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Optimal Production Planning of a Power Plant

Martin Kragelund, Ulf Jönsson, John Leth, and Rafał Wisniewski

Abstract— **This paper addresses the problem of planning the usage of actuators optimally in an economic perspective. The objective is to maximize the profit of operating a given plant during 24 hours of operation. Models of two business objectives are formulated in terms of system states and the monetary value of these objectives is established. Based on these and the cost of using the different actuators a profit function has been formulated. The optimization of the profit is formulated as an optimal control problem where the constraints include the dynamics of the plant as well as a requirement to reference tracking. A power plant is considered in this paper, where the fuel system consists of three different fuels; coal, gas, and oil.**

I. Introduction

The requirements for a complex process control system are usually derived from a top level (business) requirement to the entire system which is to maximize the income or profit of a company. However, the requirements specification for a process control system rarely includes profit maximization directly. Instead the designer works with requirements on settling time, rise time, bandwidth, disturbance rejection and so on, because these are easy to evaluate through simulation and are well defined with respect to transfer functions and the pole placement of the closed loop system. All of these measures assume that a set of actuators and sensors is given. The choice of this set of actuators and sensors does, however, influences the operating cost and performance of the system greatly - this will be addressed in this paper.

The economical cost of instrumenting a plant with sensors and actuators has, on the other hand, been considered in the selection method presented in [1], where the precision of a sensor or an actuator is assumed to be proportional to its cost. By introducing a bound on the economical cost of the instrumentation it is possible to formulate the design problem as convex optimization. This helps the designer to select the right instrumentation. However, this method only considers the implementation cost and not the operational cost which in many cases is the main concern for minimization [2].

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As the requirements for a process control system usually are derived from business objectives it would be natural to include these business objectives when configuring the sensor/actuator layout of a plant. An attempt of this has been presented in [3] where functionals describing the business objectives are maximized. The functionals have been established using data from nordpool[1] which is a marketplace for trading power contracts. This marketplace has also been used by [4] where the control of water resources in Norway is considered. An optimization of how to use different hydro plants is performed on basis of market prices and commitments.

This work will extend the work in [5] where notions from production economics have been used to formulate the objectives of a Danish power plant company. The outputs of the system are measures of the business objectives and the input is the flow of fuels. The optimization performed in [5] does not consider the dynamics of the plant and assumes that it is possible to switch from one fuel to another instantaneously.

In this work the dynamics of the fuel systems are included in the optimization and it is shown that the dynamics influence the gain in profit. Our result is a production strategy which maximizes the profit during 24 hours of operation.

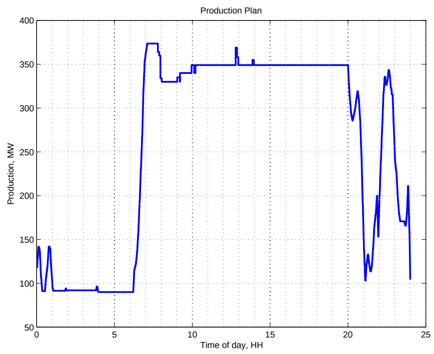
A. Outline

A description of the problem considered in this paper is presented in Section II and the relevant models are then developed in Section III. These include the time varying parameters, the dynamics of the plant, and measures of the business objectives. In Section IV the problem is stated in mathematical terms as an optimal control problem. The optimal control problem is discretized using zero-order hold sampling and the resulting optimization problem is approximated by a linear program. The numerical results are presented in Section V and some final remarks are made in Section VI.

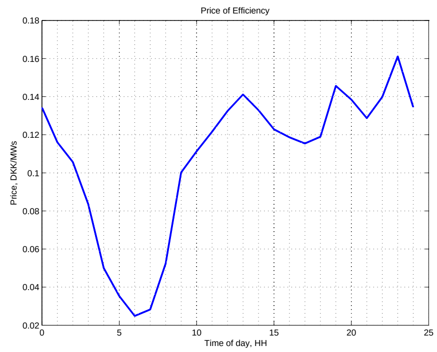
II. Problem Description

The problem in this work has been formulated in collaboration with DONG Energy - a Danish power provider. The goal of any company is to maximize its profit and for DONG Energy the profit maximization has been divided into four individual business objectives; efficiency, controllability, availability, and life time. However, to simplify the model, only the two first objectives

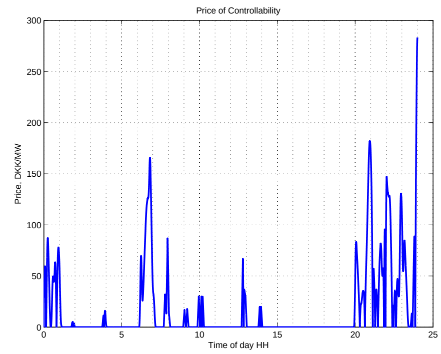
[1]www.nordpool.dk



1 The production during June 29th, 2008. The data used to generate this plot has been provided by DONG Energy.



2 The electricity price during June 29th, 2008. The data used to generate this plot has been found on www.nordpool.dk



3 Modelled controllability price during the 29th of June, 2008. The data in this plot has been established in collaboration with DONG Energy.

are considered in this work. The problem formulation is based on a model of a coal fired boiler - a vital component of a power plant - which is augmented with two additional fuel systems; gas and oil. The three different fuels have certain advantages and disadvantages e.g. gas is easy to control but is an expensive fuel. Some of the characteristics of the different fuels are:

Coal is advantageous when considering the price per stored energy, however, it is difficult to control as unmeasurable fluctuations in the coal flow are introduced by the coal mill when the coal is ground to coal dust. This implies that changing the operating point of the system should be done slowly. Furthermore, the coal mills use some electrical energy to grind the coal which needs to be considered.

Gas is more expensive than coal and energy is not converted to steam as efficiently with gas as with coal due to the layout of the chosen boiler. However, gas arrives at the power plant under high pressure which is lowered using a turbine generating electrical energy. Furthermore, gas is much easier to control as it is possible to measure the flow.

Oil is, with the current market prices, the most expensive of the three fuels and has to be heated before entering the boiler. This process demands energy itself. Nevertheless, oil is considered in this work as it is possible to measure the oil flow into the boiler and this makes it easy to control. Furthermore, oil is present in most existing coal fired plants as oil is used to start up the plant.

The focus of this work is to derive a plan for optimal usage of the three fuels described above during 24 hours of operation. Optimal usage is defined as maximizing the profit in terms of the two considered objectives; efficiency and controllability. Efficiency is a measure of how efficient a fuel is converted into electricity and controllability is a measure of the plant's capability to change the production level. Furthermore, the production level of the plant should follow a time varying reference as closely as possible.

III. Plant Model

Due to changes in demand the electricity production of a power plant is not constant during the year or even during 24 hours. It is, however, possible to make a prediction of the demands in the future and each power plant therefore knows the expected production plan 24 hours ahead. Besides the production plan the prices of electricity and controllability are also known in advance. Using these three parameters, and how they change, it is possible to plan the usage of fuels. In the following a description is given of how the prices and production changes (a description of the planning can be found in [6]).

A. Production Plan

An example of a production plan for the considered plant is depicted in Figure 1. The graph depicts the production from midnight the 29th of June, 2008 and 24 hours forward. As seen in the figure the production is low during the night but at 6:00-7:00 in the morning there is a steep gradient caused by the increase in consumption when people and companies start to use electricity. The production plan is modelled as an smooth approximation of the graph depicted in Figure 1 and is denoted by[2]

$$t \mapsto y_r(t) \quad [MW]. \tag{1}$$

The smoothness assumption is purely theoretical (see (3)). In simulation the production plan (1) will be replaced by the non-smooth function defined by the graph in Figure 1.

B. Efficiency Price

The price of electricity, p_{R1}, changes during the day as the demand changes, i.e., during the middle of the day when the demand is greatest the price is also higher than during the early morning. The electricity price from the 29th of June 2008 is depicted in Figure 2. In this work[3]

$$t \mapsto p_{R1}(t) \quad [DKK/MWs]. \tag{2}$$

denote the efficiency price defined by the graph in Figure 2.

[2] $[\cdot]$ indicates the units and in this case $y_r(t)$ is measured in Mega Watt.

[3] DKK is the Danish currency, kroner.

C. Controllability Price

Large gradients in the production plan, as seen in Figure 1 around 6:00-7:00, yield a high price on controllability as it is likely that some plants are not capable of generating the gradients needed. In general this would be related to the derivative of the production plan and thus the price is higher during the periods in the morning and afternoon/evening where there exists steep gradients in Figure 1. The controllability price is defined as

$$t \mapsto p_{R2}(t) = \beta \left| \frac{d}{dt} y_r(t) \right|, \quad [DKK/MW], \tag{3}$$

where $\beta = 1000$ is a factor which has been determined in collaboration with DONG Energy.

In the simulations the differential quotient in (3) is replaced by a difference due to the non-smooth properties of (1). The resulting graph of the simulated version of (3) is depicted in Figure 3.

D. Fuel Price

Obviously the fuel prices change over time, however, these changes are slow compared to the changes in the efficiency and controllability prices as the time span is a matter of weeks. Therefore, the fuel prices are considered as constants and the fuel prices are given as

$$\boldsymbol{p}_C = (p_{C1}, p_{C2}, p_{C3}) = (1.20, 3.74, 6.00) \tag{4}$$

with unit in $[DKK/kg]$.

E. Input-Output Mapping

Let $\mathbb{R}^3_+$ denote the set of positive elements in $\mathbb{R}^3$, i.e., $\mathbb{R}^3_+ = \{\boldsymbol{v} \in \mathbb{R}^3 | \boldsymbol{v} \geq 0\}$ where the inequality is to be understood coordinate-wise (this notation will be used throughout this work). The input space, U, and the flow space, X, are now given by

$$\begin{aligned} U &= \{\boldsymbol{v} \in \mathbb{R}^3_+ | 0 \leq \boldsymbol{v}^T \boldsymbol{e}_u \leq c_u\}, \\ X &= \{\boldsymbol{v} \in \mathbb{R}^3_+ | 0 \leq \boldsymbol{v}^T \boldsymbol{e}_x \leq c_x\}, \end{aligned} \tag{5}$$

where the vector $\boldsymbol{e}_j = (e_{j_1}, e_{j_2}, e_{j_3}) \in \mathbb{R}^3$ with $\boldsymbol{e}_j > 0$ and scalar $c_j \in \mathbb{R}$ for $j \in \{u, x\}$ are to be determined later where their physical interpretation also will be given. Note that U (resp. X) is the 3-simplex in $\mathbb{R}^3_+$ with vertices $\boldsymbol{0}$, $(c_u/e_{u_1}, 0, 0)$, $(0, c_u/e_{u_2}, 0)$, and $(0, 0, c_u/e_{u_3})$, (resp. $\boldsymbol{0}$, $(c_x/e_{x_1}, 0, 0)$, $(0, c_x/e_{x_2}, 0)$, and $(0, 0, c_x/e_{x_3})$). Each (flow) state

$$\boldsymbol{x} = (x_c, x_g, x_o) \in X, \qquad ([kg/s], [kg/s], [kg/s]),$$

in the system describe the flow of coal, gas, and oil, respectively. In the sequel we let $\mathcal{I} = \{c, g, o\}$ where the elements of the index set $\mathcal{I}$ refers to the three different fuels. Occasionally the identification $(c, g, o) = (1, 2, 3)$ will be used.

The output space $Y = Y_1 \times Y_2$ is a subset of $\mathbb{R}^2$ where each output

$$\boldsymbol{y} = (y_e, y_c) \in Y, \qquad ([MW], [MW/s]),$$

of the system describe the two objectives; efficiency and controllability, respectively. Both of these quantities contain contributions from coal, gas, and oil as they will be defined as functions of the fuels later.

Furthermore, a state space, Z, is defined as

$$Z = \left\{ \boldsymbol{z} = (z_1, z_2, ..., z_9) \in \mathbb{R}^9 | (z_1, z_4, z_7) \in X \right\},$$

which is used when describing the dynamics of the fuel flows.

1) Plant Dynamics: The fuel flow, $\boldsymbol{x}(t)$, into the power plant is governed by third order differential equations (these equations also include the power plant dynamics). The control signal to the valves controlling these flows is denoted $\boldsymbol{u} = (u_c, u_g, u_o) \in U$ and the dynamics is given by

$$\begin{aligned} \dot{\boldsymbol{z}}(t) &= \boldsymbol{A}\boldsymbol{z}(t) + \boldsymbol{B}\boldsymbol{u}(t) \\ \boldsymbol{x}(t) &= \boldsymbol{C}\boldsymbol{z}(t), \end{aligned} \tag{6}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{A}_c & \boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & \boldsymbol{A}_g & \boldsymbol{0}_{3x3} \\ \boldsymbol{0}_{3x3} & \boldsymbol{0}_{3x3} & \boldsymbol{A}_o \end{bmatrix}, \quad \boldsymbol{A}_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ h_{i_1} & h_{i_2} & h_{i_3} \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} \boldsymbol{B}_c & \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{B}_g & \boldsymbol{0}_{3x1} \\ \boldsymbol{0}_{3x1} & \boldsymbol{0}_{3x1} & \boldsymbol{B}_o \end{bmatrix}, \quad \boldsymbol{B}_i = \begin{bmatrix} 0 \\ 0 \\ h_{i_0} \end{bmatrix},$$

$$\boldsymbol{C} = \begin{bmatrix} \boldsymbol{C}_1 & \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} \\ \boldsymbol{0}_{1x3} & \boldsymbol{C}_1 & \boldsymbol{0}_{1x3} \\ \boldsymbol{0}_{1x3} & \boldsymbol{0}_{1x3} & \boldsymbol{C}_1 \end{bmatrix}, \quad \boldsymbol{C}_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

and $h_{i_j}, i \in \mathcal{I}$, are constants describing the dynamics of the three fuel systems which are obtained from transfer functions of the form

$$H_i(s) = \frac{1}{(\tau_i s + 1)^3},$$

where $\tau_i, i \in \mathcal{I}$, is 90, 60, and 70, respectively. The three fuel systems may have some shared dynamics but to simplify the model in this work the systems are assumed decoupled.

Functions describing the two business objectives are derived in the following.

2) Efficiency: The efficiency objective, $y_e = y_e(\boldsymbol{z})$, deals with how much electricity is produced from a certain amount of fuel. Three affine functions describing the contribution of the individual fuels to the efficiency objective have been established using measurement data from two Danish power plants and can be expressed as

$$\tilde{\boldsymbol{y}}_e(\boldsymbol{z}) = \boldsymbol{Q}\boldsymbol{z} + \boldsymbol{b}, \tag{7}$$

where

$$\begin{aligned} \boldsymbol{Q} &= \text{diag}(\boldsymbol{e}_x)\boldsymbol{C}, \quad \boldsymbol{e}_x = (10.77, 18.87, 15.77), \\ \boldsymbol{b} &= (-1.76, 1.85, -0.37), \end{aligned}$$

and $\boldsymbol{C}$ defined in (6). The values of $\boldsymbol{e}_x$ and $\boldsymbol{b}$ have been established using measurement data and are measured in $[MJ/kg]$ and $[MW]$ respectively. The energy used for preprocessing the individual fuels is expressed by the b_i's and the e_{x_i}'s are conversion factors which are a combination of the boiler efficiency and energy storage in the different fuels. Note the constant $\boldsymbol{e}_x$ in (5) is now defined.

The total amount of efficiency is described by the function

$$Z \to Y_1;\ \boldsymbol{z} \mapsto y_e(\boldsymbol{z}) = \boldsymbol{\gamma}^T \tilde{\boldsymbol{y}}_e(\boldsymbol{z}),$$

where

$$\boldsymbol{\gamma} = (1, 1, 1).$$

The constant c_x in (5), can now be determined by $c_x = 400 - \boldsymbol{\gamma}^T \boldsymbol{b}$, where 400 refers to the maximum efficiency (in $[MW]$) produced by the plant and $\boldsymbol{\gamma}^T \boldsymbol{b}$ is the total own-consumption of the plant used for preprocessing the three fuels. We let $c_u = c_x$ and $\boldsymbol{e}_u = \boldsymbol{e}_x$ in (5) since (6) has negative real eigenvalues and the steady state gain is 1 which guarantees that $\boldsymbol{x}(t) \in X$ during any steady state operation.

3) Controllability: The controllability objective, $y_c = y_c(\boldsymbol{z})$, deals with a measure of how fast the production of electricity can be changed. Allowed changes in the production is limited to a certain gradient depending on the current efficiency. The reason for this limit is a compliance to maximum temperature gradients in the boiler (these have not been explicitly modelled and are therefore indirectly considered by limiting the allowed changes). When using coal it is allowed to change production with 0.133 $[MW/s]$ when running the plant at low and high production and 0.267 $[MW/s]$ in the middle range from 200 $[MW]$ to 360 $[MW]$. When using oil or gas the values are 0.133 $[MW/s]$ and 0.534 $[MW/s]$. If a mixture of the three fuels are used it is assumed that the allowed change is a linear combination of the allowed change of the individual fuels. The controllability objective is, therefore, modelled as

$$Z \to Y_2;\ \boldsymbol{z} \mapsto y_c(\boldsymbol{z}) = \begin{cases} 0.133 & y_e(\boldsymbol{z}) \in S_1 \\ \frac{\boldsymbol{\xi}^T \tilde{\boldsymbol{y}}_e(\boldsymbol{z})}{y_e(\boldsymbol{z})} & y_e(\boldsymbol{z}) \in S_2 \\ 0.133 & y_e(\boldsymbol{z}) \in S_3, \end{cases} \quad (8)$$

where

$$\boldsymbol{\xi} = (0.267, 0.534, 0.534), \quad S_1 = \{s \in \mathbb{R} | 0 \le s \le 200\},$$
$$S_2 = \{s \in \mathbb{R} | 200 < s < 360\},\ \text{and}$$
$$S_3 = \{s \in \mathbb{R} | 360 \le s \le 400\}.$$

F. Prices

The cost of using the fuel, $\boldsymbol{x}$, revenue from production of output, $\boldsymbol{y}$, and the profit of operating the power plant can now be determined. The above constructions yields a product (or output) function, y_P, of the system given by

$$y_P : Z \to Y;\ \boldsymbol{z} \mapsto (y_e(\boldsymbol{z}), y_c(\boldsymbol{z})).$$

The growth of cost and growth of revenue for the system are defined by the following functions (both with units in [DKK/s])

$$g_C : Z \to \mathbb{R};\ \boldsymbol{z} \mapsto \boldsymbol{z}^T \boldsymbol{C}^T \boldsymbol{p}_C,$$
$$g_R : Y \times \mathbb{R}_+ \to \mathbb{R};\ (\boldsymbol{y}, t) \mapsto \boldsymbol{y}^T \boldsymbol{p}_R(t), \quad \boldsymbol{p}_R(t) > 0,$$

where $\boldsymbol{p}_C$ is as defined in (4) and

$$\boldsymbol{p}_R(t) = (p_{R1}(t), p_{R2}(t))$$

with the coordinate functions as defined in (2) and (3).[4]

The growth of profit is hence defined by

$$Z \times Y \times \mathbb{R}_+ \to \mathbb{R};\ (\boldsymbol{z}, \boldsymbol{y}, t) \mapsto g_R(\boldsymbol{y}, t) - g_C(\boldsymbol{z}),$$

which for the system yields the function

$$g_P : Z \times \mathbb{R}_+ \to \mathbb{R};\ (\boldsymbol{z}, t) \mapsto g_R(\boldsymbol{y}_P(\boldsymbol{z}), t) - g_C(\boldsymbol{z}).$$

Therefore, the profit is given by

$$P : \mathbb{R}_+ \to \mathbb{R};\ t \mapsto \int_0^t g_P(\boldsymbol{z}(\tau), \tau) d\tau. \quad (9)$$

IV. Optimization

The objective of the company is to maximize its profit over the planning horizon, T, such that the production plan is fulfilled with the available fuel systems. This optimization is stated as

$$\begin{aligned} &\max_{u \in U} P(T) \\ &\text{subject to} \\ &\dot{\boldsymbol{z}} = \boldsymbol{A}\boldsymbol{z} + \boldsymbol{B}\boldsymbol{u}, \\ &h(\boldsymbol{z}(t), t) = \boldsymbol{\Upsilon} \boldsymbol{z}(t) + \boldsymbol{\psi}(t) \ge 0, \end{aligned} \quad (10)$$

where

$$\boldsymbol{\Upsilon} = \begin{bmatrix} \boldsymbol{\gamma}^T \boldsymbol{Q} \\ -\boldsymbol{\gamma}^T \boldsymbol{Q} \end{bmatrix}, \quad \boldsymbol{\psi}(t) = \begin{bmatrix} \boldsymbol{\gamma}^T \boldsymbol{b} - y_r(t) + \alpha \\ -\boldsymbol{\gamma}^T \boldsymbol{b} + y_r(t) + \alpha \end{bmatrix}.$$

Hence the function $h(\boldsymbol{z}(t), t)$ is constructed such that the efficiency, $y_e(\boldsymbol{z})$, follows the production plan, $y_r(t)$, within a bound α. We have omitted the constraint on $\boldsymbol{x}(t)$, i.e., $\boldsymbol{x}(t) \in X$. It is easy to include in the optimization but here we have decided to just verified this a posteori.

The growth of profit function can be simplified when the reference is followed perfectly, i.e., $\alpha = 0$. Then $y_e(\boldsymbol{z}(t)) = y_r(t)$ which yields

$$g_P(\boldsymbol{z}(t), t) = \boldsymbol{\Theta}(t) \boldsymbol{z}(t) + \varphi(t), \quad (11)$$

where

$$\boldsymbol{\Theta}(t) = p_{R1}(t) \boldsymbol{\gamma}^T \boldsymbol{Q} - \boldsymbol{p}_C^T \boldsymbol{C} + p_{R2} \boldsymbol{\vartheta}(t),$$
$$\varphi(t) = p_{R1}(t) \boldsymbol{\gamma}^T \boldsymbol{b} + p_{R2} \zeta(t),$$

[4] The prices used in this work corresponds to the market prices the 29th of June, 2008 and has been established using internal DONG Energy documents and the archive of power price at www.nordpool.dk, which is a marketplace for trading power contracts.

and $\boldsymbol{\vartheta}(t)$ and $\zeta(t)$ makes up for the switching function in (8), i.e.,

$$\boldsymbol{\vartheta}(t) = \begin{cases} 0 & y_r(t) \in S_1 \\ \frac{\boldsymbol{\xi}^T \boldsymbol{Q}}{y_r(t)} & y_r(t) \in S_2, \\ 0 & y_r(t) \in S_3 \end{cases} \quad \zeta(t) = \begin{cases} 0.133 & y_r(t) \in S_1 \\ \frac{\boldsymbol{\xi}^T \boldsymbol{b}}{y_r(t)} & y_r(t) \in S_2, \\ 0.133 & y_r(t) \in S_3 \end{cases}$$

with the functions, constants, and sets as previously defined. The assumption $\alpha = 0$, might not be feasible because the demand might change quicker than what is possible with the dynamics of the fuel systems. However, (11) will be used as an approximation for the real $g_p(\boldsymbol{z}(t))$ when $\alpha \neq 0$.

A. Discrete optimization

In this section the cost, constraint, and system from the previous section will be converted into discrete time. From the discrete time cost, constraint, and system a linear program formulation of the problem will be obtained.

First, however, some assumptions about the problem will be made. The time period T is divided into N equally sized time units, h, i.e., $T = Nh$. It is assumed that $\boldsymbol{\Theta}(t)$, $\varphi(t)$, and $\boldsymbol{\psi}(t)$ can be approximated by piecewise constant functions for each time step, i.e.,

$$\begin{aligned} \boldsymbol{\Theta}(t) &= \boldsymbol{\Theta}_k, \quad kh < t < (k+1)h, \\ \varphi(t) &= \varphi_k, \quad kh < t < (k+1)h, \\ \boldsymbol{\psi}(t) &= \boldsymbol{\psi}_k, \quad kh < t < (k+1)h. \end{aligned}$$

Furthermore, the control will be assumed piecewise constant as customary when digital to analogue conversion is performed using sample-hold circuits.

Using a fact from [7] the continuous time state $\boldsymbol{z}(t)$ in the dynamic system in (6) can be described by

$$\begin{aligned} \boldsymbol{z}(t) &= e^{\boldsymbol{A}t}\boldsymbol{z}_0 + \int_0^t e^{\boldsymbol{A}(t-s)}\boldsymbol{B}\boldsymbol{u}_0(s)ds \\ &= \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \exp\left\{ \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} t \right\} \begin{bmatrix} \boldsymbol{z}_0 \\ \boldsymbol{u}_0 \end{bmatrix}, \end{aligned} \tag{12}$$

where $\boldsymbol{I}$ is an identity matrix with appropriate dimension. Using (12) it is possible to derive the following formula which is used during the discretization of the cost and constraint.

$$\begin{aligned} \int_0^h e^{\boldsymbol{A}t}dt &= e^{\boldsymbol{A}h} \int_0^h e^{-\boldsymbol{A}(h-t)}dt \\ &= e^{\boldsymbol{A}h} \left(e^{-\boldsymbol{A}h} \cdot 0 + \int_0^h e^{-\boldsymbol{A}(h-t)}\boldsymbol{I}dt \right) \\ &= e^{\boldsymbol{A}h} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} \exp\left\{ \begin{bmatrix} -\boldsymbol{A} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} h \right\} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix}. \end{aligned} \tag{13}$$

B. System

The system equation is sampled forming the well known discrete system equations

$$\boldsymbol{z}_{k+1} = \boldsymbol{\Phi}\boldsymbol{z}_k + \boldsymbol{\Gamma}\boldsymbol{u}_k,$$

where

$$\boldsymbol{\Phi} = e^{\boldsymbol{A}(t_{k+1}-t_k)} \text{ and } \boldsymbol{\Gamma} = \int_0^{t_{k+1}-t_k} e^{\boldsymbol{A}s}ds\boldsymbol{B}.$$

C. Cost

When deriving a sampled version of the cost the integral is split into a sum of N integrals and then (12) and (13) are used to derive a discrete cost function, i.e.,

$$\begin{aligned} P(T) &= \sum_{k=0}^{N-1} \int_{kh}^{(k+1)h} (\boldsymbol{\Theta}(t)\boldsymbol{z}(t) + \varphi(t))\, dt \\ &= \sum_{k=0}^{N-1} \boldsymbol{\Theta}_k \int_0^h \left(e^{\boldsymbol{A}t}\boldsymbol{z}_k + \int_0^t e^{\boldsymbol{A}(t-s)}\boldsymbol{B}ds\boldsymbol{u}_k \right) dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \boldsymbol{\Theta}_k \int_0^h \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\tilde{\boldsymbol{A}}t} \begin{bmatrix} \boldsymbol{z}_k \\ \boldsymbol{u}_k \end{bmatrix} dt + h\varphi_k \\ &= \sum_{k=0}^{N-1} \boldsymbol{\Theta}_k \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\tilde{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\hat{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{z}_k \\ \boldsymbol{u}_k \end{bmatrix} + h\varphi_k \\ &= \sum_{k=0}^{N-1} \boldsymbol{C}_k\boldsymbol{z}_k + \boldsymbol{D}_k\boldsymbol{u}_k + \boldsymbol{E}_k, \end{aligned}$$

where

$$\begin{aligned} \boldsymbol{C}_k &= \boldsymbol{\Theta}_k \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\tilde{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\hat{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{0} \end{bmatrix}, \\ \boldsymbol{D}_k &= \boldsymbol{\Theta}_k \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\tilde{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} \end{bmatrix} e^{\hat{\boldsymbol{A}}h} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix}, \\ \boldsymbol{E}_k &= h\varphi_k,\ \hat{\boldsymbol{A}} = \begin{bmatrix} -\tilde{\boldsymbol{A}} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \tilde{\boldsymbol{A}} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}, \end{aligned}$$

and I denoting identity matrices of appropriate dimensions.

D. Constraint

The constraint needs to be satisfied at all times which, of course, is not guaranteed by ensuring the constraint is satisfied at each sample time. In order to approximate this, the constraint is sampled L times between each sample of the cost. The discrete version of the constraint is described by

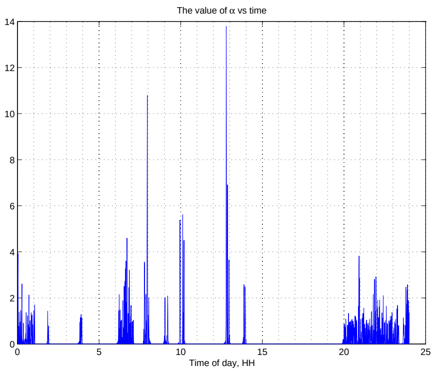
$$\begin{aligned} &h(\boldsymbol{z}(t), t) = \boldsymbol{\Upsilon}\boldsymbol{z}(t) + \boldsymbol{\psi}(t) \\ &= \boldsymbol{\Upsilon} \left(e^{\boldsymbol{A}\frac{l}{L}h}\boldsymbol{z}_k + \int_0^{\frac{l}{L}h} e^{\boldsymbol{A}(\frac{l}{L}h-s)}\boldsymbol{B}ds\boldsymbol{u}_k \right) + \boldsymbol{\psi}(\frac{l}{L}h + kh) \\ &= \boldsymbol{\Psi}_l\boldsymbol{z}_k + \boldsymbol{\Pi}_l\boldsymbol{u}_k + \boldsymbol{\Omega}_{k,l}, \end{aligned}$$

where

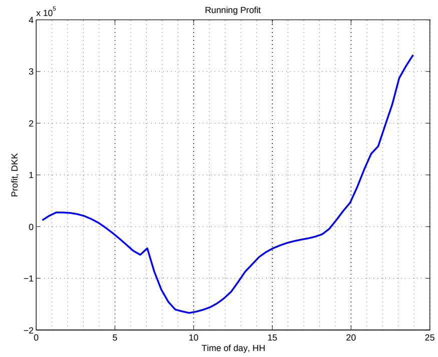
$$\boldsymbol{\Psi}_l = \boldsymbol{\Upsilon}e^{\boldsymbol{A}\frac{l}{L}h}, \quad \boldsymbol{\Pi}_l = \boldsymbol{\Upsilon}\int_0^{\frac{l}{L}h} e^{\boldsymbol{A}(\frac{l}{L}h-s)}\boldsymbol{B}ds, \text{ and}$$

$$\boldsymbol{\Omega}_{k,l} = \boldsymbol{\psi}(\frac{l}{L}h + kh).$$

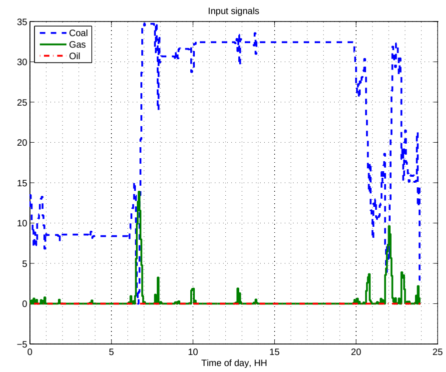
Now, the problem in (10) can be approximated by a linear program where the constraint is not guaranteed to be satisfied at all times but it is, however, satisfied at LN equally spaced points in time. Furthermore, the cost function is approximated by (11) which is a good approximation when α is small. To ensure this α is made



4 The efficiency is also equal to the production plan reference at all times as α is small.



5 Optimal profit during 24 hours of operation June 24th, 2008.



6 The input signal to the fuel systems shows that only coal and gas is used.

time-varying and the cost function is augmented with an α-term (and appropriate weight W_k). The linear program can thus be stated as

$$\max_{\substack{u \in U \\ \alpha \geq 0}} \sum_{k=0}^{N-1} \left(\boldsymbol{C}_k \boldsymbol{z}_k + \boldsymbol{D}_k \boldsymbol{u}_k + \boldsymbol{E}_k - W_k \alpha_k \right)$$

$$\text{subject to} \quad \boldsymbol{z}_{k+1} = \boldsymbol{\Phi} \boldsymbol{z}_k + \boldsymbol{\Gamma} \boldsymbol{u}_k, \qquad \boldsymbol{\Psi}_l \boldsymbol{z}_k + \boldsymbol{\Pi}_l \boldsymbol{u}_k + \boldsymbol{\Omega}_{k,l} \geq 0.$$

V. Results

The linear program stated in the previous section has been formulated using YALMIP [8] and solved using SeDuMi[5]. In this section the results will be presented where the following values have been used

$$T = 86400s,\ N = 432,\ h = 200s,\ L = 5,\ W_k = \frac{500000}{NL}.$$

Figure 4 depicts α vs time which shows that the approximation of the cost function is good as the values are below 14 at all times, which is within 3.5% of full production (and less at most times).

The profit over time, $P(t)$, is depicted in Figure 5 and is low during most of the morning. Actually, from approximately 2:00-10:00 the gain in profit is negative which is caused by the low price on efficiency, p_{R1}. At 10:00 the profit starts to grow and at the end of the day the total profit is approximately $330000 DKK$.

The usage of the three fuel systems is illustrated in Figure 6, where the input signals to the coal, gas, and oil systems are depicted. Coal is used as the primary fuel during the day, but at times the gas system is used to compensate for the slow coal system during transients. This can especially be observed around 6:00-7:00 and at the evening.

VI. Discussion

Comparing the results of this work with the results from [5], where no dynamics were present, it can, as expected, be concluded that the dynamics should be considered as the profit is different. However, the usage of fuels are comparable as gas is used during periods with large gradients in the reference. The profit found in Section IV is smaller than the profit obtained without dynamics in [5], but it also greater than the profit obtained when running the plan only with coal. Thus, mixing fuels is beneficial under consideration of the two business objectives presented in this paper.

Furthermore, the usage of the fuels does not switch completely from one fuel to another and thus the gas and oil systems are not fully used - the oil system is actually not used at all. This would suggest that a new plant should only be instrumented with a full coal system and a partial gas system.

Future work could include expanding the business models to include more detail about the bidding and settling of prices performed at Nordpool. In particular, the controllability model and price have been simplified in this work.

[5]SeDuMi is a software package used to solve optimization problems (see http://sedumi.ie.lehigh.edu/content/view/17/53/)

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