by

$$\delta = \frac{I}{M\omega_d}e^{-\xi\omega_n t} \sin \omega_d t$$

where the damped eigenfrequency is $\omega_d = \omega_n \sqrt{1 - \xi^2}$. For impulses of finite time duration $t_p$, the shape of the force time function has to be taken into account. For different impulse shapes figure 2 (left) shows the dynamic magnification factor $\kappa$ corresponding to the ratio between the maximum dynamic and static response. The magnification factor has been calculated using a damping ratio of $\xi = 0.05$, but could conservatively be calculated for zero damping.

An impulse correction factor $\alpha$ can be found by normalizing the maximum dynamic response with the Dirac impulse response. Figure 2 (right) shows the impulse correction factor for different impulse shapes. An approximation of the impulse response can thus be obtained by using the Dirac impulse response (7) multiplied by the correction factor $\alpha$. 


Figure 2: Dynamic magnification factor $\kappa$ and impulse shape correction factor $\alpha$.

**Periodic impulse response**

For a Dirac impulse acting periodically with the period $T_p$ on the damped single degree of freedom system, the response has been found analytically as

$$\delta = \frac{I}{M\omega_d} e^{-t\omega_n} \left( \frac{A}{C} \sin \omega_d t + \frac{B}{C} \cos \omega_d t \right)$$  \hspace{1cm} (8)

where the constants are determined as

$$A = 1 - e^{-t\omega_n T_p} \cos \omega_d T_p$$
$$B = e^{-t\omega_n T_p} \sin \omega_d T_p$$
$$C = 1 - 2e^{-t\omega_n T_p} \cos \omega_d T_p + e^{-2t\omega_n T_p}$$

The maximum displacement response is found at the time

$$t_{ext} = \frac{1}{\omega_d} \arctan \left( \frac{\xi \omega_n B - \omega_d A}{\xi \omega_n A + \omega_d B} \right) + \frac{n\pi}{\omega_d}$$  \hspace{1cm} (9)

where $n$ is the lowest integer for which $t_{ext} > 0$. For multiple periodic Dirac impulses superposition can be used and $t_{ext}$ would have to be found for the superimposed responses.

Using a Fourier series solution to find the "correct" response of the periodic half-sine impulse it is possible to compare the maximum response with that of the periodic Dirac impulse response multiplied by the impulse shape correction factor.
Computing the results for a variety of $t_p/T_n$ ratios and for $T_p/T_n$ ratios in the interval $[0,4]$ shows that the scaled periodic Dirac impulse response is a good approximation. For $t_p/T_n = 0.8$ the results are shown in figure 3. It is seen that the agreement is good even for this large contact duration compared to the natural period. For the case $t_p/T_n = 0.8$ shown in figure 3, it is worth noting that below $T_p/T_n = 0.8$ the impulse time durations overlap, resulting in a static response from the constant reaction part $F_d$ of the load. This has currently not been further investigated.

**Experimental ideas**

The experimental part of this research project will monitor the movements of the person or persons participating in the experiment, so that it will be possible to estimate the movement of the center of mass. A simple model of a person could consist of 2 parts for each leg and arm, one part for the body and one for the head, thus giving 10 rigid moving parts. The center of mass would have to be confirmed with medical research.

The experiments will be carried out in two main phases, one in the laboratory and another at a grandstand at a rock concert or a football match. In the laboratory there will probably be three stages. In the first stage the body motion and the load as a function of time will be measured on a small platform mounted on a stiff laboratory floor for jumping and the special wave motion ("the wave" seen at football matches). In the second stage the load measuring platform will be mounted
on a simple beam structure and the measurements repeated on the beam both on
and off the platform. The eigenfrequency of the beam structure may be varied by
altering support conditions. In the third stage the effect of multiple persons on the
beam will be measured.

At the grandstand measurements will be performed for one person jumping, for
multiple persons jumping and for real situations either at concerts or at football
matches.

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