MC-Parameter Calibration of Baskarp Sand No. 15

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MC-Parameter Calibration for Baskarp Sand No. 15

Parameters for the Mohr Coulomb constitutive model is determined for Baskarp Sand No. 15 (Aalborg University Sand No. 1). The parameters have been determined based on a combination of cone penetration tests with a mini-CPT cone and drained triaxial tests.

1 Introduction

The soil parameters are dependent on both confining pressure and density index. The density index of Baskarp Sand can be determined by conducting cone penetration tests with the mini-CPT cone used in the Geotechnical Engineering Laboratory at Aalborg University:

\[
I_D = 5.14 \left( \frac{\sigma'_e}{q_c} \right)^{0.42}
\]

where

- \(\sigma'_e\) is the vertical effective stress [MPa]
- \(q_c\) is the cone resistance [MPa]

The unit weight of Baskarp Sand No. 15 is estimated to 20 [kN/m³] and used to calculate the vertical effective stress through the soil. The variation of the density index is then determined from the measured cone resistance of the mini-CPT cone used in the Geotechnical Engineering Laboratory. All the expressions are derived from several conventional drained triaxial tests at two different density indices performed on Baskarp Sand No. 15 (Ibsen & Bødker, 1994). The used triaxial tests are listed in Table 1 and Table 2.
2 Strength Parameters

In the following, the strength parameters for Baskarp Sand No. 15 are determined.

2.1 Friction Angle and Cohesion

The friction angle and cohesion for Baskarp Sand No. 15 are determined by the following three expressions:

- Curved Coulomb Criterion

- Modified Schmertmann

- Linear Coulomb Criterion

Curved Coulomb Criterion

The expression is derived by fitting the Curved Coulomb Criterion to the triaxial test data and then expressing a linear relation between the asymptotic parameters $\varphi_a$ and $c_a$ and the density index. The Curved Coulomb Criterion is fitted to the failure values from the triaxial tests in a $(\sigma_3 - q)$-plot, see Figure 1.
Figure 1  Confining pressure and deviatoric stress at failure for the triaxial tests and
the Curved Coulomb Criterion at density indices of 51% and 80%.

The expression is modified to be density index dependent by expressing a linear relation between the
asymptotic parameters, $\varphi_a$ and $c_a$, and the density index:

$$\varphi_a = 0.091 \cdot I_D + 30.6 \quad [^\circ]$$  \hspace{1cm} (2)

$$c_a = 0.43 \cdot I_D + 7.5 \quad [kPa]$$  \hspace{1cm} (3)

The friction angle and the cohesion can now be determined by (4) and (5).

$$\varphi = \sin^{-1} \left( \frac{\delta \varphi}{\delta \sigma'} \right) \quad [^\circ]$$  \hspace{1cm} (4)

$$c = \frac{q_0}{\frac{2 \sin \varphi}{1-\sin \varphi} \cot \varphi} \quad [kPa]$$  \hspace{1cm} (5)

**Schmertmann**

After fitting the Modified Schmertmann expression to the series of triaxial tests the following variation of
the secant friction angle with density index and confining pressure is found:

$$\varphi_s = 0.152 \cdot I_D + 27.4 \cdot (\sigma'_3)^{0.28} + 23.2$$  \hspace{1cm} (6)

where

- $I_D$ is the density index, determined by (1) [%]
- $\sigma'_3$ is the confining pressure [kPa]

The expression is not valid for very loose sands and it goes towards infinity for very small confining
pressures. The expression is in Figure 2 plotted against the results of the triaxial tests. The Modified
Schmertmann expression yields a secant friction angle, hence the cohesion equals zero.
Linear Coulomb Criterion

When using the Linear Coulomb Criterion, the tangent friction angle and the cohesion are taken as pressure independent. Thus, the Linear Coulomb Criterion is only density index dependent. The expression assumes a linear relation between the strength parameters and the density index.

\[
\varphi_t = 0.11 \cdot I_D + 32.3 \quad [\degree] \tag{7}
\]

\[
c = 0.032 \cdot I_D + 3.52 \quad [\text{kPa}] \tag{8}
\]

The tangent friction angle and cohesion are fitted for triaxial tests with a confining pressure below 100 kPa, cf. Figure 3.

2.2 Dilatancy Angle

The dilatancy angle also depends on the friction angle and the density index. A linear relation between the dilatancy angle and the density index is found to fit the results from the triaxial tests. The stress dependency is expressed by a power function, see (9).
\[ \psi = 0.195 \cdot I_D + 14.9 \cdot \left(\sigma_3^{'}\right)^{0.976} - 9.95 \]  

where

- \( \psi \) is the dilatancy angle \([\cdot]\)
- \( I_D \) is the density index, determined by (1) \([\%]\)
- \( \sigma_3^{'} \) is the confining pressure \([\text{kPa}]\)

The expression is not valid for very loose sands and it goes towards infinity for very small confining pressures. The expression is fitted from the dilatancy angles determined from triaxial tests, which are plotted in Figure 4 together with the derived expression. The variation of the dilatancy angle is plotted for density indices of 51\% and 80\%, respectively.

![Figure 4 Dilatancy angles from the triaxial tests and the Modified Schmertmann expression at density indices of 51\% and 80\%.

3 Elastic Parameters

As elastic parameters Young’s modulus and Poisson’s ratio are used.

3.1 Young’s Modulus

Young’s modulus is chosen as the secant modulus at 50\% strength and determined based on the triaxial tests on Baskarp Sand No. 15.

\[ E_{50} = E_{50}^{ref} \left( \frac{c \cdot \cos (\varphi_t) + \sigma_3^{'} \cdot \sin (\varphi_t)}{c \cdot \cos (\varphi_t) + \sigma_3^{ref} \cdot \sin (\varphi_t)} \right)^{\mu} \]  

where
\( \varphi_t \) is the tangent friction angle [\(^\circ\)]

\( \sigma_3 \) is the confining pressure [kPa]

\( \sigma_3^{ref} \) is the reference confining pressure [kPa]

\( m \) is the amount of stress dependency [-]

\( E_{50}^{ref} \) is the reference secant modulus corresponding to \( p_{ref} \) [kPa]

In Figure 5, \( E_{50} \) determined by means of (10) is plotted against the triaxial tests.

![Graph showing the relationship between \( E_{50} \) and \( \sigma_3 \) for two different density indices, 51% and 80%.]

Figure 5 \( E_{50} \) determined from the triaxial tests and the derived expression at density indices of 51% and 80%.

For a reference confining pressure of 100 kPa, the reference secant modulus, \( E_{50}^{ref} \) and the amount of stress dependency, \( m \), are determined. The parameter \( m \) accounting for the stress dependency is determined to 0.58. The reference secant modulus can be determined by:

\[
E_{50}^{ref} = 0.6322 \cdot I_D^{2.507} + 10920 \quad [kPa]
\]

3.2 Poisson’s Ratio

The elastic parameter Poisson’s ratio, \( \nu \), is determined in (Andersen et al. 1998) where it is found to, \( \nu = 0.25 \), with a standard deviation of 0.06.

4 Verification of MC-parameters

The derived expressions for the MC-parameters are verified by simulating triaxial tests in ABAQUS and comparing it with results from the performed triaxial tests. The estimates of the secant modulus, \( E_{50} \), Poisson’s ratio and the dilatancy angle are the same for all plots, see Table 3, but the friction angle and the cohesion are estimated as described above in three different ways, see Table 4.
Table 3 E₅₀, ν, and ψ input parameters for simulation of triaxial tests at a density index of 80%.

<table>
<thead>
<tr>
<th>σ₃ [kPa]</th>
<th>E₅₀ [10³ kPa]</th>
<th>ν [-]</th>
<th>ψ [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>13.0</td>
<td>0.25</td>
<td>18.0</td>
</tr>
<tr>
<td>10</td>
<td>17.1</td>
<td>0.25</td>
<td>17.7</td>
</tr>
<tr>
<td>20</td>
<td>22.1</td>
<td>0.25</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 4 Friction angle and cohesion determined by the Curved Coulomb Criterion, the Modified Schmertmann, and the linear Coulomb Criterion, Iₚ=80%.

<table>
<thead>
<tr>
<th>σ₃ [kPa]</th>
<th>Modified Schmertmann</th>
<th>Curved Coulomb Criterion</th>
<th>Linear Coulomb Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>φᵣ [°] c [kPa]</td>
<td>φ [°] c [kPa]</td>
<td>φᵣ [°] c [kPa]</td>
</tr>
<tr>
<td>5</td>
<td>52.5 0.1</td>
<td>48.9 1.9</td>
<td>40.8 6.0</td>
</tr>
<tr>
<td>10</td>
<td>49.8 0.1</td>
<td>46.5 3.4</td>
<td>40.8 6.0</td>
</tr>
<tr>
<td>20</td>
<td>47.3 0.1</td>
<td>44.0 5.8</td>
<td>40.8 6.0</td>
</tr>
</tbody>
</table>

The triaxial tests performed at 5, 10 and 20 kPa with a density index of 80% are shown in Figure 6.
Figure 6 Stress-strain relations and normal- and volume strain relations for triaxial tests.
5 Conclusion

The very high friction angles at low stress levels determined from the Modified Schmertmann expression cause problems for the numerical calculations in ABAQUS, which are shown by the scatter in Figure 6(a) and Figure 6(c). In spite of that, the parameters fit the triaxial tests good. Both the Linear Coulomb Criterion and the Curved Coulomb Criterion fits the triaxial tests closely.

6 References


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