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Soft Time-Suboptimal Controlling Structure for Mechanical Systems

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Abstract

The paper presents conception of a soft control structure based on the time-optimal approach. Its parameters are selected in accordance with the rules of the statistical decision theory and additionally it allows to eliminate rapid changes in control values. The object is a basic mechanical system, with uncertain (also non-stationary) mass treated as a stochastic process. The methodology proposed here is of a universal nature and may easily be applied with respect to other uncertainty elements of time-optimal controlled mechanical systems.

1 Introduction

The main constraint of application possibilities of systems based on the principles of the classical optimal control theory [1] has been their excessive sensitivity to the object dynamics modeling inaccuracy, identification of object parameters, as well as perturbations and noise naturally accompanying real processes. However, the very idea of optimal control often turns out to be a proper basis to design a suboptimal structure in which excessive sensitivity would be eliminated.

In this paper, an object described using the second principle of Newton's dynamics, i.e. from physical point of view, representing mass subjected to force, will be considered. Such a mechanical system is a basic element accompanying all considerations in robotics [4]. The uncertainty problem will be considered on the example of the main parameter of such an object, i.e. the value of mass (or the moment of inertia). This problem will be solved here by the introduction of a random factor. Namely, a mass will be treated as a realization of a stochastic process with almost all realizations being piecewise continuous and jointly bounded. The introduction of a random factor makes it possible to take into account errors in the identification of mass, whereas the fluctuations of the particular realizations describe its changes, including also those of a discontinuous nature.

A controlling structure based on the time-optimal approach will be proposed in this paper. It is of soft a character, i.e. allows to eliminate sliding trajectories, which should be avoided in contemporary mechanical systems, since they have negative impact on the endurance of a device and user comfort. The parameters of the designed structure are selected in accordance with the rules of statistical decision theory [2]. The conception presented here is universal and may be supplemented by and generalized with a number of various aspects occurring in such tasks.

The material of this paper will be presented with details in article [13] soon. It carries on research concerning robust control, published already in works [5-10, 12, 16-17].

2 Main Results

Consider a single degree of freedom mechanical system, whose dynamics are described by the second law of Newtonian mechanics

$$m\,\ddot{s}(t) = u(t) \quad , \tag{1}$$

where m, s, u mean the load (mass or moment of iner-

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tia), position (linear or angular), control (force or torque), respectively. If the parameter m is treated as a realization of a stochastic process M, then denoting by $\omega \in \Omega$ a random factor, and by X_1, X_2, U real stochastic processes which represent the position, velocity and control respectively, the dynamics of the system under consideration can be now described by the following random differential equation:

$$\dot{X}_1(\omega, t) = X_2(\omega, t) \tag{2}$$

$$\dot{X}_2(\omega, t) = \frac{1}{M(\omega, t)} U(\omega, t) \quad, \tag{3}$$

with the initial condition

$$\begin{bmatrix} X_1(\omega, t_0) \\ X_2(\omega, t_0) \end{bmatrix} = x_0 \quad \text{for almost all} \quad \omega \in \Omega \quad, \tag{4}$$

given these assumptions

(A1) $t_0 \in \mathbb{R}$, $T = [t_0, \infty)$;

(A2) $x_0 = [x_{01}, x_{02}]^T \in \mathbb{R}^2$ and $x_f = [x_{f1}, x_{f2}]^T \in \mathbb{R}^2$ constitute initial and target states, respectively;

(A3) the values of admissible controls are limited to the interval $[-1, 1]$;

(A4) (Ω, Σ, P) denotes a complete probability space;

(A5) M is a real stochastic process with almost all realizations being piecewise continuous and satisfying the boundary condition $M(\omega, t) \in [m_-, m_+]$ for $t \in T$, where $0 < m_- \leq m_+$.

Introduce also the following subdivision of the state space $\mathbb{R}^2$ into the disjoint sets R_+, R_-, Q_+, Q_-, $\{x_f\}$; see Fig. 1. First, let $\hat{m}$ and $\Delta\hat{m} > 0$ be given such that $\hat{m}, \hat{m} + \Delta\hat{m} \in [m_-, m_+]$. Let also K_{+-}, K_{++} denote sets of all states which can be brought to the target by the control $U \equiv +1$, if $M \equiv \hat{m}$ or $M \equiv \hat{m} + \Delta\hat{m}$, respectively; likewise K_{--} and K_{-+} for $U \equiv -1$, if $M \equiv \hat{m}$ or $M \equiv \hat{m} + \Delta\hat{m}$. Moreover, let:

$$Q_+ = \{[x_1, x_2]^T \in \mathbb{R}^2 \quad \text{such that there exist}$$
$$[x_1', x_2]^T \in K_{+-} \quad \text{and} \quad [x_1'', x_2]^T \in K_{++} \quad \text{with}$$
$$x_1' \leq x_1 \leq x_1'' \quad \text{or} \quad x_1'' \leq x_1 \leq x_1' \} \tag{5}$$

$$Q_- = \{[x_1, x_2]^T \in \mathbb{R}^2 \quad \text{such that there exist}$$
$$[x_1', x_2]^T \in K_{-+} \quad \text{and} \quad [x_1'', x_2]^T \in K_{--} \quad \text{with}$$
$$x_1' \leq x_1 \leq x_1'' \quad \text{or} \quad x_1'' \leq x_1 \leq x_1' \} \tag{6}$$

$$R_+ = \{[x_1, x_2]^T \in \mathbb{R}^2 \setminus Q \quad \text{such that there exists}$$
$$[x_1', x_2]^T \in Q \quad \text{with} \quad x_1 < x_1' \} \tag{7}$$

$$R_- = \{[x_1, x_2]^T \in \mathbb{R}^2 \setminus Q \quad \text{such that there exists}$$
$$[x_1', x_2]^T \in Q \quad \text{with} \quad x_1' < x_1 \} \quad, \tag{8}$$

where $Q = Q_+ \cup \{x_f\} \cup Q_-$. Therefore, the sets K_{+-}, K_{++} represent all those states which can be brought to the target by the control $+1$, at the minimum and maximum possible values of a mass. The set Q_+ contains intermediate points. The sets K_{-+}, K_{--}, and Q_- may be interpreted analogously for the control -1. For illustration, see Fig. 1.
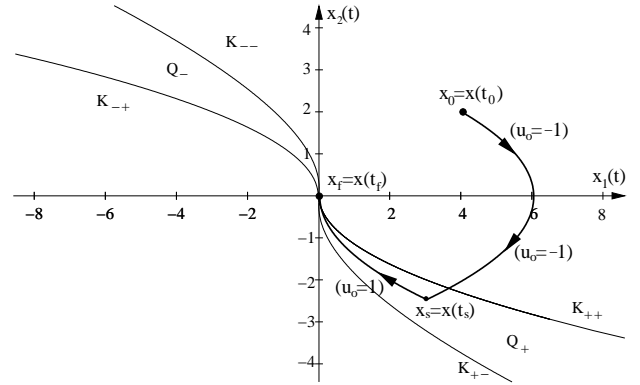

Fig. 1. Illustration of introduced notations.

To define the feedback controller, one should specify rules to calculate the values of the parameters $\hat{m}$ and $\Delta\hat{m}$, and define the control for particular sets R_+, R_-, Q_+, Q_-, $\{x_f\}$. Based on the dedicated mathematical theorem and detailed sensitivity analysis with elements of the statistical decision theory [2] (in particular Bayes and minimax rules), the solutions satisfying these goals are proposed below. Details of this methodology can be found in paper [13].

The case $x_{f2} = 0$ will be considered first.

If over-regulations can be allowed in the controlled object, it is worthwhile using the flexible Bayes rule with the loss function given in the linear and nonsymmetrical form:

$$l(\hat{m}, m) = \begin{cases} -p(\hat{m} - m) & \text{if} \quad \hat{m} - m < 0 \\ 0 & \text{if} \quad \hat{m} - m = 0 \quad, \\ q(\hat{m} - m) & \text{if} \quad \hat{m} - m > 0 \end{cases} \tag{9}$$

where $p, q > 0$ and m can be interpreted here as a real (but unknown) value of a mass. Then the parameter $\hat{m}$ should be calculated as a solution of the following equation:

$$F(\hat{m}) = \frac{p}{p+q} \quad , \qquad (10)$$

where F denotes the distribution function of the random variable characterizing a mass. The values of such random variable can be interpreted here as the mean values of particular realizations of the stochastic process M and may be estimated on the base of the experimentally obtained values of the mass of an object. The practical algorithm to solve equation (10) is presented in paper [11]. For this purpose, one can also use artificial neural networks, according to the procedure presented in article [15].

In turn, if over-regulations are not allowed, this determination needs to be carried out on the basis of the minimax rule for the loss function (9) however with $p = \infty$, realized by

$$\hat{m} = m^+ \quad , \qquad (11)$$

where m^+ means maximal experimentally obtained value of a mass.

Let now $x_{f2} \neq 0$. In this case, the value of the parameter $\hat{m}$ should be determined using the minimax rule, i.e. by dependence (11).

Besides the parameter $\hat{m}$, the additional positive constant $\Delta\hat{m}$ should be specified. The value of the parameter $\Delta\hat{m}$ influences the speed of the control fluctuations in the set Q: the greater the value, the fluctuations are milder. It should be fixed experimentally according to needs of particular problems. To the primary researches one can suggest $\Delta\hat{m} = \hat{m}/10$.

If one possesses the values $\hat{m}$ and $\Delta\hat{m}$, the feedback controller equations can be defined.

As before, the case $x_{f2} = 0$ will be considered first. Let a feedback controller be as follows

$$u(t) = \begin{cases} -1 & \text{if} \quad [x_1(t), x_2(t)]^{\mathrm{T}} \in R_- \\ z(x_1(t), x_2(t)) & \text{if} \quad [x_1(t), x_2(t)]^{\mathrm{T}} \in Q_- \\ 0 & \text{if} \quad [x_1(t), x_2(t)]^{\mathrm{T}} \in \{x_f\} \\ z(x_1(t), x_2(t)) & \text{if} \quad [x_1(t), x_2(t)]^{\mathrm{T}} \in Q_+ \\ +1 & \text{if} \quad [x_1(t), x_2(t)]^{\mathrm{T}} \in R_+ \end{cases} \quad (12)$$

with the function $z : \mathbb{R}^2 \to \mathbb{R}$ continuously and strictly increasing from the value -1 on the sets K_{--} and K_{++} to the value $+1$ on the sets K_{-+} and K_{+-} (see also Fig 1). If the solution $X(\omega, \cdot)$ is "too close" – with respect to real value of the mass – to the set K_{+-}, then control (12) is "too great" and it makes this solution further from the

set K_{+-} to the interior of the set Q_+. And inversely, if the solution is "too far" to the set K_{+-}, then control (12) is "too small" and brings the trajectory closer to this set (see Figs. 1 and 2). The obtained in such a manner result is similar to the effect reached on a bob-sled track thanks to the appropriate modeling of its shape. It is a fluid movement, therefore, allowing such a structure to be named "soft". The analogous situation occurs between the sets K_{-+} and K_{--}.

Having the value $\hat{m}$ following the ideas presented above, and assuming the constant $\Delta\hat{m}$, one can calculate the equation of the set K_{+-}

$$x_1 = \frac{\hat{m}}{2}{x_2}^2 + x_{f1} - \varepsilon \quad \text{for} \quad x_2 \in (-\infty, 0) \qquad (13)$$

and for the set K_{++}

$$x_1 = \frac{\hat{m} + \Delta\hat{m}}{2}{x_2}^2 + x_{f1} + \varepsilon \quad \text{for} \quad x_2 \in (-\infty, 0) \quad , \quad (14)$$

where the additional parameter $\varepsilon \geq 0$ is closer (but is not greater than) to precise positioning (i.e. assumed in practice precision of reaching the target state) and has been introduced to avoid the over-increasing of the function z near the axis x_1. The function z can be proposed in the following manner:

$$z(x_1, x_2) = a(x_2)[x_1 - c(x_2)]^d - 1 \quad \text{for} \quad x_2 \in (-\infty, 0) \qquad (15)$$

with

$$a(x_2) = \frac{4}{\Delta\hat{m}\,{x_2}^2 + 4\varepsilon} \qquad (16)$$

$$c(x_2) = \frac{\hat{m} + \Delta\hat{m}}{2}{x_2}^2 + x_{f1} + \varepsilon \quad , \qquad (17)$$

while the value of the positive parameter d presents a compromise between speed of action of the time-suboptimal control system and its robustness. Namely, $d = 1$ can be treated as neutral; the values $d < 1$, results in making the solutions nearer to the curves K_{-+} or K_{++} that which slows down the process but increases robustness; the inverse when $d > 1$. For primary experimental research $d = 0.25$ is proposed.

The analogous dependencies are outlined in the sets K_{--} and K_{-+}, respectively

$$x_1 = -\frac{\hat{m}}{2}{x_2}^2 + x_{f1} + \varepsilon \quad \text{for} \quad x_2 \in (0, \infty) \qquad (18)$$

$$x_1 = -\frac{\hat{m} + \Delta\hat{m}}{2}x_2{}^2 + x_{f1} - \varepsilon \quad \text{for} \quad x_2 \in (0, \infty) \quad . \quad (19)$$

The function z is proposed here as

$$z(x_1, x_2) = a(x_2)[x_1 - c(x_2)]^{1/d} - 1 \quad \text{for} \quad x_2 \in (0, \infty) \quad (20)$$

with

$$a(x_2) = \frac{-4}{\Delta\hat{m}\, x_2{}^2 + 4\varepsilon} \qquad (21)$$

$$c(x_2) = -\frac{\hat{m}}{2}x_2{}^2 + x_{f1} + \varepsilon \quad . \qquad (22)$$

Let now $x_{f2} \neq 0$. The concept introduced in the preceding paragraph should be transferred in a natural way. For simplicity of notation, the case $x_{f2} > 0$ will be investigated below; if $x_{f2} < 0$ considerations are symmetrical. A feedback controller is also defined in this case by formula (12).

The sets K_{+-} and K_{++} in the part between the target and the axis x_1, should be both defined by the equation

$$x_1 = \frac{\hat{m} + \Delta\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} \quad \text{for} \quad x_2 \in [0, x_{f2}) \qquad (23)$$

with

$$z(x_1, x_2) = 1 \quad \text{for} \quad x_2 \in [0, x_{f2}) \quad . \qquad (24)$$

For the part lying in lower half-plane, the set K_{++} is defined by

$$x_1 = \frac{\hat{m} + \Delta\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} \quad \text{for} \quad x_2 \in (-\infty, 0) \qquad (25)$$

and the set K_{+-} by

$$x_1 = \frac{\hat{m}}{2}x_2{}^2 - \frac{\hat{m} + \Delta\hat{m}}{2}x_{f2}{}^2 + x_{f1} - \varepsilon \quad \text{for} \quad x_2 \in (-\infty, 0) \quad . \qquad (26)$$

The function z is given as

$$z(x_1, x_2) = a(x_2)[x_1 - c(x_2)]^{d} - 1 \quad \text{for} \quad x_2 \in (-\infty, 0) \qquad (27)$$

with

$$a(x_2) = \frac{-4}{\Delta\hat{m}\, x_2{}^2 + 4\varepsilon} \qquad (28)$$

$$c(x_2) = \frac{\hat{m} + \Delta\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} + \varepsilon \quad , \qquad (29)$$

Finally, the sets K_{--} and K_{-+} are defined by

$$x_1 = -\frac{\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} + \varepsilon \quad \text{for} \quad x_2 \in (x_{f2}, \infty) \qquad (30)$$

$$x_1 = -\frac{\hat{m} + \Delta\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} - \varepsilon \quad \text{for} \quad x_2 \in (x_{f2}, \infty) \,, \qquad (31)$$

respectively, and the function z is given as

$$z(x_1, x_2) = a(x_2)[x_1 - c(x_2)]^{1/d} - 1 \quad \text{for} \quad x_2 \in (x_{f2}, \infty) \qquad (32)$$

with

$$a(x_2) = \frac{-4}{\Delta\hat{m}(x_2{}^2 - x_{f2}{}^2) + 4\varepsilon} \qquad (33)$$

$$c(x_2) = -\frac{\hat{m}}{2}(x_2{}^2 - x_{f2}{}^2) + x_{f1} + \varepsilon \quad . \qquad (34)$$

An illustration of the control structure thus obtained, along with the trajectories it generates, is provided in Fig. 2. Rapid changes in control values have been eliminated, according to the assumed goal of the soft structure. The control changes its value fluently in full range of the interval $[-1,1]$.

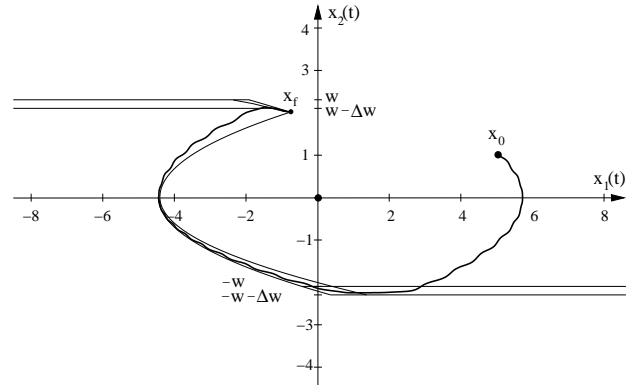

Fig. 2. Soft structure (12) and the trajectories it generates.

3 Final Suggestions and Remarks

The subject presented in this paper is of a universal nature and owing to its clear interpretation it may be easily sup-

plemented by a number of auxiliary aspects frequently occurring in robust control tasks. As a representative example, the problem of velocity limitation, described by the condition

$$\left| X_2(\omega, t) \right| \le w \tag{35}$$

for almost every $\omega \in \Omega$ and every $t \in [t_0(\omega), t_f(\omega)]$, while $w > 0$ and $-w < x_{f2} < w$, will be investigated. Let also an auxiliary parameter Δw, such that $0 < \Delta w \le w$ and $\Delta w - w \le x_{f2} \le w - \Delta w$, be introduced. By defining the function $v : \mathbb{R}^2 \to \mathbb{R}$ (similar to the function z) con-

tinuously and strictly increasing from the value -1 on the set $\mathbb{R} \times \{w\}$ to the value $+1$ on the set $\mathbb{R} \times \{w - \Delta w\}$, with the formula

$$v(x_1, x_2) = 2 \left(\frac{w - x_2}{\Delta w} \right)^D - 1 \quad \text{for} \quad x_2 \in [w - \Delta w, w] \ , \tag{36}$$

where the parameter $D > 0$ plays the same role like d introduced in dependence (15), one can obtain soft structure (12) supplemented with the problem of velocity limitation:

$$u(t) = \begin{cases} -1 & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{R}_- \cup \{\mathbb{R} \times (w, \infty)\} \\ v(x_1(t), x_2(t)) & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{R}_+ \cap \{\mathbb{R} \times [w - \Delta w, w]\} \\ \min\{v(x_1(t), x_2(t)), z(x_1(t), x_2(t))\} & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{Q}_- \cap \{\mathbb{R} \times [w - \Delta w, w]\} \\ z(x_1(t), x_2(t)) & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{Q}_- \cap \{\mathbb{R} \times (\Delta w - w, w - \Delta w)\} \\ 0 & \text{if} & [x_1(t), x_2(t)]^T \in \{x_f\} \\ z(x_1(t), x_2(t)) & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{Q}_+ \cap \{\mathbb{R} \times (\Delta w - w, w - \Delta w)\} \\ \max\{-v(x_1(t), -x_2(t)), z(x_1(t), x_2(t))\} & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{Q}_+ \cap \{\mathbb{R} \times [-w, \Delta w - w]\} \\ -v(-x_1(t), -x_2(t)) & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{R}_- \cap \{\mathbb{R} \times [-w, \Delta w - w]\} \\ +1 & \text{if} & [x_1(t), x_2(t)]^T \in \mathrm{R}_+ \cup \{\mathbb{R} \times (-\infty, -w)\} \end{cases} \tag{37}$$

For interpretation of the above formula, see Fig. 2.

The presented concept can also be applied for many other similar issues appearing in optimal control, e.g. modeling of motion resistance [5-9]. As an example, consider initial system (1) supplemented with the discontinuous model of motion resistance $-b \, \mathrm{sgn}(\dot{s}(t))$, i.e.

$$m \ddot{s}(t) = u(t) - b \, \mathrm{sgn}(\dot{s}(t)) \ , \tag{38}$$

where $b \in [0,1)$; then under- or overestimating the value of the parameter b will entail similarly raising or lowering the parameter m, and further considerations are analogous to that presented above.

The correct functioning of the suboptimal structure investigated in this paper has been verified by numerical simulation [6]. The object is a mechanical system (1) with unknown (random) and/or varying mass. In the case $x_{f2} = 0$, if it is assumed that over-regulations are undesirable, then they did not occur in the controlled object. Rapid changes in control values, in particular switchings along sliding trajectories, were completely eliminated in the object controlled by soft structure.

Typical trajectories generated by control structure (12) are shown in Fig. 2. Tables 1 and 2 show times to reach the target set when $x_{f2} = 0$ and $x_{f2} \ne 0$, respectively. The results are shown for the optimal control (under practically unrealistic assumption that the true value of the mass m is known exactly) and the suboptimal structures: hard (hypothetically obtained for $\Delta \hat{m} = 0$, with possibility of commonness existence of sliding trajectories) and soft ones. It is not surprising that the shortest times to reach the target were obtained for optimal control (owing to hypothetical assumption of exactly known mass), followed by the hard structure (although at the cost of frequent and arduous switches on sliding trajectories), while the longest times for the soft structure, inversely proportional to the value of the parameter d. If, however, each value of m was supplemented by perturbation, with the value of $0.5m \sin(25t)$, the results favored the soft structure at small values of the parameter d, as the most robust. Note that in the case of the soft structure, the results were satisfying even when temporarily $m \notin [m_-, m_+]$.

The material of this paper will be presented with details in article [13] soon.

Tab. 1. Times to reach the target set for $x_0 = [5,0]^T$, $x_f = [0,0]^T$, $\hat{m} = 1.5$, $\Delta\hat{m} = 0.3$.

control structures:	optimal	hard	soft		
			$d = 0.2$	$d = 1$	$d = 5$
$m = 0.6$	3.446	4.537	4.805	4.638	4.560
$m = 1.0$	4.442	4.955	5.154	5.010	4.966
$m = 1.4$	5.250	5.340	5.430	5.356	5.343

Tab. 2. Times to reach the target set for $x_0 = [5,0]^T$, $x_f = [2,2]^T$, $\hat{m} = 1.5$, $\Delta\hat{m} = 0.3$.

control structures:	optimal	hard	soft		
			$d = 0.2$	$d = 1$	$d = 5$
$m = 0.6$	4.3635	7.0253	7.8483	7.6609	7.5716
$m = 1.0$	6.4588	7.9402	8.7259	8.5706	8.5218
$m = 1.4$	8.4851	8.7839	9.4997	9.4164	9.4009

References

[1] M. Athans and P.L. Falb, *Optimal Control.* McGraw-Hill, 1966.

[2] J.O. Berger, *Statistical Decision Theory*. Springer-Verlag, 1980.

[3] P. Kowalski, K. Krawiec, *Własności silnego i słabego trybu sterowania poślizgowego"*. Cracow University of Technology, Faculty of Electrical and Computer Engineering, Department of Control Engineering, Cracow, 2003 (unpublished).

[4] K. Kozłowski, P. Dutkiewicz, W. Wróblewski, *Modelowanie i sterowanie robotów*. PWN, 2003.

[5] P. Kulczycki, "Time-Optimal Stabilization of a Discontinuous and Non-Autonomous Dynamic Object," *Control and Cybernetics*, vol. 25, no. 4, pp. 707-720, 1996.

[6] P. Kulczycki, "Almost Certain Time-Optimal Positional Control," *IMA Journal of Mathematical Control & Information*, vol. 13, no. 1, pp. 63-77, 1996.

[7] P. Kulczycki, "Some Remarks on Solutions of Discontinuous Differential Equations Applied in Automatic Control," *Industrial Mathematics*, vol. 46, no. 2, pp. 119-128, 1996.

[8] P. Kulczycki, "Random time-optimal control for mechanical systems," *European Journal of Automation*, vol. 33, no. 1-2, pp. 115-140, 1999.

[9] P. Kulczycki, "A Random Approach to Time-Optimal Control," *Journal of Dynamic Systems, Measurement, and Control*, vol. 121, no. 3, pp. 542-543, 1999.

[10] P. Kulczycki, "Fuzzy Controller for Mechanical Systems," *IEEE Transactions on Fuzzy Systems*, vol. 8 no. 5, pp. 645-652, 2000.

[11] P. Kulczycki, "An Algorithm for Bayes Parameter Identification," *Journal of Dynamic Systems, Measurement, and Control*, special issue "Identification of Mechanical Systems," vol. 123, no. 4, pp. 611-614, 2001.

[12] P. Kulczycki and R. Wisniewski, "Fuzzy Controller for a System with Uncertain Load," *Fuzzy Sets and Systems*, vol. 131. no. 2, pp. 185-195, 2002.

[13] P. Kulczycki, R. Wisniewski, P. Kowalski, and K. Krawiec, "Hard and Soft Sub-Time-Optimal Controllers for a Mechanical System with Uncertain Mass," in press.

[15] H. Schiøler and P. Kulczycki, "Neural Network for Estimating Conditional Distributions," *IEEE Transactions on Neural Networks*, vol. 8, no. 5, pp. 1015-1025, 1997.

[16] R. Wisniewski and P. Kulczycki, "Rotational Motion Control of a Spacecraft," *IEEE Transactions of Automatic Control*, vol. 48, no. 4, pp. 643-646, 2003.

[17] R. Wisniewski and P. Kulczycki, "Slew Maneuver Control for Spacecraft Equipped with Star Camera and Reaction Wheels," *Control Engineering Practice*, special issue on Aerospace Control, 2004, in press.