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Strategies on 3-Connected Graphs Optimization Search For Future ICT Infrastructure *

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ABSTRACT
Usually, for economic reasons, the physical infrastructure for transmission networks is designed as 2-connected graphs, i.e. interconnected rings. However, the increment of both, the traffic and the dependency of our professional and personal lives on ICT, demands more reliable transmission and distribution networks. Network interconnection problems are in general costly and complex to solve, categorized as NP-hard computational complexity. In addition, structural constraints such as 3-connectivity increase the time and resources utilized in the optimization search process. This paper presents and evaluates novel strategies in order to simplify the optimization process when planning the physical network interconnection as 3-connected with nodal degree 3. These strategies are applied in a case study and the results, using this and other approaches, are compared in terms of path distances, path length, network length, processing time, and number of iterations to find the solution.

Categories and Subject Descriptors
C.2.1 [**Network Architecture and Design**]: Network topology; G.2.2 [**Discrete Mathematics**]: Graph Theory

General Terms
Algorithms

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Keywords
3-connected graphs, network interconnection, optical backbone

1. INTRODUCTION
Currently, our daily lives are, in both professional and personal aspects, highly dependent on the proper functioning of communications networks. Failures might cause significant losses, especially if they occur at the optical backbone networks due to the huge volume of traffic traversing their links [1]. In addition, this type of networks is expensive to deploy due to their size and it might take several years to complete them. Therefore, they should be planned to be able to provide the demanded reliability in a middle and long term.

In relation to the physical interconnection of such networks, usually they are conceived as 2-connected graphs, implying that when a failure occurs, there is always an alternative path available. However, this feature might not be enough for future demands, and 3-connected graphs could be the natural evolution in reliability terms to be able to support dual failures situations [2].

In this work we propose to design the interconnection of optical backbone networks with 3-connected graphs. In addition, due to the expensive deployment of the lines, the minimum possible number of links is used in order to form the networks, resulting in degree 3 topologies.

The authors have been working with similar problems in [3] and [4], by designing the interconnection following specific topologies, i.e. degree 3, 3-connected among others. The adjacency matrix was an input to the problem and the solution was the location of each specific node in the matrix. This method is limited to a single topology for each search process.

This paper takes a step forward regarding the solution search and optimization process when using degree 3, 3-connected graphs for network interconnection by removing the topological limitation commented above.

The main constraint is the computational complexity of the problem that is higher than solving the problem with the adjacency matrix as an input. The combination of links problem is itself NP hard [5], and in addition, the 3-connected constraint (or minimum cut calculation) implies another problem to be solved in polynomial time for each of the iterations in the search process [6]. The main goal is to present and evaluate novel techniques and procedures to design degree 3, 3-connected graphs without the adjacency matrix as an input.

These techniques are implemented and discussed in a case study. In addition, the same problem is solved for several degree 3, 3-connected topologies with their adjacency matrix as an input. The resulting interconnections using both approaches are compared in terms of path distances, path length, network length, processing time, and number of iterations to find the solution.

The rest of the document is as follows: Section 2 presents the background of this work. Section 3 describes the procedure to follow for solving this type of problems. Section 4 presents a case study applying the described procedure and the results are compared to other degree 3 solutions. Finally, Section 5 presents the conclusion of this work.

2. BACKGROUND

2.1 Concepts and Definitions

Nodal degree: Nodal degree is referred to as the number of directed or undirected links of a node. If all the nodes in a network have the same degree, then the network is regular.
k-connected graph: A graph is k-connected when any $k-1$ elements, nodes or links, can be removed from the network and still maintain a connected graph.
Minimum links for 3-connected graphs: The minimum number of links for a network to be 3-connected is $3N/2$, each node is connected exactly to 3 links.
Organized Topologies: Referred as topologies presenting some sort of pattern in their interconnection i.e. symmetric topologies.
Length and Distance: Define two different concepts in this paper. Length is always used in physical terms (km) and distance is always used in transmission path terms (hops).

2.2 Routing Schemes:

Formally, a set of precomputed paths, $P_h(s,d)$, can be defined as the set of all the possible paths from a source node s to a destination node d, with a maximum hop distance of h. Let $Hp_i(s,d)$ be the distance and $Lp_i(s,d)$ the length of each path $p_i(s,d) \in P_h(s,d)$. And let $P'_h(s,d) \in P_h(s,d)$ be a set of k disjoint paths. Two different ways of selecting $P'_h(s,d)$ are covered in this work[1].

Minimum Hop Routing, MH: $p_{i'}(s,d)$ is a selected disjoint path in $P'_h(s,d)$ if $Hp_{i'}(s,d) \in min^k(Hp_i(s,d))$, for $0 < i \leq size(P_h(s,d))$ and $0 < i' \leq k$.

In case of multiple selection options, $Hp_a(s,d) = Hp_b(s,d)$, the priority can be based on the minimum length, if $Lp_a(s,d) < Lp_b(s,d)$ $p_a(s,d) \in P'_h(s,d)$ being $p_a(s,d), p_b(s,d) \in P_h(s,d)$.

Minimum Length Routing, ML: $p_{i'}(s,d)$ is a selected disjoint path in $P'_h(s,d)$ if $Lp_{i'}(s,d) \in min^k(Lp_i(s,d))$, for $0 < i \leq size(P_h(s,d))$ and $0 < i' \leq k$.

[1] $min^k(X)$ is the set feasible k minimum values of X.

2.3 Related Work: Interconnection and Topology Based Approach to the Problem

In relation to the design of 3-connected telecommunications networks, an interesting approach has been developed in [7]. The paths between pairs of nodes are encoded as the chromosomes of a genetic algorithm and constraints such as maximum diameter and nodal degree are given. However, this work solves the problem for only 20 demands. To solve the complete backbone interconnection problem following this procedure, all-to-all demands must be considered, implying $N \cdot (N-1)/2$ individual problems to be solved.

The authors have been solving the 3-connected graph problem using a topological approach. It consists of one or several adjacency matrices given as the input to the problem, and the objective is to arrange the nodes according to these matrices in the optimization process. The optimized solution always follows the given adjacency matrix.

This approach is explained in depth and applied in [3]. In this work, these input matrices correspond to degree 3, 3-connected organized topologies. In this way, it is guaranteed that the solution is always 3-connected without the need of minimum-cut or path calculations. In this way, the solution to the problem is the location of each of the nodes in the adjacency matrix that optimizes the objective function.

Therefore, the problem solving process is much faster than the one proposed in this work due to the reduction of the topologies in the solution space. On the other hand, this procedure does not guarantee an overall optimization of any objective function in 3-connected graphs since it is constrained to a limited number of specific topologies.

In the case study in Section 4, the results of using this and the proposed approaches are presented and discussed.

3. SOLVING STRATEGIES

This section describes the proposed procedure to simplify the commented interconnection problem and the solving techniques. This approach is defined as *Link Based Approach*, where the adjacency matrix is the objective of the problem. The number of links is given as an input and the optimization is the result of properly distributing the links.

3.1 Size of the Problem

The optimization of any interconnection can be performed working with the adjacency matrix. The adjacency matrix consists of NxN elements taking values of "0" or "1". Therefore, the largest search space when there are no constraint to the problem is all the possible combinations of $0's$ and $1's$, 2^{N^2} potential solutions. A "1" corresponds to a link between two nodes.

However, the search space can be considerably reduced by applying some of the following constraints and conditions when the problem is focused on degree 3, 3-connected graphs:

No loopback links: Nodes cannot be connected to themselves; this reduces the matrix elements to work with to $N \cdot (N-1)$.
Unidirectional Links: If there is a link between i and j,

there is a link between j and i. The number of matrix elements to work with is reduced to $N \cdot (N-1)/2$.
Constant Nodal Degree: All nodes have the same number of incoming/outgoing links. The number of links in the network is constant, L. Consequently, L matrix elements of the total of $N \cdot (N-1)/2$ must be $1's$ and, if all the possible combinations are considered, the total number of theoretical solutions is given by Eq. (1).

$$Total\ Sol. = \binom{\frac{N\cdot(N-1)}{2}}{L} = \frac{\frac{N\cdot(N-1)}{2}!}{(\frac{N\cdot(N-1)}{2} - L)! \cdot L!} \tag{1}$$

Depending on the specific problem, some of these possibilities might be infeasible, but these cannot be avoided/discarded before their evaluation.

3.2 Search Process

The following paragraphs describe the heuristic approach for solving the described problem.

Initial solution

The search process must start from a feasible solution. In this particular case, any of the well-known regular degree 3 topologies' adjacency matrices can be used. For example, a Double Ring matrix C can be used following Eq. (2) for its construction.

$$C_{ij} = \begin{cases} 1 & if\ i + \frac{N}{2} = j \\ 1 & if\ i = (j \pm 1)_{mod(\frac{N}{2})}\ \ \&\ \ i < \frac{N}{2} \\ 1 & if\ i - \frac{N}{2} = (j - \frac{N}{2} \pm 1)_{mod(\frac{N}{2})}\ \ \&\ \ i \leq \frac{N}{2} \\ 0 & Rest \end{cases} \tag{2}$$

Generation Process

The generation of feasible solutions is one of the main points to focus when using heuristics to solve complex problems. In this specific case, traditional offspring generation used in Genetic Algorithms is not very well suited since it is very unlikely that the new solutions are also degree 3 graphs.

Instead, a procedure based on the "moves" in Simulated Annealing (SA) is proposed. These moves are specially designed to always obtain new degree 3 solutions. Each move consists of the swap of 2+2 bits in the matrix as follows:

1. Select random row, i.
2. Select random cell (ij) in row i where $C_{ij} = 1$.
3. Select random cell (ij') in row i where $C_{ij'} = 0$.
4. Swap C_{ij} and $C_{ij'}$ values.
5. Adjust symmetry $C_{ji} = C_{ij}$ and $C_{j'i} = C_{ij'}$.
6. Select random row i' where $C_{i'j} = 0$ and $C_{i'j'} = 1$.
7. Swap $C_{i'j}$ and $C_{i'j'}$ values.
8. Adjust symmetry $C_{ji'} = C_{i'j}$ and $C_{j'i'} = C_{i'j'}$.

Fig. 1 illustrates an example of one of these moves.

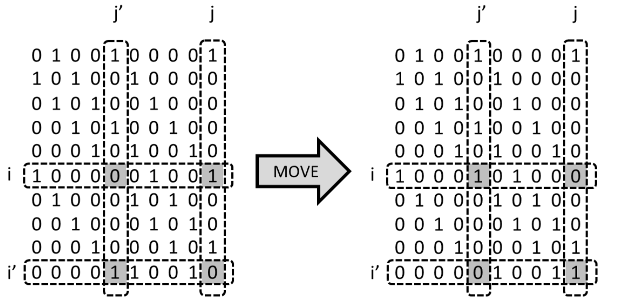

Figure 1: Move Example

Each new generation consists of a number of these moves. The exact number of moves for each generation is given by randomly taking values between 1 and M. The value of M should be reduced following the cooling strategy in SA [8]. The cooling down function depends on the specific problem, and for the case study, M follows a simple linear function defined by Eq. (3), k_i being the iteration number, and K the maximum number of iterations.

$$M = N - \left\lfloor \frac{N \cdot k_i}{K} \right\rfloor \tag{3}$$

Feasibility and Evaluation

The new solution generation strategy guarantees degree 3 graphs but does not cover the second constraint, 3-connected graph solutions.

To check if a graph is k-connected it is usually solved calculating its *Minimum Cut* which can be solved by polynomial time algorithms such as the Ford-Fulkerson [6]. This implies solving a complex problem for each of the iterations in the NP-hard problem.

However, some advantages can be taken for the concrete problem described in this paper. If 3 disjoint paths exist between each pair of nodes, then the graph is at least 3-connected, and since all nodes are degree 3, it can never be 4-connected. Moreover, due to the distribution of the links, if 3 disjoint paths exist from a random node to the rest, then 3 disjoint paths exist between all pairs of nodes. This leads to the following theorem:

THEOREM 1. *A graph with N nodes and exactly $L = 3N/2$ links is 3-connected if 3 disjoint paths exist from one node to the rest.*

This feature considerably reduces the time resources spent in the feasibility evaluation. The number of sets of k disjoint paths to be calculated is reduced from $N \cdot N - 1/2$ to $N - 1$ pairs of nodes.

In relation to the evaluation of the objective function, once each of the new solutions is identified as feasible, the procedure is the same as the traditional combinatorial problems.

The relevant parameters are calculated and the solution is kept or discarded depending on the results of the objective function and the search methods used.

In this case, the objective function used is the minimization of the network's physical length since it is rather simple to obtain and evaluate. In addition, the network's length can be directly related to the required deployment investment and, usually, to minimize the length implies a minimization of this investment.

Let l_{ij} be the length of a link between nodes i and j nodes. Then the objective function is formally defined as Eq. (4).

$$min \left(\sum l_{ij} \cdot C_{ij} \right) \quad 0 < i, j < N \tag{4}$$

The objective function is independent of methodology to generate new solutions, and consequently, does not affect the proposed procedure to solve this type of problems.

Solution Selection

Each feasible solution must be kept/discarded after its evaluation. Let $S = s_1, s_2, s_3, ..., s_n$ be the population of solutions, and $Cost(s_i)$ the cost of each s_i solution, $0 < i \leq n$. A new solution s' is kept if $Cost(s') < max(Cost(S))$. In order to keep a solution another one from S must be discarded to take its place. A solution $s'' \in S$ is discarded when $Cost(s'') = max(Cost(S))$.

4. CASE STUDY

The case study consists of designing the physical interconnection for 20 European cities, $N = 20$. The resulting graph must be 3-connected using the minimum number of links, $L = 3N/2$. Applying these values to Eq. (1), there are $2.46 \cdot 10^{15}$ different potential solutions.

As commented above, the objective function consists of the minimization of the network's total length, all the links being disjoint among each other. In addition, the same problem is solved using the topological approach, summarized in Section 2, using three well-known degree 3 topologies: Double Ring DR, Chordal Ring CR, and $N2R$, see Fig. 2.

The resulting interconnections and the procedure to achieve them are evaluated in terms of the following factors: Shortest network length obtained, average time consumption[2] to find the solution, and average number of iterations required to find the solution. In addition, two types of routing are applied to the resulting interconnections *Minimum Hop Routing* (MH) and *Minimum Length Routing* (ML), already introduced in Section 2. The average path length and distance is calculated in each of the cases for a primary path and two alternative disjoint paths.

The topologies used are the following: DR, $CR(20,5)$, $CR(20,9)$, $N2R(10,3)$, and $D3$ for the topology found using the newly introduced approach. $CR(20,3)$ is equivalent to DR and $CR(20,7)$ is equivalent to $CR(20,5)$.

[2]AMD 64X2 Dual Core Processor 6000+, 3.0GHz. RAM 2GB

Each of the five cases is repeated 10 times, the population is $2N = 40$, and the maximum number of iterations is 1000. The best results, shortest network length in each case in km, is presented in Table 1, together with the information regarding the number of iterations required to reach the solution and, the processing time in seconds.

	DR	**CR(20,5)**	**CR(20,9)**	**N2R**	**D3**
Length	17252	18776	17495	19180	16209
Avg Iter.	744	977	746	800	696
Avg Time	10,6	14,3	10,9	11,9	22478

Table 1: Resulting Interconnections

The main conclusion when following the new approach is that the network's total length can be significantly reduced. In fact, this optimization provides the shortest degree 3, 3-connected graph possible. On the other hand, the procedure is much more complex due to the calculation of paths for each of the iterations in order to evaluate the 3-connected constraint. The difference on the number of iterations required is not significant; however, the time consumption is 2000 times higher.

In connection with the routing and path analysis, Table 2 presents the results obtained when 3 disjoint paths are provided. The use of $D3$ outperforms all the rest when ML routing is applied, obtaining the shortest primary path average length. In any of the other cases, the feasibility of the new solution should be evaluated considering the trade-off deployment investment-topological properties.

Rt	**Path**	**DR**	**CR(20,5)**	**CR(20,9)**	**N2R**	**D3**
MH	1st (hops)	3,16	2,53	2,84	2,63	2,83
	2nd (hops)	4,26	4,11	4,53	3,89	4,34
	3rd (hops)	7,58	5,18	7,05	5,48	6,59
	1st (km)	1634	1396	1437	1336	1422
	2nd (km)	2527	2135	2158	2333	2260
	3rd (km)	4480	3588	4108	3411	3856
ML	1st (hops)	3,63	2,81	3,15	2,87	3,25
	2nd (hops)	4,27	4,32	4,8	4,31	4,35
	3rd (hops)	7,59	5,24	6,61	5,31	6,55
	1st (km)	1551	1313	1331	1291	1213
	2nd (km)	2395	2110	2146	2130	2137
	3rd (km)	4572	3666	4100	3657	3993

Table 2: Path Analysis

The resulting topology following the proposed approach is illustrated in Fig. 3. Apart from the degree 3 regularity, the network does not follow any other interconnection pattern as the rest of the organized topologies. This asymmetry affects some of the path parameters, as it can be identified by the values displayed in Table 2.

These are the results in connection with the type of routing applied to each of the topologies:

- The primary paths using MH are between 3-17% longer but the average hop count is between 8-15% lower than when using ML.
- The first alternative paths are between 1-10% longer when applying MH but the hop count is between 0-10% lower than when using ML.
- No pattern can be defined for the third path. The hop

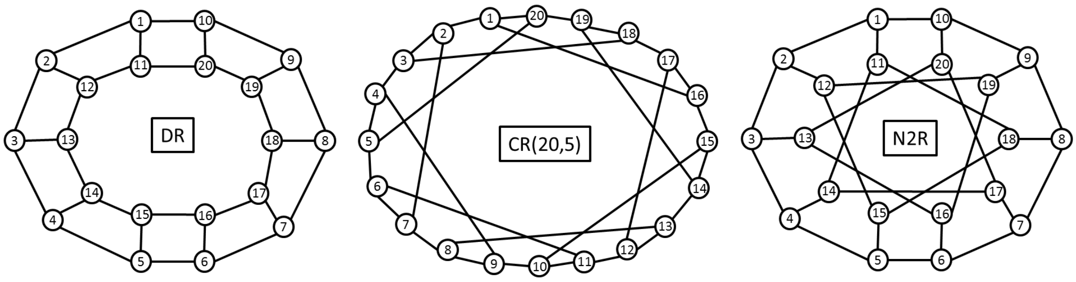

Figure 2: Organized Topologies

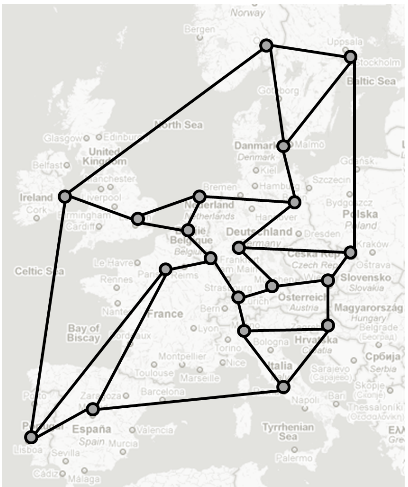

Figure 3: Degree 3 New Solution

count is only lower for the DR and $CR(20,5)$ when applying MH, and the paths are for none of the topologies shorter when applying ML.

5. CONCLUSION

This paper presents a number of strategies to simplify the procedures of solving the network interconnection problem when using degree 3, 3-connected graphs. Previously, the problem was solved by using fixed adjacency matrices as an input to the problem. This procedure limits the solution to a few topologies, it is called the topology based approach.

Instead, this new technique allows to search in all the solution space in order to guarantee an overall solution at the cost of increasing the complexity of the problem solving. The new proposal is specifically designed to simplify this complexity by generating degree 3 solutions to be evaluated. Furthermore, due to the structural properties of the desired interconnection, the number of set of calculated paths is reduced from $N \cdot (N-1)/2$ to $N-1$ node pairs.

Based on the case study results, this methodology allows solving the problem in a similar number of iterations as the topology based approach. The cost of the link based approach is the time consumption due to the evaluation of the 3-connected constraint in every new solution generated. It takes approximately 2000 times longer to solve the problem in the case study scenario.

On the other hand, the proposed method can lead to better solutions than using the topology based approach. More specifically, in the case study, the network length of $D3$ is more than 1000 km shorter (-6%) than the best solution using the topological approach, the DR.

However, this solution does not provide the overall best path parameters when the two routing schemes are applied, Minimum Hop and Minimum Length Routing. Therefore, as in many planning and design problems, the final decision must be taken based on the trade-off between the relevant network parameters for each specific scenario.

This new approach enlarges the space of possibilities when designing the physical network interconnection as degree 3 3-connected graphs in a more complex but still feasible way.

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