Assessment of Time Functions for Piles Driven in Clay

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Assessment of Time Functions for Piles Driven in Clay

A. H. Augustesen, L. Andersen, and C. S. Sørensen

The vertical bearing capacity of piles situated in clay is studied with regard to the long-term set-up. A statistical analysis is carried out on the basis of data from numerous static loading tests. The database covers a wide range of both soil and pile properties, which ensures a general applicability of the results. Firstly, it is validated that set-up leads to a linear increase of the capacity with the logarithm of time. This property is a basic assumption in most set-up models. Secondly, three different models suggested in the literature are assessed, and a comparison is made with two alternative models. In the first of these models, the rate of set-up is independent of the soil properties, whereas the second function depends on the undrained shear strength. Based on the available data, there is no statistical evidence that the magnitude of set-up depends on the properties of the soil. Hence, it is suggested that a constant set-up factor should be applied for the prediction of pile capacities at a given time after initial driving.

1 Introduction

For piles located in clay, sand or a combination of different soil types, experience shows that engineering time has an important effect on pile capacity. Thus, Wendel (1900) documented that the bearing capacity of timber piles located in clay continued to increase for two to three weeks after driving. This phenomenon is also known as set-up and has later been discussed extensively in the literature by, for example, Bullock et al. (2005a,b), Long et al. (1999) and Augustesen et al. (2005b). In particular, an attempt has been made to identify the causes of set-up, and empirical relations for quantifying the set-up have been offered. Such relations, in the following denoted time functions, may advantageously be employed in the design phase, since they will lead to economic savings through fewer, shorter or thinner piles.

This paper focus on assessment of time functions for axially loaded piles in clay. Time functions proposed in the literature are discussed and the possibility of introducing an alternative time function is investigated. The analysis is based on 18 cases with a total of 27 piles reported in the literature. In total, 88 pile tests are included in the database and only static tests are considered. The time between initial driving and the final static test on each pile varies from 22 to 9778 days. Further, the number of tests on each pile ranges from two to six.

2 Background

Skov and Denver (1988) described the relation between time, \( t \), and vertical bearing capacity, \( Q \), by a semi-logarithmic time function in the form

\[
Q = Q_0 \left[ 1 + \Delta_{10} \log_{10} \left( \frac{t}{t_0} \right) \right]
\]

Here \( Q_0 \) is the reference capacity measured at the reference time \( t_0 \), and \( \Delta_{10} \) is a factor providing the capacity increase corresponding to a ten-fold increase in time. In the following, \( \Delta_{10} \) is referred to as the set-up factor.

Short-term effects regarding the bearing capacity of piles are related to both real time effects (ageing) and the equalisation of excess pore pressures built up during driving. In contrast, long-term effects are only due to ageing. Hence, different values of \( \Delta_{10} \) are expected when either short- or long-term effects are investigated (Figure 1). The definition “short-term” can be misleading in connection with piles in clay because it may cover up a long period of time. If set-up is considered, the short-term component of \( \Delta_{10} \) is greater than the long-term component, i.e. \( \Delta_{10} \text{short-term} > \Delta_{10} \text{long-term} \). When relaxation, defined as a drop in capacity with time, takes place, \( \Delta_{10} \text{short-term} \) becomes negative. Relaxation have been reported and discussed by, for example, Davie and Bell (1991), Thompson and Thompson (1985), and York et al. (1994).

According to Skov and Denver (1988), the values of \( \Delta_{10} \) in Eq. (1) for piles located in sand, clay and chalk are 0.2, 0.6 and 5.0, respectively. Correspondingly, the reference time, \( t_0 \), is assumed to be 0.5, 1.0 and 5.0 days. These values of \( t_0 \) ensure a stabilized increase of the capacity with time. Before this, the pore pressure has not reached the stationary state and soil remoulding continues to take place. Furthermore, Skov and Denver (1988) point out that there should be an upper limit to \( t \) for which Eq. (1) is used. However, no guidelines are given for this upper limit.

The assumption \( t_0 = 0.5 \), 1.0 or 5.0 for piles in sand, clay, and chalk, respectively, may be inconvenient if \( Q_0 \) is to be measured at the reference time. Therefore, Svinkin and Skov (2000) gave an alternative definition of \( Q_0 \) as the capacity at the end of initial driving and suggested the reference time \( t_0 = 0.1 \) days. Based on dynamic and static tests performed within a period of...
132 days after driving, $\Delta_{10}$ was found to vary between 1.14 and 3.50 for piles located in clayey soils.

Equation (1) concerns the total bearing capacity of a pile. However, Bullock et al. (2005a,b) report on similar development of the side-shear capacity with time for concrete piles driven into a variety of coastal plain soils in Florida. $\Delta_{10}$ is found to lie in the range 0.12 to 0.32. Piles located in clays generally experience higher values of $\Delta_{10}$ than piles in sand; but $\Delta_{10}$ does not depend significantly on the properties of the soil within each category. For design purposes Bullock et al. (2005b) recommend a value of $\Delta_{10} = 0.1$ ($t_0 = 1$ day) for piles in clay when no test results are available for the specific site.

Whereas Skov and Denver (1988) and Bullock et al. (2005a,b) propose a constant value of $\Delta_{10}$, Clausen and Aas (2000) postulate that the long-term set-up depends on the soil properties. Thus, $\Delta_{10}$ is a function of the plasticity index, $I_p$, and the over-consolidation ratio, OCR,

$$\Delta_{10} = 0.1 + 0.4 \left(1 - \frac{I_p}{50}\right) \text{OCR}^{-0.8},$$

NGI

0.1 $\leq \Delta_{10} \leq 0.5$

Equation (2) is based on very few tests. The reference time, $t_0$, is chosen to be 100 days. The time function based on Eq. (2) is denoted NGI because it has been developed at the Norwegian Geotechnical Institute.

Other semi-empirical relations have been presented in the literature. Guang-Yu (1988) proposes a relation where the capacity of piles in soft ground corresponding to $t = 14$ days depend on the sensitivity, $S_s$ and $Q_{\text{void}}$ (capacity at the "end of initial driving"). Huang (1988) postulates that the capacities for piles in soft Shanghai soils are a function of $Q_{\text{void}}$, the logarithm to time and a quantity denoted the maximum pile capacity. Svinkin et al. (1994) propose two exponential functions where the capacity is a function of $Q_{\text{void}}$ and time. The study is based on testing five pre-stressed concrete piles driven in predominantly silty sands and dense soil at the lower third of the piles’ embedded lengths.

In this study the assessment of time functions consists of validating the semi-logarithmic relation between capacity, $Q$, and time after driving, $t$, i.e. Eq. (1). Another objective is to compare existing expressions of $\Delta_{10}$, and thereby the time functions, proposed by Skov and Denver (1988), Bullock et al. (2005a,b) and Clausen and Aas (2000). Thirdly, the possibility of introducing a new expression for $\Delta_{10}$ is investigated. However, firstly a common reference time should be chosen, and a careful interpretation of the test results forming the basis of the analyses has to be made.

### 2.1 Choice of reference time

As indicated in the former section, Skov and Denver (1988) and Bullock et al. (2005a,b) use an arbitrary, but practical, reference time of $t_0 = 1$ day. However, in the present study, $t_0$ is chosen to be 100 days. This value was applied by Clausen and Aas (2000). It is noted that the choice of reference time affects the value of both $Q_0$ and $\Delta_{10}$, cf. Eq. (1). However, if the value of $\Delta_{10}$ for a different reference time is required, this may be found easily. Thus, defining two consistent sets of parameters $(t_0, 1, Q_{0,1}, \Delta_{10,1})$ and $(t_0, 2, Q_{0,2}, \Delta_{10,2})$, the following relationship is obtained from Eq. (1):

$$Q = Q_{0,1} \left[1 + \Delta_{10,1} \log_{10} \left(\frac{t}{t_{0,1}}\right)\right]$$

(3)

$$= Q_{0,2} \left[1 + \Delta_{10,2} \log_{10} \left(\frac{t}{t_{0,2}}\right)\right] \forall t$$

$$\Rightarrow Q_{0,1} \cdot \Delta_{10,1} \cdot \log_{10} (t) = Q_{0,2} \cdot \Delta_{10,2} \cdot \log_{10} (t)$$

$$\Rightarrow Q_{0,1} \cdot \Delta_{10,1} = Q_{0,2} \cdot \Delta_{10,2}$$

**Figure 1** Influence of short- and long-term effects on $\Delta_{teoc}$. The time for equalisation of pore pressures due to pile installation is denoted $t_{teoc}$. Short-term effects are related to pore pressure dissipation and ageing whereas long-term effects are only due to ageing. It should be noted that both $\Delta_{teoc}$ and $Q_{0}$ depend on whether $t < t_{teoc}$ or $t > t_{teoc}$ are considered.
Now, substitution of $Q_{0,2}$ with

$$Q_{0,2} = Q_{0,1} \left[ 1 + \Delta_{10,1} \log_{10} \left( \frac{t_{0,2}}{t_{0,1}} \right) \right]$$

in Eq. (3) yields the relation

$$\Delta_{10,1} - \Delta_{10,2} = \Delta_{10,1} \cdot \Delta_{10,2} \cdot \log_{10} \left( \frac{t_{0,2}}{t_{0,1}} \right)$$

Thus, $t_0$ may be chosen freely without loss of generality; but in order to compare the values of $\Delta_{10}$ suggested in the literature, they need to be converted to the same reference time in accordance with Eq. (5).

### 2.2 Choice of reference capacity

The reference capacity, $Q_0$, may be determined by some design method such as the API procedure proposed by the American Petroleum Institute (API, 1993) or the NGI-99 method which is developed at the Norwegian Geotechnical Institute (Clausen and Aas, 2000). However, as shown by Clausen and Aas (2000) and Augustesen et al. (2005a), even some of the widely accepted methods involve great amounts of uncertainty. Therefore, in the present work $Q_0$ is instead determined on the basis of the available test results by linear regression of $Q(t)$ versus $\log_{10}(t)$ for each pile, see Section 4. $Q_0$ is then the point on the regression line corresponding to $\log_{10}(t_0)$.

By choosing a small value of $t_0$, e.g. 1 day as proposed by Skov and Denver (1988) for clayey soils, small or even negative values of $Q_0$ may be obtained for piles which are only tested twice. In these circumstances, erroneous capacities are predicted. With $t_0 = 100$ days this problem is avoided. Hence, this reference time has been employed in the present analysis.

### 2.3 Interpretation of loading tests

A reliable measurement of set-up requires that the uncertainties related to the test procedure and the site conditions are minimized. First of all, the soil and pile conditions should be clearly defined. Further, the strata should be homogeneous and of such horizontal extent that several similar piles can be installed at approximately the same time in the same type of soil without group action taking place. Pile tests should be arranged as sketched in Figure 2. Thereby the effects of time can be separated from the effects of previous load testing of the same pile. If ageing is of interest, the first pile should be tested after equalisation of excess pore pressures. In contrast, if the goal is to establish the maximum set-up no considerations should be paid to excess pore pressures. Furthermore, the piles should be tested by the same procedure and the failure criterion should be defined uniquely. Though, the data in the present database are treated consistently and in most cases are of high quality, they do not punctually fulfil the above-mentioned recommendations.

#### 2.3.1 Soil and pile conditions

In most of the cases constituting the present database, the strata are highly non-homogeneous and in some cases sand layers interbed the clay layers. This is not taken into consideration, i.e. the soil is assumed to consist solely of clay. The specific influence of the sand layers could be taken into consideration by measuring the side shear forces during testing but this is far from common in practice.

The properties of the soil are not determined in the same way in all cases in the database. For example, the undrained shear strength, $S_u$, may be measured by a vane shear test in some cases and by means of unconsolidated undrained triaxial tests in other cases. This complicates the application of time functions in which $\Delta_{10}$ depends on the soil properties, e.g. the NGI model, cf. Eq. (2). However, in order to obtain a consistent treatment of the available data, a unique set of rules based on Clausen and Aas (2000) that allow any strength to be calculated from another has been employed. This strength conversion is a controversial matter within the profession of soil mechanics. Further, if the plasticity index, $I_p$, and the overconsolidation ratio, OCR, are not provided, it is assumed that $I_p = 25\%$ and OCR is calculated by means of the SHANSEP relation (Ladd et al., 1977),

$$\frac{S_u}{\rho_0} = \beta \cdot OCR^{\Lambda}$$

where $S_u$ is the undrained shear strength, $\rho_0$ is the vertical effective stress, $\beta$ is the normally consolidated undrained shear strength ratio $(S_u/\rho_0)_{nc}$, and $\Lambda$ is a strength rebound parameter [-]. In this study, it is chosen to make use of the parameters $\beta = 0.25$ and $\Lambda = 0.85$, and $S_u$ is assumed to be the unconsolidated undrained shear strength, $S_{uu}$. As is the case for $S_u$ conversions, the dependence of $S_u$ on OCR is also of controversial matter within geotechnical engineering. Limits on $\beta$ and $\Lambda$ as function of shear strength are discussed in Mayne (1988). It should be mentioned that in none of the cases associated with this study, OCR is measured.

Bullock et al. (2005b) postulate that $\Delta_{10}$ does not depend significantly on the pile length for penetration depths smaller than 25m. By contrast, the pile diameter
Figure 2 Ideal test series for studying the influence of time on the bearing capacity. The arrangement of pile tests implies that the effects of time are separated from the effects of previous load testing.

\[ t_d = 0 \text{ Time of installation.} \quad n \text{ is the number of piles.} \]
\[ t_1 \text{ Pile 1 is tested.} \quad t_d = 0 < t_1 < t_2 < \ldots < t_n \]
\[ t_2 \text{ Pile 1 and 2 are tested.} \]
\[ t_n \text{ Pile 1 - n are tested.} \]

has an influence on the time development of set-up. In particular, excess pore pressures are induced by the penetration of a displacement pile with a maximum value of the pore pressure near the pile surface and diminishing to zero at some radial distance. As these pressures dissipate, the soil consolidates. The duration of this process is approximately proportional to the square of the pile diameter. Assuming that the pile displacement causes a destructuring gradient in the soil similar to the pore pressure gradient, the restructuring (ageing) may develop over time in a similar manner (Bullock et al., 2005b). Hence, the cross-sectional geometry of a pile affects the development of the capacity with time due to both consolidation and ageing. However, the geometry of the pile is not taken into consideration in the present analysis, i.e. piles are not divided into groups according to their cross-sectional geometry.

Since the pile and soil conditions are important parameters in set-up analyses, a quality ranking, \( Q_r \), is specified for all available cases, cf. Table 1. Five categories are applied for the quality, namely \( Q_r = 0 \): not known; \( Q_r = 1 \): low; \( Q_r = 2 \): average; \( Q_r = 3 \): high; and \( Q_r = 4 \): very high.

2.3.2 Group action and staged loading

The interpretation of loading tests, and thereby the magnitudes of the measured capacities, may be influenced by group action and pre-shearing effects (staged loading). Thus, if a pile has been tested more than once, previous loading tests may have an effect on the capacity. However, in the light of their test results Bergdahl and Hult (1981) postulate that it is not possible to show any change in capacity as a result of previous loading tests for piles in clay.

In contrast to this, Karlsrud and Haugen (1986) as well as Bullock et al. (2005a,b) report that pre-shearing effects may be substantial. Thus, staged loading results in higher bearing capacities compared to the “intact equivalents”, i.e. the capacities obtained by unstaged loading. Based on two examples in the literature and a research programme conducted at the University of Florida, Bullock et al. (2005b) recommend that the ratio \( C_{st} = \frac{\Delta 10,\text{Unstaged}}{\Delta 10,\text{Staged}} = 0.4 \) should be applied to convert the results of staged to unstaged tests when using \( t_0 = 1 \text{ day} \).

Unfortunately, only few results for unstaged loading are available in the literature. Hence, the starting point of this study is staged loading tests. However, employing the guidelines provided by Bullock et al. (2005b) the corresponding results for unstaged loading are readily obtained. Furthermore, when only staged loading tests are considered the data can be treated in a consistent manner. Finally, sufficiently many tests are available in the literature to ensure that statistically significant conclusions can be drawn.

Group action leads to an increase in set-up magnitudes as reported by Camp et al. (1993). Even if group effects are substantial they are not considered in the present work, simply because little information related to group action is provided in the cases forming the basis of this study.

2.3.3 Testing procedure

In this study attention is entirely paid to pile capacities based on static loading tests. Cases including dynamic tests could advantageously be incorporated in the database to improve the statistical foundation of the analyses. However, this implies that the capacities obtained from dynamic and static loading tests are strictly comparable. By focussing on static tests, uncertainty regarding this subject is neglected. The data are thereby treated in a consistent manner if the same failure criterion is applied in all cases. It should be mentioned that the influence of loading rate is not taken into consideration.
According to Bullock et al. (2005b) virtually all pile setup research is based on piles being unloaded between consecutive tests. Those few tests carried out for piles that remain loaded between tests indicate that such piles exhibit conservatively more set-up than piles which are unloaded between tests. This has not been taken into consideration in the present study, i.e. no distinction is made with regard to the loading conditions between two subsequent tests.

In the database forming the basis of this study no distinction is made between piles loaded in compression and tension. As indicated in Table 2 only four of the 27 piles constituting the database are loaded in tension. Based on a study, the results of which are not included in this manuscript, it has been found that the results for three of these piles do not differ significantly from the results for the piles loaded in compression with respect to the formulation and calibration of a time function. However, results for the fourth pile, which belongs to the case with ID 9, differ significantly ($\Delta_{10} = -0.06$), cf. Table 1 and Table 2. Therefore, this case has been omitted in all of the following analyses.

2.3.4 Failure criterion
The measured capacities associated with each case in the database are based on failure loads corresponding to settlements equal to $0.1d$, where $d$ is the equivalent pile diameter referring to an equivalent circle diameter for square and hexagonal piles. Experience shows that both the toe and shaft resistance are fully mobilised at this displacement (Vijayvergiya, 1977; API, 2000).

3 Presentation of cases
Key data for the cases that form the basis of this study are summarized in Table 1 and Table 2. The majority of the cases have been found in the literature and further data have been provided by the Danish company COWI A/S and the Norwegian Geotechnical Institute. The following comments are given regarding the database:

1. 18 cases including 27 piles constitute the database.
2. All piles have been subjected to staged static tests.
3. In total, 88 pile tests are included in the database.
4. The time elapsed between initial driving and the final static test of each pile varies from 22 to 9778 days. Further, the number of tests on each pile ranges from two to six.
5. Four piles are loaded in tension and 23 piles are loaded in compression.
6. Eleven piles are made of steel, three of timber, and four of concrete.
7. The diameters range from 0.1 m to 1.372 m and four piles are driven open-ended. The tip penetration range from five to 49 m.
8. The average unconsolidated undrained shear strength, $S_{uu}$, varies between approximately 12 and 136 kPa, the overconsolidation ratio, OCR, varies between 1.1 and 25.2, and the average plasticity index, $I_p$, varies between 15 and 47.

Both offshore and onshore piles are included and the different cases are grouped, specified by the quality ranking, $Q_r$, cf. Subsection 2.3.1.

4 Semi-logarithmic time function
The bearing capacity and the time scale for different piles and test sites may be very different, cf. Table 2. In order to test linearity between time, $t$, elapsed since initial driving and capacity, $Q$, as expressed in Eq. (1) and in order to compare results from different cases, the normalised versions of $Q$ and $t$ are investigated by plotting

\[
\left(\frac{Q_j}{Q_{0j}} - 1\right) / \Delta_{10j} \text{ versus } \log_{10}\left(\frac{t}{t_0}\right)
\]

Here $Q_j$ is the measured capacity for pile $j$ at time $t$ after installation and $Q_{0j}$ is the reference capacity for pile $j$ corresponding to the reference time $t_0 = 100$ days, cf. Section 2.1.

Firstly, for each individual pile the Method of Least Squares is adopted for a linear regression analysis of $Q_j(t)$ versus $\log_{10}(t)$. $Q_{0j}$ is then defined as the point on the regression line corresponding to $\log_{10}(t_0)$. Further, in accordance with Eq. (7), the set-up factor, $\Delta_{10j}$, for pile $j$ is determined as the inclination of the regression line obtained when plotting

\[
\frac{Q_j(t)}{Q_{0j}} - 1 \text{ versus } \log_{10}\left(\frac{t}{t_0}\right)
\]

Employing the values of $\Delta_{10j}$ obtained in this manner, Eq. (7) provides ideally a number of lines with the inclination $\beta_j = 1$ and going through origo. The deviation of the normalised test data from this line forms the basis for testing the validity of the semi-logarithmic time function.

4.1.1 Linear regression
Based on Eq. (7) and the data presented in Table 1 and Table 2, the normalised capacities are plotted against normalised time in Figure 3. Every dot corresponds to one measured capacity, i.e. one pile test. By visual inspection it is concluded that the pile tests all fit into the assumed relation between $Q$ and $t$, i.e. Eq. (1). However, cases including only two tests on the same pile
### Table 1 Site specifications.

<table>
<thead>
<tr>
<th>ID</th>
<th>Name</th>
<th>Reference</th>
<th>ID</th>
<th>Name</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Houston</td>
<td>O’Neill et al. (1982a,b)</td>
<td>2</td>
<td>Cowden</td>
<td>Powell et al. (2003)</td>
</tr>
<tr>
<td>3</td>
<td>Drammen</td>
<td>Eide et al. (1961)</td>
<td>4</td>
<td>St. Alban</td>
<td>Konrad and Roy (1987)</td>
</tr>
<tr>
<td>7</td>
<td>Canons Park</td>
<td>Powell et al. (2003)</td>
<td>8</td>
<td>Bothkennar</td>
<td>Clausen and Aas (2000)</td>
</tr>
<tr>
<td>13</td>
<td>Nitsund</td>
<td>Flaate (1972)</td>
<td>14</td>
<td>Skå-Edeby</td>
<td>Bergdahl and Hult (1981)</td>
</tr>
<tr>
<td>15</td>
<td>Haga</td>
<td>Karlsrud and Haugen (1986)</td>
<td>16</td>
<td>Florida</td>
<td>Bullock et al. (2005a,b)</td>
</tr>
<tr>
<td>17</td>
<td>Northwestern</td>
<td>Finno et al. (1989)</td>
<td>18</td>
<td>-</td>
<td>Svinink et al. (1994)</td>
</tr>
</tbody>
</table>

#### Soil conditions

- **ID**: Identification number.
- **Name**: Description of the site.
- **Reference**: Referencing the original source.
- **Pile name**: Name of the pile.
- **$Q_k$**: Pile load capacity.
- **$I_p$**: Average plasticity index. If not given, $I_p = 25\%$ and marked with asterisk.
- **OCR**: Overconsolidation ratio based on $S_u$-strengths.
- **$S_u$**: Average unconsolidated undrained shear strength.
- **Open/Closed**: Indicates if the pile is open or closed-ended.
- **Type**: Pile material indicator: S = steel, C = concrete, T = timber.
- **Diam.**: Diameter of the pile.
- **Wall****: Wall thickness at pile tip.
- **Taper**: Taper denotes pile wall taper.
- **Tippen**: Tip penetration.

### Soil conditions

- **Quality ranking of soil and pile data**: 0 = not known, 1: low, 2: average, 3: high, 4: very high.
- **Tip penetration**.

### Pile conditions

- **Diam.**: Diameter of the pile.
- **Wall**: Wall thickness at pile tip.
- **Taper**: Taper denotes pile wall taper.
- **Tippen**: Tip penetration.

---

- **Material provided by Kampsax Geodan, which is part of COWI A/S.**
- **Cases marked with ● include piles belonging to the Group “Super Piles”.**
- **Cases marked with ● include piles belonging to the Group “Super Piles”.**
- **Cases marked with ● include piles belonging to the Group “Super Piles”.**

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**Notes:**

- **d)** Data taken directly from Clausen and Aas (2000).
- **e)** Data taken directly from Clausen and Aas (2000).
- **f)** Data taken directly from Clausen and Aas (2000).
- **g)** Quality ranking of soil and pile data: 0 = not known, 1: low, 2: average, 3: high, 4: very high.
- **h)** **i)** $I_p$ is the average plasticity index. If not given $I_p = 25\%$ and marked with asterisk.
- **j)** OCR is the average overconsolidation ratio based on $S_u$-strengths.
- **k)** $S_u$ is the average unconsolidated undrained shear strength.
- **l)** Pile material indicator: S = steel, C = concrete, T = timber.
- **m)** Diameter and wall thickness at pile tip, respectively. Wall thickness is only given in cases where the piles are driven open-ended. Taper denotes pile wall taper.
- **n)** Tip penetration.
Table 2: Measured capacities.

<table>
<thead>
<tr>
<th>ID - Pile #</th>
<th>Pile Name</th>
<th>CMP / TNS</th>
<th>Qm [$\text{kN}$]</th>
<th>$\Delta t_a$</th>
<th>Time / Cap $^d$ [days / kN]</th>
<th>Time / Cap $^d$ [days / kN]</th>
<th>Time / Cap $^d$ [days / kN]</th>
<th>Time / Cap $^d$ [days / kN]</th>
<th>Time / Cap $^d$ [days / kN]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>- C</td>
<td>784</td>
<td>0.20</td>
<td>18 / 670</td>
<td>180 / 192</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>A C</td>
<td>1252</td>
<td>0.15</td>
<td>30 / 1140</td>
<td>396 / 1390</td>
<td>9125 / 1608</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>- C</td>
<td>259</td>
<td>0.19</td>
<td>31 / 220</td>
<td>71 / 270</td>
<td>799 / 300</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>A C</td>
<td>103</td>
<td>0.36</td>
<td>4 / 47</td>
<td>8 / 67</td>
<td>20 / 77</td>
<td>33 / 83</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>- C</td>
<td>1892</td>
<td>0.20</td>
<td>1.7 / 1225</td>
<td>10.5 / 1555</td>
<td>20.5 / 1670</td>
<td>32.5 / 1670</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>D C</td>
<td>174</td>
<td>0.34</td>
<td>1.9 / 189</td>
<td>496 / 200</td>
<td>1130 / 231</td>
<td>6200 / 291</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>B C</td>
<td>189</td>
<td>0.15</td>
<td>74 / 194</td>
<td>217 / 197</td>
<td>683 / 200</td>
<td>1312 / 221</td>
<td>6200 / 249</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>A C</td>
<td>160</td>
<td>0.20</td>
<td>31 / 159</td>
<td>134 / 161</td>
<td>248 / 163</td>
<td>525 / 170</td>
<td>1154 / 188</td>
<td>6200 / 231</td>
</tr>
<tr>
<td>9</td>
<td>T C</td>
<td>36</td>
<td>0.18</td>
<td>4 / 27.34</td>
<td>32 / 32.91</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>C C</td>
<td>741</td>
<td>0.13</td>
<td>14 / 660</td>
<td>9778 / 930</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>P1-16</td>
<td>2699</td>
<td>0.26</td>
<td>16 / 2150</td>
<td>140 / 2800</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11.2</td>
<td>P2-16</td>
<td>2082</td>
<td>0.41</td>
<td>14 / 1350</td>
<td>141 / 2210</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>P1</td>
<td>1572</td>
<td>0.44</td>
<td>21 / 1100</td>
<td>153 / 1700</td>
<td>-</td>
<td>-</td>
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</tr>
<tr>
<td>13</td>
<td>I C</td>
<td>287</td>
<td>0.26</td>
<td>32 / 243</td>
<td>207 / 321</td>
<td>357 / 336</td>
<td>641 / 350</td>
<td>1043 / 350</td>
<td>-</td>
</tr>
<tr>
<td>13.2</td>
<td>II C</td>
<td>281</td>
<td>0.44</td>
<td>34 / 228</td>
<td>209 / 314</td>
<td>357 / 343</td>
<td>637 / 378</td>
<td>1023 / 414</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>B C</td>
<td>69</td>
<td>0.61</td>
<td>39 / 52</td>
<td>75 / 64</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14.2</td>
<td>B C</td>
<td>43</td>
<td>0.33</td>
<td>42 / 36</td>
<td>456 / 56</td>
<td>1116 / 54</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14.3</td>
<td>C C</td>
<td>75</td>
<td>0.54</td>
<td>30 / 54</td>
<td>75 / 70</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14.4</td>
<td>C C</td>
<td>48</td>
<td>0.32</td>
<td>42 / 41</td>
<td>96 / 48</td>
<td>456 / 60</td>
<td>1116 / 62</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14.5</td>
<td>D C</td>
<td>69</td>
<td>0.47</td>
<td>30 / 52</td>
<td>75 / 65</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>14.6</td>
<td>D C</td>
<td>44</td>
<td>0.18</td>
<td>96 / 42</td>
<td>171 / 47</td>
<td>456 / 49</td>
<td>1116 / 51</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>15</td>
<td>T C</td>
<td>80</td>
<td>0.24</td>
<td>7 / 59</td>
<td>20 / 65</td>
<td>36 / 73</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>16</td>
<td>AUC</td>
<td>1616</td>
<td>0.17</td>
<td>3 / 1197</td>
<td>16.1 / 1427</td>
<td>65.1 / 1528</td>
<td>265 / 1712</td>
<td>1727 / 1982</td>
<td>-</td>
</tr>
<tr>
<td>16.2</td>
<td>V1W</td>
<td>751</td>
<td>0.20</td>
<td>3 / 519</td>
<td>19 / 635</td>
<td>157 / 783</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>17</td>
<td>Pipe</td>
<td>871</td>
<td>0.35</td>
<td>14 / 623</td>
<td>35 / 712</td>
<td>301 / 1024</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>18</td>
<td>TP2</td>
<td>4034</td>
<td>0.31</td>
<td>0.2 / 1913</td>
<td>9 / 2789</td>
<td>22 / 3189</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

a) ID refers to the case (see Table 1) and Pile # refers to the pile number, if there is more than one pile associated with the given case. Piles belonging to the Group "Super Piles" are marked with ●.
b) CMP and TNS denote piles loaded in compression and tension, respectively.
c) Estimated from the measured capacities by linear regression for every single case; the reference time $t_0 = 100$ days.
d) Measured capacity based on static test at the given time after installation.

will automatically create two points on the bisectional line with the inclination $\beta_1 = 1$. Thus, only cases involving at least three tests qualify for the verification of Eq. (1). In Table 3 the results of the regression analysis are presented. Since the regression coefficients are $\beta_1 = 1.0$ and $\beta_9 = 0$ there is a linear relation between normalised capacity and normalised time as expressed by Eq. (7). $R^2$ is loosely interpreted as the proportion of the total variation in the data that can be accounted for or explained by the regression line (Walpole and Myers, 1993). The range of $R^2$ is 0 to 1, with larger values indicating a better correlation. Thus, $R^2 = 0.96$, cf. Table 3, indicates an acceptable correlation.

4.1.2 Hypothesis testing

A reliability check of the estimated regression line can be performed by testing the two-sided hypothesis.
Figure 3 Normalised capacity versus normalised time for piles tested more than two times. The dots are test results (one for each pile test), the solid line is the regression line, and the dotted lines mark the 95% confidence interval on the regression line. Dots deviating from the regression line imply that a semi-logarithmic relation does not describe the development of pile capacity with time.

\[
H_0 : \beta_1 = 1.0 \\
H_1 : \beta_1 \neq 1.0
\]

for e.g. a 1% level of significance (\(\alpha = 0.01\)). Analyses show that the null hypothesis cannot be rejected for a stated level of significance of 1%. Another common output of hypothesis testing is the P-value, which ranges from 0 to 1. It is defined as the smallest level of significance \(\alpha\) that would lead to rejection of the null hypothesis \(H_0\) (Walpole and Myers, 1993). Therefore, if the P-value is less than \(\alpha\), \(H_0\) is rejected. In other words, the P-value is the probability of observing the given sample result under the assumption that the null hypothesis is true. A very small P-value casts doubt on the truth of the null hypothesis and the higher P-value the stronger evidence for accepting \(H_0\). Therefore, the P-value contains more information than “reject” or “do not reject” (Walpole and Myers, 1993). When testing the hypothesis expressed by Eq. (9) on the available data, the P-value is approximately equal to 1. This strongly indicates that \(\beta_1 = 1.0\), i.e. the validity of Eq. (1) has been verified.

### 4.1.3 Model adequacy checking

To identify possible statistical outliers with respect to the estimated regression line with the inclination \(\beta_1 = 1\), the standardized residuals and the R-student are computed for each pile test (Montgomery, 2001). The standardized residuals are the raw residuals normalised by an estimate of their standard deviation. By contrast, the R-student is normalised by a so-called independent estimate of the standard deviation. Hence, the R-student is more sensitive to outliers.

If the raw residuals are normally distributed with zero mean and variance \(\sigma^2\), i.e. \(N(0,\sigma^2)\), the standardized residuals should be normally distributed with zero mean and unit variance (Montgomery, 2001). Hence, R-students and standardized residuals greater than 3 or less than -3 are potential outliers. From Figure 5 it is concluded that there are no such data in the present database, though a few of the data are close to the limit.

| Table 3 Linearity between capacity and the logarithm to time. |
|---|---|---|---|
| \(\beta_{10} / [\text{CI}]^b\) | \(\beta_{00} / [\text{CI}]^b\) | \(R^2\) |
| 1.0 / [0.9538;1.0462] | 0 / [-0.0409;0.0409] | 0.96 |

a) Inclination of the regression line.  

b) 95% confidence interval.  

c) Intersection with the axis of the ordinate.  

d) Sample coefficient of determination.

### 4.1.4 Violation of assumptions

The analyses presented in the former sections assumes that the raw residuals 1) have zero mean, 2) have a constant variance across all values of normalised time, 3) are normally distributed, and 4) are independent (Ayyub and McCuen, 1997).

In Figure 5 the ordinary least square residuals are plotted as function of normalised time. In the actual case the mean is zero. Further, the ordinary least-square residuals show approximately constant variance when plotted...
Figure 5 Residuals versus normalised time. Δ = ordinary raw least square residuals, • = standardized residuals, and x = externally studentized residuals (R-student). The solid line corresponds to residuals equal to zero.

Figure 4 Check of normality of the raw residuals. A normal probability density function is superimposed on the discrete probability density function for the present data (left). The dotted line on the normality plot (right) is the line joining the first and third quartiles and hereafter extrapolated out to the ends of the sample to help evaluate the linearity of the data.

against the normalised time. A similar conclusion can be drawn if they are plotted against the normalised capacity. Other analyses, not presented here, indicate that for each pile there is no systematic “over”- and “under-shooting”, i.e. there is no tendency that the residuals systematically increase or decrease with normalised time.

Next, the normality assumption has been checked in Figure 4 by plotting a histogram of the ordinary raw least square residuals and the corresponding superimposed normal probability density function and by showing the normal probability plot. As seen, the tails of the residuals in the normal probability plot do not fit into a linear relation (dotted line) dictated by the first and third quartiles. The reason can be found by inspecting the histogram; the two columns located nearest to the mean contain too many pile tests, which implies that the values of the residuals corresponding to the first and third quartiles are close to the mean. Hence, according to Figure 4 the normality condition is not exactly fulfilled. However, a Lilliefors test (Conover, 1980) shows that the hypothesis, that the residuals have a normal distribution, cannot be rejected at a level of significance of 4%, which is acceptable. Therefore, it is concluded that the residuals are normally distributed.

The analyses forming the basis of testing the semi-logarithmic relation between capacity and time also assume that the residuals are independent. Since piles are tested more than ones, and since group action may influence pile capacities, the independence criterion is
not exactly fulfilled. In spite of this, there is no reason to suspect the model assumptions.

4.1.5 Tests to be included

When estimating \( Q_0 \), for a given pile, it has to be determined whether some test results should be omitted. According to Figure 1 and the discussion in Section 2, different rates of set-up are recorded before and after the end of primary consolidation. Since the present study concerns the long-term set-up, it is obvious to include tests performed after \( t_{oc} \). However, this instant is usually not known, and for each pile it has to be evaluated if tests performed earlier than, for example, 10 days should be excluded. Further, the lower limit on \( t \) should be as small as possible to include as many tests as possible in the calibration of the model, thereby minimising the statistical uncertainties. Analyses, not presented here, indicate that one day is the optimal choice in this study, i.e. test performed more than one day after installation have been employed. The outlier diagnostics also show that there are no significant outliers, which justify including tests, performed one day after installation.

5 Existing time functions

The time functions, proposed by Skov and Denver (1988), Bullock et al. (2005a,b) and Clausen and Aas (2000), are compared by examining the residuals obtained when applying the respective time functions to the available data, cf. Table 1 and Table 2. As mentioned previously, the measured capacities for the different piles and test sites are very different. Hence, in order to explicitly compare the residuals obtained for every single case and pile test, the residuals must be dimensionless, i.e. by normalizing the measured and the predicted capacities with respect to the reference capacity, the residual, \( r \), defined as

\[
(10) \quad r = \frac{Q_{\text{meas}}}{Q_0} - \frac{Q_{\text{pred}}}{Q_0}
\]

becomes dimensionless and is a measure of how well a time function predicts an observed capacity. \( Q \) is the measured capacity at time \( t \), \( Q_0 \) is the reference capacity at the reference time \( t_0 = 100 \) days, and \( Q_{\text{pred}} \) is the predicted capacity corresponding to the time function in consideration.

Now, substitution of \( Q_{\text{pred}} \) with Eq. (1) yields

\[
(11) \quad r = \frac{Q_{\text{meas}}}{Q_0} - 1 - \Delta_{10,\log} \log_{10} \left( \frac{t}{t_0} \right)
\]

By defining \( Q_{\text{meas}} \) and \( Q_{\text{est}} \) as

\[
(12) \quad Q_{\text{meas}} = \frac{Q}{Q_0}, \quad Q_{\text{est}} = \Delta_{10,\log} \log_{10} \left( \frac{t}{t_0} \right)
\]

The residual, \( r \), can also be defined as

\[
(13) \quad r = Q_{\text{meas}} - Q_{\text{est}}
\]

where \( Q_{\text{meas}} \) and \( Q_{\text{est}} \) are expressions of the measured and predicted capacities, respectively. In Eq. (12) \( \Delta_{10,\log} \) is the set-up factor corresponding to the time function in consideration. Further, \( Q, t, \) and \( Q_0 \) for every single pile in the database are shown in Table 2.

Since the \( \Delta_{10,\log} \) proposed by Skov and Denver (1988) and Bullock et al. (2005a,b) are based on \( t_0 = 1 \) day rather than \( t_0 = 100 \) days, they must be converted in order to obtain a consistent comparison of the suggested time functions. This is done by applying Eq. (5). Hence, \( \Delta_{10,\log} \) when \( t_0 = 100 \) days whereas \( \Delta_{10,\log} \) is 0.6 for \( t_0 = 1 \) day, cf. Section 2. For piles subjected to un-staged loading, Bullock et al. (2005a,b) recommend \( \Delta_{10,\log} \) for \( t_0 = 1 \) day. This corresponds to \( \Delta_{10,\log} = 0.25 \) for \( t_0 = 1 \) day, cf. Section 2, which implies that \( \Delta_{10,\log} = 0.17 \) when the reference time is \( t_0 = 100 \) days and staged loading is considered.

5.1.1 Comparison of existing time functions

In Figure 6, the residuals obtained by applying the time functions to the available data are plotted as functions of normalised time. Since \( \Delta_{10,\log} \) is based on the plasticity index, \( I_p \), and overconsolidation ratio, \( OCR \), cf. Eq. (2), and since the soil conditions are not provided in the cases with IDs 16-18, cf. Table 1, these have been omitted. Further, the case with ID 9 has also been omitted, because \( \Delta_{10,\log} \) in that case is less than zero, cf. Table 2. The models suggested by Clausen and Aas (2000) and by Bullock et al. (2005a,b) provide a skew distribution of the residuals, i.e. they are negative for \( t < t_0 \) and positive for \( t > t_0 \). On the other hand, the residuals in the model proposed by Skov and Denver (1988) are apparently independent of time, which characterises an adequate time function.

Next, the box plots in Figure 7 indicate that the three models produce almost symmetric distributions of the residuals; but the variation differs. This is also indicated in Table 4 and Figure 6. However, the standard deviations of the residuals are not significant different at a 1% level of significance, cf. Table 5.

Another measure of the time functions ability to predict the observed behaviour is the sum of squared residuals, \( SSR \), defined as
Figure 6 Residuals plotted as function of normalised time and time function. The solid line corresponds to residuals equal to zero. The dotted lines mark the mean and the residuals corresponding to three times the standard deviation with respect to the mean.

\[
SSR = \sum_{i=1}^{n} \sum_{j=1}^{k} \left( \frac{Q_j}{Q_{0i}} - 1 - \Delta_{10} \cdot \log_{10} \left( \frac{t_j}{t_0} \right) \right)^2
\]

where \(Q_j\) is the measured capacity at time \(t_j\), \(Q_{0i}\) is the reference capacity for pile \(i\), \(t_0\) is the reference time, \(k\) is number of static loading test on pile \(i\), and \(n\) is the number of piles included in the analysis. An SSR-value equal to zero implies a perfect match between measured and predicted capacities, i.e. a small SSR-value indicates a good prediction of the pile capacity. The SSRs obtained by applying the time functions proposed by Skov and Denver (1988), Bullock et al. (2005a,b) and Clausen and Aas (2000) to the available data are shown in Table 4. The time function proposed by Skov and Denver (1988) provides the smaller SSR-value and therefore the better estimate of the pile capacity. Compared to this model, the SSRs obtained by the NGI model (Clausen and Aas, 2000) and the model proposed by Bullock et al. (2005a,b) are approximately 77% and 35% greater, respectively. This is primarily due to the relatively large residuals occurring at the tails of the normalised time range, see Figure 6. It should be mentioned that the largest residuals in the upper end of the time interval are associated with the piles with ID 6.2, 7, 14.6, and 13.2, see Table 2. These piles belong to cases of high quality, i.e. \(Q_{r,soil} = Q_{r,pile} = 4\) (Table 1). This indicates that NGI (Clausen and Aas, 2000) and Bullock et al. (2005a,b) generally underestimate the long-term capacities, i.e. \(\Delta_{10}\) is too small for these models. It is further noted that the diameters and penetration depths associated with the mentioned piles are relatively small, see Table 2.

Generally, the mean of the residuals, \(\mu_r\), should be zero. Therefore, the hypothesis

\[
H_0 : \mu_r = 0 \\
H_1 : \mu_r \neq 0
\]

with unknown variance has been tested for a 1% level of significance (\(\alpha = 0.01\)). The calculated \(\mu_r\) does not differ significantly from zero for any of the time functions. However, there are great differences in the \(P\)-values (cf. Section 4) listed in Table 4. The large \(P\)-value obtained
by the Bullock time function is due to the small value of the mean, i.e. \( \mu_r = -0.003 \), see Table 4.

### 5.1.2 Model assumptions

The hypothesis tests require independent and normally distributed residuals. As mentioned in Section 4 the independence criterion is not exactly fulfilled. However, a Lilliefors test, not shown here, indicates that the hypothesis, that the residuals based on any of the three models have a normal distribution, cannot be rejected at a level of significance ranging from 1 to 20%, which is acceptable.

### 6 Calibration of time functions

The starting point, when investigating the possibilities of introducing an alternative time function to the models proposed by Skov and Denver (1988), Bullock et al. (2005a,b) and Clausen and Aas (2000), is to assume that \( \Delta_{10} = \text{constant} \). Subsequently it is investigated whether it is advantageous to make \( \Delta_{10} \) a function of relevant soil parameters.

#### 6.1 Constant set-up factor

In order to check the robustness of the calibrated time function, \( \Delta_{10} = \text{constant} \) is calibrated based on all tests, for which \( \Delta_{10,\text{meas}} > 0 \), cf. Table 2, and a special subset of piles. The quality in terms of soil and pile conditions, including number and time range for the tests, are especially high for these piles. The subset is denoted “Super Piles”, abbreviated \( SP \), and they are marked with a ● in Table 1 and Table 2. It should be mentioned that \( SP \) consists of 9 cases including 13 piles and 48 pile tests, i.e. \( SP \) constitutes approximately 50% of the available data. For the case with ID 14, Skå-Edeby, piles tested more than two times are included in \( SP \). The reason for not employing the other tests is that they reflect tests on the same piles initially located in other depths. Further, by not employing all tests, case 14 is not weighted as high in the calibration process. Thereby, the proposed time function reflects to a greater extent the trends observed in connection with relatively many cases instead of just a single case.

Figure 9 shows the sum of squared residuals, cf. Eq. (14), for different values of \( \Delta_{10} = \text{constant} \). Evidently the time function based on \( \Delta_{10} = 0.24 \) provides the better estimate of the measured capacities regardless of whether all tests or only the \( SP \) are employed in the calibration of \( \Delta_{10} \). This further implies that the time function based on the constant set-up factor \( \Delta_{10} = 0.24 \) is robust.

<table>
<thead>
<tr>
<th>Table 4 Comparison of the time functions – residual statistics.</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGI</td>
</tr>
<tr>
<td>( \mu_r )</td>
</tr>
<tr>
<td>( \sigma_r )</td>
</tr>
<tr>
<td>SSR</td>
</tr>
<tr>
<td>( P )-value</td>
</tr>
</tbody>
</table>

a) Mean of the residuals.

b) Standard deviation of the residuals.

c) Squared Sum of the Residuals as defined in Eq. (14).

d) \( P \)-value associated with the test: \( H_0: \mu_r = 0 \), \( H_1: \mu_r \neq 0 \)

e) Time function based on \( \Delta_{10} = 0.24 \), cf. Section 6.1.

f) Time function based on undrained shear strength, cf. Section 6.2.

### Figure 7

Box plot of the residuals. The box has lines at the lower quartile, median, and upper quartile values. Lines (whiskers) extend from the ends of the box to the minimum and maximum residuals.

#### Table 5 Testing the equality of residual variances as function of applied time functions, i.e. \( \sigma^2_r,1 = \sigma^2_r,2 \). \( \text{H}_1: \sigma^2_r,1 \neq \sigma^2_r,2 \), where 1 and 2 refer to one of the time functions: NGI, Skov, Bullock, Best fit, or AAU. 0 indicates that the null hypothesis cannot be rejected at a 1% significance level whereas 1 symbolizes that the null hypothesis can be rejected.

<table>
<thead>
<tr>
<th></th>
<th>NGI</th>
<th>Skov</th>
<th>Bullock</th>
<th>Best fit</th>
<th>AAU</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGI</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Skov</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bullock</td>
<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Best fit</td>
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<td>0</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>AAU</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
</tbody>
</table>

a) Time function based on \( \Delta_{10} = 0.24 \), cf. Section 6.1.

b) Time function based on undrained shear strength, cf. Section 6.2.
6.1.1 Confidence interval for $\Delta_{10}$

It is useful to obtain an estimate of the 95% confidence interval for $\Delta_{10}$. Assume that $\Delta_{10} = 0.24$ is the estimate of the mean of $\Delta_{10}$ and the variance is unknown. A set of sample statistics, i.e. sample size, standard deviation etc., is needed to establish the confidence interval. Since such a set does not exist for $\Delta_{10}$, the following assumptions are made:

1. The distribution of $\Delta_{10}$ is similar to the distribution of the measured $\Delta_{10}$s.
2. The estimate of the standard deviation of $\Delta_{10}$ equals the standard deviation of the measured $\Delta_{10}$s.
3. The sample size equals the number of measured $\Delta_{10}$s. This is also equal to the number of piles associated with the database.

The measured $\Delta_{10}$s, also denoted $\Delta_{10,\text{meas}}$, are the real set-up factors associated with every single pile. They are determined by means of regression analysis, cf. Section 4, and shown in Table 2. $\Delta_{10,\text{meas}}$ for case with ID 9 is negative. Therefore, it is omitted in the following and the sample size equals 26.

Figure 8 shows a histogram of the logarithm to the measured $\Delta_{10}$s and the corresponding superimposed normal density function. It turns out that a lognormal distribution fits the measured $\Delta_{10}$s better than a normal distribution; a Lilliefors test (Conover, 1980) shows that the hypothesis, that the $\Delta_{10,\text{meas}}$ have a lognormal distribution, cannot be rejected at a level of significance of 9%. The same conclusion can be drawn at a level of significance of 4% when testing the $\Delta_{10,\text{meas}}$ for normality. Therefore, it is assumed that $\Delta_{10}$ is lognormally distributed with a mean equal to $\log_{10}(0.24) = -0.62$. The estimated standard deviation of the logarithm to $\Delta_{10,\text{meas}}$ and thereby the logarithm to $\Delta_{10}$ equals 0.19 and the sample size is 26. By means of basic statistics, e.g. Montgomery (2001) or Walpole and Myers (1993), a 95% confidence interval for $\log_{10}(\Delta_{10})$ can be found to $[-0.6975; -0.5439]$. This implies that the 95% confidence interval for $\Delta_{10}$ is $[0.20; 0.29]$ when the reference time is $t_0 = 100$ days and staged loading is considered.

If Eq. (5) and $C_{st} = 0.4$ ($t_0 = 1$ day) also hold true for confidence intervals, the upper and lower bounds of a 95% confidence interval for $\Delta_{10}$ can be determined for all other combinations of $t_0$ and loading conditions, i.e. staged or unstaged. Examples are shown in Table 6. The lower confidence limit when considering unstaged loading and $t_0 = 1$ day is approximately 30% higher than the design set-up factor, $\Delta_{10,\text{Bullock}} = 0.1$, recommended by Bullock et al. (2005a,b). The mean is 80% higher. When considering staged loading and $t_0 = 1$ day, the mean and the upper confidence limit are approximately 23% lower.

<table>
<thead>
<tr>
<th>Loading</th>
<th>$t_0$ [days]</th>
<th>$\Delta_{10}$ [-]</th>
<th>LL [-]</th>
<th>UL [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Staged</td>
<td>100</td>
<td>0.24</td>
<td>0.20</td>
<td>0.29</td>
</tr>
<tr>
<td>Staged</td>
<td>1</td>
<td>0.46</td>
<td>0.33</td>
<td>0.69</td>
</tr>
<tr>
<td>Unstaged</td>
<td>1</td>
<td>0.18</td>
<td>0.13</td>
<td>0.28</td>
</tr>
<tr>
<td>Unstaged</td>
<td>100</td>
<td>0.13</td>
<td>0.10</td>
<td>0.18</td>
</tr>
</tbody>
</table>

a) Reference time.
b) Mean value of the set-up factor.
c) Lower limit of a 95% confidence interval for $\Delta_{10}$.
d) Upper limit of a 95% confidence interval for $\Delta_{10}$.
Figure 10 Relation between $\Delta_{10,\text{meas}}$ and average soil parameters such as the plasticity index, $I_p$, unconsolidated undrained shear strength, $S_{uu}$, and overconsolidation ratio, OCR, depicted for every single pile in the database. Piles marked with a dot belong to SP. The first number in each plot label refers to the case and the second, if there is more than one pile associated with the given case, to pile number (see Table 1 and Table 2).
and 30% higher, respectively, than the set-up factor, $\Delta_{10,\text{Skov}} = 0.6$, proposed by Skov and Denver (1988).

The confidence limits presented in Table 6 are only intended as a guideline. They are encumbered with great uncertainty because the true distribution of $\Delta_{10}$ is unknown and so is a set of sample statistics. Further, the limits of the confidence intervals when inspecting un-staged loading or staged loading for the set-up factor. As such, the benefit is small compared to the model with a constant value of $\Delta_{10}$.

### 6.2 Set-up factor as function of undrained shear strength

The soil parameters influencing $\Delta_{10}$ are assumed to be the plasticity index, $I_p$, the overconsolidation ratio, $OCR$, and the unconsolidated undrained shear strength, $S_{uu}$. In Figure 10, the measured $\Delta_{10}$, denoted $\Delta_{10,\text{meas}}$, are depicted as function of these soil parameters. The values are listed in Table 1 and Table 2.

A natural starting point for the formulation of an enhanced time function appears to be a recalibration of the coefficients in the NGI model, cf. Eq. (2). However, Augustesen et al. (2005b) postulate that the form of $\Delta_{10}$ that best fits the observed behaviour is

$$\Delta_{10} = 1.24 \left(\frac{S_{uu}}{60}\right)^{0.03}, \quad \text{AAU}$$

which is superimposed on Figure 10. Thus, $\Delta_{10}$ depends entirely on the average unconsolidated undrained shear strength, $S_{uu}$. The time function based on Eq. (16) is denoted AAU. As illustrated in Figure 10, $\Delta_{10}$ varies between 0.22 and 0.29 for the $S_{uu}$ range examined. Considering the scatter of the data and the power of 0.03 in Eq. (16), there is no distinctive correlation between $S_{uu}$ and the set-up factor. As such, the benefit is small compared to the model with a constant value of $\Delta_{10}$.

### 6.3 Comparison of time functions

The AAU model and the time function based on the constant value $\Delta_{10} = 0.24$ have been applied to the same data as the existing models, cf. Section 5. The residuals obtained and relevant sample statistics are shown in Figure 6, Figure 7, Figure 11, and Table 4. Generally, the AAU model provides the better fit of the data in terms of SSR. In fact, the SSR-value obtained by AAU is 11% and 54% less than the SSR-values provided by the time function based on $\Delta_{10} = 0.24$ and the NGI model, respectively. For the AAU model and the time function based on $\Delta_{10} = 0.24$, the distributions of the residuals around $r = 0$ are symmetric and similar to the trends observed for Skov and Denver’s model, cf. Figure 6, Figure 7, and Figure 11. The minimum standard deviation is obtained by the AAU model, see Table 4. However, the standard deviations and thereby the variances provided by the other models (except the NGI-model) are not significantly different at a 1% level of significance as indicated in Table 5.

Generally, the mean of the residuals, $\mu_r$, should be zero. Compared to the other models, the time function proposed by Bullock et al. (2005a,b) results, on the average, in residuals closest to zero. However, based on hypothesis test like the one described by Eq. (15), it can be concluded that the estimated means of the residuals do not differ significantly from zero at a 1% level of significance regardless of the time function employed. This is also indicated by the $P$-values shown in Table 5.

### 6.4 Choice of time function

The time functions capability to predict the capacities associated with the piles in the database have been measured based on:

1. the magnitude of the sum of squared residuals, SSR,
2. the magnitude of the standard deviation of the residuals, $\sigma_r$,
3. the magnitude of the mean of the residuals, $\mu_r$,
4. the visual distribution of the residuals, $r$, when plotted against normalised time,
5. the outcome of the test $H_0: \mu_r = 0$, $H_1: \mu_r \neq 0$,
6. the outcome of the hypothesis test $H_0: \sigma_r^2 = \sigma_{r,1}^2$, $H_1: \sigma_r^2 \neq \sigma_{r,2}^2$, where 1 and 2 refer to one of the time functions NGI, Skov, Bullock, Best fit (constant $\Delta_{10} = 0.24$), or AAU.

Based on these investigations, it can be concluded that AAU provides the better estimate of the available data,
Figure 12 Visualisation of how the time function based on $\Delta_{10} = 0.24$ (solid line) and NGI (dotted line) predict the observed behaviour for some selected cases in the database. Information regarding the cases is given in Table 1 and Table 2. EOD denotes End Of Driving.
which is natural since the model is calibrated based on these data. The NGI-model provides the least suitable fit, whereas the time function based on $\Delta_{10} = 0.24$ is slightly better than the time function proposed by Skov and Denver (1988).

The time function based on $\Delta_{10} = 0.24$ almost fits the observed behaviour as well as AAU, which is also indicated by Figure 10. It is primarily in the tails of the $S_{uu}$ range that the two models differ. Considering:

1. the non-distinctive correlation between $S_{uu}$ and $\Delta_{10}$ for AAU,
2. that the variance of the residuals based on the two time functions do not differ significantly,
3. that the distribution of residuals around the mean is similar,
4. that SSR obtained for AAU is only 11% less than the SSR-value provided by the time function based on $\Delta_{10} = 0.24$,
5. that the time function based on $\Delta_{10} = 0.24$ provides a slightly better fit to the available data compared to the models proposed by Skov and Denver (1988), Bullock et al. (2005a,b), and by Clausen and Aas (2000),

it is recommend to make use of the time function based on $\Delta_{10} = 0.24$ when estimating the development of capacity with time based on staged loading and the reference time $t_0 = 100$ days. According to Table 6, $\Delta_{10}$ equals 0.13 for unstaged loading and $t_0 = 100$ days, whereas $\Delta_{10}$ equals 0.46 and 0.18 for staged and unstaged loading, respectively, when $t_0 = 1$ day.

Finally, in Figure 12 it is visualised how the time function based on $\Delta_{10} = 0.24$ predicts the actual behaviour compared to the NGI model.

7 Conclusions

The vertical bearing capacity of piles in clay has been assessed with the focus on its long-term development. The primary aim has been to quantify the rate of set-up. Further, it has been analysed whether the magnitude of set-up is related to the properties of the soil surrounding the pile. The analyses are based on 88 static pile tests, and the data represent a great variety of soil and pile properties. Therefore, the findings in this paper are of general applicability to piles in clay.

In the literature it has been suggested that the pile capacity increases with the logarithm of time after initial driving. Based on the available data, there is statistical evidence that this semi-logarithmic relationship is valid.

Concerning the set-up models proposed in the literature, the time function proposed by Skov and Denver (1988) provides the better fit of the available data. The model proposed by Bullock et al. (2005a,b) systematically under-predicts the capacity a long time after installation, which is also the case for the model proposed by Clausen and Aas (2000). Further, the maximum difference between the measured and predicted capacities is significantly smaller in the model suggested by Skov and Denver (1988) than in the two other models.

Skov and Denver (1988) as well as Bullock et al. (2005a,b) employ a constant value of the set-up factor, whereas Clausen and Aas (2000) propose that $\Delta_{10}$ depends on the properties of the soil. However, the present study indicates that neither of the undrained shear strength, the plasticity index or the overconsolidation ratio has a significant influence on the set-up.

Hence, in conclusion the set-up factor for piles situated in clay is constant and independent on the soil properties. The following relation may be applied to predict the bearing capacity at time $t$ after initial driving:

$$Q = Q_0 \left[1 + \Delta_{10} \log_{10}\left(\frac{t}{t_0}\right)\right]$$

where $t_0$ and $Q_0$ are the reference time and capacity of the pile, respectively, while $\Delta_{10}$ is the set-up factor. For $t_0 = 1$ day and unstaged loading it has been found that $\Delta_{10} = 0.18$, whereas $\Delta_{10} = 0.13$ for staged loading. The listed values of the set-up factor are characteristic values determined as the lower limits of a 95% significance interval. It is worthwhile to note that the suggested value of $\Delta_{10} = 0.13$ is higher than the value proposed by Bullock et al. (2005a,b).

Finally, if another reference time than $t_0 = 1$ day is preferred, the set-up factor and reference capacity should be adjusted accordingly as described in Section 2.

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9 References


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