DILUTION OF BUOYANT SURFACE PLUMES

by

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List of symbols

$u_s$ m/s surface flow velocity cm/s
$u_f$ m/s bottom friction velocity $= \sqrt{\frac{T}{\rho}}$
$\Delta T_o$ °C temperature difference between outlet and flume
$T_a$ °C ambient temperature
$T_u$ °C outlet temperature
$z$ m vertical coordinate
$x$ m stream wise coordinate
$y$ m transverse coordinate
$k$ mm equivalent roughness
d m water depth
$v$ m$^2$/s viscosity of water at 20°C $1 \cdot 10^{-6}$ m$^2$/s
$V$ m/s bulk flow velocity
$D_y$ m$^2$/s transverse bulk diffusion coefficient of plume
$D_z$ m$^3$/s vertical bulk diffusion coefficient of plume
$\nu_e$ m$^2$/s eddy viscosity
$\sigma_z^2$ m$^2$/s vertical spatial variance
$\sigma_y^2$ m$^2$/s transverse spatial variance
$h$ m height
$b$ m width
t sec time
$\beta$ dimensionless transverse diffusion coefficient
$R$ m hydraulic radius
c kg/m$^3$ concentration
$Q_n$ m$^3$/s outlet flow
$A_n$ m$^2$ outlet area
\( c_p \) specific heat capacity
\( K \) sec time constant of thermo-couple
\( \rho \) kg/m\(^3\) density
\( \rho_o \) kg/m\(^3\) ambient density
\( \Delta \rho \) kg/m\(^3\) \( \rho - \rho_o \)
\( M_n^y \) nth moment of density distribution around centerline
\( M_n^z \) nth moment of density distribution around water surface
\( M_n^T \) nth moment of temperature distribution
\( h_0 \) m height of outlet
\( b_0 \) m width of outlet
\( \Delta T \) °C mean temperature difference
\( R \) Reynolds number
\( g \) m/s\(^2\) acceleration of gravity

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1. Introduction

Sewage from treatment plants, located at marine recipients, is often discharged through a sea outfall.

A common design criterion for outfalls is that the concentration of E.coli does not exceed 100 coli/ml in more than 5% of the time, in some specific distance from the outfall.

In designing sea outfalls it is therefore essential to be able to predict the dilution and the spatial development of the sewage plume.

Due to differences in salinity and temperature there often are differences in density between the recipient and the sewage.

Previous work by /Weil, 1973, Schröder, 1980/ have shown that even small density differences strongly affect the behaviour of the plume as it tends to be wide and thin.

The purpose of the present work is to establish a quantitative description of a surface plume which is valid for the range of density differences occurring in relation to sewage outfalls.

This report is divided into two parts.

The first part deals with an experimental investigation of the surface plume.

The second part is an integral description of surface plumes, based on the experiments.

2. Experiments

2.1 Experimental programme

In the experiments the buoyant surface plume is obtained by discharging heated water through an outlet arrangement to the surface in a hydraulic flume.

To avoid jet entrainment, the flow velocity in the flume and outlet velocity are set to equal.

The fundamental measured parameter is the spatial distribution of density differences in subsequent cross sections of the plume.

As independent variables is chosen the outlet temperature difference, $\Delta T_0$, the flow velocity in the flume, $u$, and bottom friction in the flume, $u_f$.

The transverse and vertical diffusion of the plume, in case of no density difference, is measured separately.

As an illustration of the experimental range are on Figs. 2.1a, 2.1b, and 2.1c shown three pictures recorded by an infra-red sensitive camera. The grey scales correspond to surface temperatures.

The experimental conditions on Fig. 2.1a are high temperature difference and low turbulence level, while Fig. 2.1c corresponds to low temperature and high turbulence level.
In Fig. 2.2 is shown the experimental conditions used in each experiment.

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Fig. 2.2. Experimental conditions.

Us = flow velocity; Uf = friction velocity;

dT0 = initial excess temperature; dRoO = initial density deficit;

Ta = ambient temperature.
2.2.1 Measurement of equivalent roughness

To determine the equivalent roughness, \( k \), of the flume bed is measured a vertical velocity profile in the center of the flume.

Velocity is measured by means of an micropropeller, diameter 0.5 cm, and an electronic device which converts the speed of rotation into an electric current, proportional to the flow speed.

For hydraulic rough flow, the following logarithmic profile can be used /Engelund, 1978/

\[
\frac{u}{u_f} = 2.45 \cdot \ln\left(\frac{z}{0.03k}\right)
\]

(2.1)

where \( u \) is velocity in distance \( z \) above some, yet undetermined level \( z_0 \), \( u_f \) is friction velocity and \( k \) is equivalent sand roughness.

From the measured profile, \( z_0 \) is determined, using linear regression on 2.1, as the level which gives the best linear dependency.

In Fig. 2.3 is shown the calculated values of \( k \), for each bed type.

<table>
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<th>Bottom type</th>
<th>( z_0 ) cm</th>
<th>( V ) cm/s</th>
<th>( u_f ) cm/s</th>
<th>( k ) cm</th>
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Fig. 2.3 Equivalent roughness. \( z_0 \) is measured above flume bed. \( V \) = bulk velocity.

As a control of the measurement and the calculations is plotted \( u/u_f \) versus \( z/k \) in Fig. 2.4 and as can be seen the profiles match the theoretical values.

![Graph](image)

Fig. 2.4 Velocity profiles. \( \star k = 4.0 \) cm \( \star k = 0.07 \) cm

2.2.2 Measurement of undisturbed diffusion coefficients

The diffusion coefficients for the plume in case of no density difference are measured in both vertical and transverse direction, by means of a fluorescent tracer and an in situ - fluorometer. (Navitronic Fluorometer Q-200).

Transverse diffusion coefficient. \( D_y \).

When measuring \( D_y \), the in situ fluorometer is mounted on the transversing waggion, and connected to a digital voltmeter.

The mean concentration, calculated from measurement in app. 10 sec, is measured 1 cm below water surface in 20 equally distributed points across the plume.

From the resulting concentration profile, a plume width is calculated as the mean dispersion around the centroid of the profile.
When the width of the plume $\sigma_y$ is somewhat greater than the lengthscale of turbulence one can assume that the diffusion coefficient, in a non-dimensional form, is constant /Engelund, 1969/

$$
\delta = \frac{1}{2d} \frac{d}{dx} \left( \frac{u}{u_f} \right)
$$

(2.2)

where $d$ = water depth, $u$ = bulk velocity, and $u_f$ = friction velocity.

In Fig. 2.5 is shown the results from the experiments together with the experimental conditions.

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A mean value of $\delta$ is calculated to

$$
\delta = 0.14
$$

with standard deviation 0.07.

Compared to other investigations conducted under similar conditions /Engelund, 1969. Fisher, 1979/ this value seems small, but within the range measured under similar conditions.

The vertical diffusion coefficient, $D_z$.

The vertical diffusion coefficient of the plume is found using a combination of numerical simulation and measurements.

The one-dimensional vertical diffusion equation was solved using a numerical random walk model. For the local eddy viscosity was assumed a parabolic variation with depth. As initial condition was used a gaussian concentration profile.

From the resulting vertical concentration profiles was calculated a height $h$ as the root mean dispersion around the water surface. The vertical diffusion coefficient could then be found as

$$
D_z = \frac{1}{2} \frac{dh^2}{dt}
$$

(2.3)

The result from the simulation was a nearly parabolic variation of $D_z$ with depth which could be described as

$$
D_z = \gamma u_f \frac{d}{dx} \left( 1/\sqrt{5} - h/d \right)
$$

(2.4)

The coefficient $\gamma$ is then determined from experiments.

In the experiments a dilution of Rosamin-B is discharged through the outlet and the vertical distribution is measured in subsequent cross sections.

Water is sampled through 13 siphons, shown on Fig. 2.6, to 1 liter bottles.
Concentration of tracer is measured using the in-situ fluorometer and a 50 ml plexi-glass cuvette.

The experiments were conducted using a water depth of 18 cm, flow velocity 10.1 cm/sec. and both roughness types.

In Fig. 2.7 is shown the results from the experiments.

Using (2.3) and (2.4) a value of \( \gamma \) is calculated between each cross section and a mean value of \( \gamma = 0.71 \) is calculated.

The spread is considerable, but a plot of plume height versus a diffusion distance, as shown in Fig. 2.8, seems to provide a satisfactory fit to the data.

The curve is (2.4) integrated with \( \gamma = 0.7 \) and an initial height of 1.0 cm.

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Fig. 2.7 Measured plume heights.

Fig. 2.8 Vertical diffusion coefficient.
2.2.3 Friction velocity

As friction velocity $u_f$ is chosen as scale of the turbulence level in the flow it is essential to determine this for each flow condition used in the experiments.

The velocity measured in each experiment is the approximate surface velocity, $u_s$ measured by means of a floater.

Using Colebrook & White's friction law

$$\frac{V}{u_f} = \frac{2}{f} = 6.4 - 2.45 \cdot \ln \left( \frac{k}{Re} \right) + 4.75 \cdot \frac{1}{Re}$$

$$R = \frac{V \cdot R}{v}$$

and a velocity defect law

$$\frac{u_s - V}{u_f} = 2.45 + 2.45 \cdot \ln \left( \frac{d}{s} \right)$$

where $R = $ hydraulic radius, $d = $ water depth, $V =$ bulk flow velocity, $k = $ Nikuradse bottom roughness.

Combining (2.8) and (2.9) the following dependency is derived

$$\frac{u_s}{u_f} = 8.84 - 2.45 \cdot \ln \left( \frac{k}{d} + \frac{3.2}{R_s} \right)$$

$$R_s = \frac{u_f \cdot d}{V}$$

from which $u_f$ can be calculated.

2.3 Experiments on plume

2.3.1 Experimental facility

The experimental facility consists of a $180^\circ$ insulated, constant head barrel, which can be filled with water at the desired temperature.

The water flows from the barrel through an adjustable valve and a flowmeter to a deareator, consisting of a closed box where air bubbles can accumulate without influencing the flow.

From the deareator the water is discharged through an outlet arrangement to the surface of the flume.

To prevent shear between the plume and the water in the flume the bulk outlet velocity and the surface velocity on the flume are set to equal.

![Outlet arrangement](Fig. 2.9)

2.3.2 Temperature measurement

Temperature is measured by means of 8 copper-constantan thermocouples mounted on a metal rod.

The rod is mounted on a transversing waggon, which can be controlled from a computer.
As reference temperature for the thermocouples is used an insulated bottle, mounted on the rod.

The voltage from each thermocouple is amplified 5000 times and registered on a digital voltmeter, with a smallest increment of 10 mV.

The principle of a thermocouple is that a temperature difference between the two solderings produces a voltage, proportional to the temperature difference.

The constant of the proportionality for each thermocouple is found by measuring voltage and temperature in a range covering the expected temperatures in the experiments.

The measured parameter in the experiments is temperature difference between plume and ambient water. Before each traversal the zero of the thermocouples are set to the temperature of the ambient water. This procedure accounts for drift in the electronic equipment and changes in ambient temperature.

Sensitivity of thermocouples are 0.05°C. Because the temperature fluctuates, it is important to know the time constant of the thermocouples. This has been measured by simulating a temperature step function.

In Fig. 2.11 is shown the registered temperature as a function of time, while the thermocouple is dropped down right in front of the outlet.

![Fig. 2.10 Thermocouples mounting.](image)

Fig. 2.10 Thermocouples mounting.

Fig. 2.11 Step response of thermocouple.

Using an energy balance for the soldering one can express the time constant $K$ as

$$T_2 - T = (T_2 - T_1) e^{-\frac{t}{K}}$$

(2.11)

where $T_1$ and $T_2$ are ambient temperature before and after the step, $T$ is the temperature of the soldering, and $t$ is the time.

From the experiment $K$ is found to 0.05 sec.

2.3.3 Temperature to density conversion

As the relationship between the temperature and the density is nonlinear, the density difference $\Delta \rho$ is calculated from the measured temperature difference $\Delta T$ as

$$\Delta \rho = \rho - \rho_0 = f(T_0 + \Delta T) - f(T_0)$$

(2.12)

where $\rho_0$ is density of the ambient water, $T_0$ is temperature of ambient water, and $f$ is a functional relationship between density of water and temperature.
The relationship is calculated using quadratic interpolation in a table taken from /Fisher, 1979, Table no. 1/.

When measuring the temperature in the plume the 8 thermocouples are scanned sequentially for 10 seconds in each vertical. Each registered temperature difference is converted using (2.12) and a mean density difference is calculated for each thermocouple.

2.3.4 Data-reduction and experimental procedure

Each cross section of the plume is divided into 15-20 verticals with equal spacing dependent on plume width.

The depth of the rod is adjusted before each experiment, so that the topmost thermocouple is 0.2 cm below water surface.

The transverse temperature profile is approximately normal, while the vertical profile is like a half normal distribution, with top of the water surface.

The resultant two-dimensional density profile is characterized by a vertical, \( h \), and a horizontal length scale, \( b \), and a mean density difference, \( \Delta \rho \).

The length scales are defined by the moments of the distribution:

\[
M_j^n = \int \int M^n \cdot x_j^n \, dx_j \, dx_1
\]  

(2.13)

where \( M_j^n \) is the n'th central moment. Index \( j \) refers to axis direction.

In vertical direction the distribution is assumed symmetric around the water surface; \( \Delta \rho \) is local time average density difference.

The following definitions appear:

\[
\rho^0 \quad \text{[kg/m}^3] \quad \text{the total mass deficit in the cross section}
\]

\[
h = \left( \frac{M_1^2}{\rho^0} \right)^{1/2} \quad \text{[m]} \quad \text{the vertical length scale}
\]

\[
b = \left( \frac{M_2^2}{\rho^0} \right)^{1/2} \quad \text{[m]} \quad \text{the horizontal length scale}
\]

\[
\frac{\Delta \rho}{2h \cdot 4b} \quad \text{[kg/m}^3] \quad \text{the mean density difference in the cross section}
\]

From this it can be seen that plume width and height equals the standard deviations of the temperature distribution.

The total flux of the excess temperature is used as a control parameter of the measurement with no heat loss to the atmosphere, conservation of heat equals

\[
\Delta T \cdot A \cdot u = \rho^0 \cdot u
\]  

(2.14)

where \( \Delta T \) is initial temperature difference, \( A \) is area of outlet.

Index \( T \) refers to substitution of \( \Delta \rho \) with \( \Delta T \) in (2.13)

Rearranging (2.14) yields

\[
\frac{\rho^0}{\Delta T} = \frac{A}{u}
\]  

(2.14a)

2.3.5 Results of experiments on plumes

In Fig. 2.12 is shown the main results of the experiments on plumes. For each cross-section is shown the distance from the outlet, the calculated height, width, and mean density difference of the cross-section together with the total temperature difference.

In Fig. 2.13 is shown the ratio of total buoyancy flux to the flux measured as a function of distance from the outlet.

There is apparently some variation, but the scatter seems random.

A linear regression shows no significant trend, which could indicate heat loss to atmosphere.
3. INTEGRAL DESCRIPTION OF BUOYANT SURFACE PLUME

The objectives of the integral description is to describe the geometrical development of the surface plume from the transition zone, just after the impingement of the surface, until density differences no longer can be recognized.

In the absence of turbulent energy in the ambient water no mixing will occur, but the buoyancy will spread the plume in a thin surface layer. Eventually some entrainment can be found near the edges of the plume.

When turbulence is present diffusion will occur through the bottom and the sides of the plume.

In the limit with no density differences the mixing will be pure diffusional.

3.1 Initial conditions

In the comparison between the integral description and the measured expansion of the plume between two cross sections, is used measured values in the up-stream cross section as initial conditions.

In practical applications, for example a sewage outfall, the known variables usually are outlet flow, \( Q_u \), density difference between recipient and sewage, \( \Delta \rho \), and initial dilution, \( S_0 \).

Assuming that densimetric Froude number is 1 in the first cross section /Larsen, 1968/, one can derive the following initial values for \( h \) and \( b \).

\[
\begin{align*}
    h_0 &= \frac{Q_u}{u p Q} = \frac{Q_u}{4 \cdot u^3} \quad \text{(3.1)}
    \\
    h_0 &= \frac{u^2 - \rho Q \cdot \Delta \rho}{2 \rho \cdot \Delta \rho} - S_0 \quad \text{(3.2)}
\end{align*}
\]
3.2 Hypotheses

The plume is described by the plume half width, b, the height of the plume, h, and the mean density difference between the plume and the ambient water, $\Delta \rho$.

In the limiting case of pure diffusional spread with constant diffusion coefficient, the following relation will hold

$$\frac{db}{dx} = \frac{h}{u \cdot b} \quad (3.3)$$

$$\frac{dh}{dx} = \frac{D_z}{u \cdot b} \quad (3.4)$$

where $h$ and $b$, respectively, are defined as the spatial variances of the density distribution. $u$ is ambient flow velocity.

In the case of pure buoyancy spread, the buoyancy induced wavefront will advance at a velocity of

$$v_f = a \cdot \frac{u}{\sqrt{g \cdot \Delta \rho \cdot b}} = \frac{1}{F_d} \quad (3.5)$$

where $\rho_a$ is density of ambient water, $g$ is gravitational acceleration = 9.81 m$^2$/s and $a$ is a dimensionless number of order one /Niel, 1973/, which is dependent on friction between the two fluids, the $h/d$ ratio, and the actual form of the density profile /Engelund, 1976. Benjamin, 1968/. $F_d$ is a densimetric Froude number defined as

$$F_d = \sqrt{\frac{g \cdot \Delta \rho \cdot h}{u}}$$

The relation (3.5) is valid only when $u \gg v_f$, which is true outside the impingement zone in the sea outfalls.

Defining the total buoyancy as

$$M = h \cdot b \cdot \Delta \rho$$

it follows from the conservation of buoyancy that when no mixing occurs

$$\frac{dh}{dx} = -\frac{dh}{dx} \cdot \frac{h}{b} \quad (3.7)$$

Assuming that the resultant dispersion of the plume represents the additional influence of diffusion and buoyancy spread it follows from (3.3) and (3.5) that

$$\frac{db}{dx} = a \cdot \frac{1}{F_d} \cdot \frac{D_z}{b} \quad (3.8)$$

and using (3.4), (3.5) and (3.6)

$$\frac{dh}{dx} = -a \cdot \frac{1}{F_d} \cdot \frac{h}{b} \cdot \frac{D_z}{u \cdot h} \quad (3.9)$$

Previous investigations (fx /Turner, 1973. Schiller, 1975/) show that the presence of density gradients highly affects the turbulence, especially the vertical movements, and thereby affects the turbulent diffusion.

It is assumed that the transverse diffusion remains unaffected. Further it is assumed that the influence on the vertical diffusion relates to a bulk Richardson $R_{10}$ number as

$$\frac{D_z}{D_{zo}} = (1 + 6 \cdot R_{10})^{-1} \quad (3.10)$$

where $D_{zo}$ is the unaffected vertical diffusion coefficient, $D_z$ is the
actual diffusion coefficient; \( \delta \) is an empirical constant, and

\[
\kappa_0 = \frac{g \Delta \rho \cdot h}{u_f} \tag{3.10a}
\]

This dependency is used, although it formally relates the total eddy viscosity to a gradient Richardson number /Munk, 1948/.

Given appropriate initial conditions and values for \( a \) and \( \delta \), the two coupled integral equations can now be solved by using a numerical forward stepping integration technique.

### 3.3 Comparison with experiments

To give a visual interpretation of the experiments is chosen two plots. The first is based on the observation that the wave front velocity is independent of vertical dilution of the plume.

Based on this (3.8) can be rearranged to a linear form

\[
\frac{1}{2d} \frac{u \cdot d b^2}{u_f \cdot dx} = \frac{D_Y}{u \cdot b} \cdot \frac{1}{d} \frac{u \cdot b}{u_f} \tag{3.11}
\]

This relation is shown in Fig. 3.2 and it is seen that the experiments fit eq. (3.11) reasonably well.

For the transverse diffusion coefficient is used the measured values directly.

The second is based on the equation for change in density difference

\[
u \cdot \frac{d \Delta \rho}{dx} = - \frac{D_y}{\rho} \cdot \frac{D_z}{b} + \frac{D_z}{b^3} \tag{3.12}
\]

Inserting the equation for \( D_z \) this yields

\[
\frac{D_z}{b_0} \cdot \frac{\rho}{D_z} \cdot \frac{d \Delta \rho}{dx} + \frac{D_y}{b^3} = \frac{1}{1 + \delta R_{10}} \tag{3.13}
\]

**Fig. 3.2 Expansion of plume width.**
As shown in Fig. 3.3 one recognizes the hyperbolic dependency, but the scatter is large, which reflects the difficulties in measuring the plume height.

Fig. 3.3 Attenuation of vertical diffusion.

The estimation of the two constants, \( a \) and \( \delta \), from the experiments could be done from (3.11) and (3.13), but this choice of weighting will merely be based on the necessity of using a linear dependency, which is a rather arbitrary choice.

One major application of this description is believed to be prediction of concentrations in the near field of sea outfall. Based on this the values of \( a \) and \( \delta \) are estimated to give the best prediction of concentration change, i.e., \( \frac{dc}{dx} \).

Using eq. (3.8) and (3.9) and a forward stepping numerical integration, a least-square estimate of predictions compared with experiments can be obtained as

\[
0 = \sum_{i=1}^{n} \left( \frac{dc}{dx_c} - \frac{dc}{dx_m} \right)^2 + \sum_{i=1}^{n} \left[ \left( \frac{c_c - h_c}{h_m} \right)^2 + \left( \frac{c_c - h_m}{h_m} \right)^2 \right] 
\]

(3.14)

where index \( c \) refers to the value obtained from the numerical integration, with the conditions measured in the previous cross-section as initial conditions, index \( m \) refers to the measured value and \( n \) is the number of plume expansions measured. \( \Delta p \) is a central value of density difference between the two cross sections.

Where it is possible the value of \( D_y \) measured under same conditions is used in the integration. Otherwise the calculated mean value is used.

For \( D_z \) is used the derived relation (2.5).

Using a gradient based optimization method \( a \) and \( \delta \) are estimated as the value which minimizes the function (3.14).

From this \( a \) and \( \delta \) is estimated to

\[
a = 1.35 \\
\delta = 1.58
\]

With these values the distribution of the errors \( \frac{dc}{dx_c} - \frac{dc}{dx_m} \) are shown in Fig. 3.4.
Due to the scatter in the measured variables, \( a \) and \( \delta \) are estimated with some uncertainty. As an analysis of sensitivity to changes on \( a \) and \( \delta \) is on Fig. 3.5 shown a plot of \( \theta \) versus respectively \( a \) and \( \delta \).

The coefficient, \( \gamma \), which appears in the equation for \( D_z \), has been included in the optimization procedure, and this seems to confirm the value of 0.71.

4. DISCUSSION

The purpose of this study has been to establish an appropriate description of the dilution of buoyant surface plumes.

The experiments seem to confirm the hypothesis that the dilution is a result of a buoyancy induced wavefront motion and turbulent diffusion. Further it seems that the description of the attenuation of vertical diffusion is reasonable.

Compared to previous investigations, the coefficient on the buoyant wavefront motion seems to be in the correct order of size, though slightly larger than the value found in /Weil, 1979/. This was expected, as the plume width in this study is defined in a depth-integrated manner.
5. SUMMARY

The purpose of the present study has been to establish a description of the dilution of buoyant surface plumes in the near-field of sea-outfalls.

The description is based on two integral length-scales, a local height and width, which characterize a cross-section of the plume.

The geometrical development of the plume is assumed to be the sum of a buoyant spread, due to the density difference and a turbulent diffusion, where turbulence is generated at the bottom of the ambient water. The density difference is assumed to attenuate vertical diffusion, while the lateral diffusion remains unaffected. At the limit of no density difference, the description has a pure turbulent diffusion as solution.

The two resulting integral equations for the buoyant surface plume are

\[
\frac{db}{dx} = a \cdot \frac{\sqrt{g \cdot \delta \rho \cdot h}}{U} + \frac{D_y}{U \cdot b}
\]

\[
\frac{dh}{dx} = -a \cdot \frac{g \cdot \delta \rho \cdot h}{U} + \frac{D_{zo}}{U \cdot h(1 + \delta \cdot R_{zo})}
\]

where \(b\) and \(h\) is a characteristic width and height,
\(U\) = ambient velocity, \(\rho\) = ambient density,
\(\delta \rho\) = local mean density deficit, \(D_y\) = lateral diffusion coefficient
\(D_{zo}\) = vertical diffusion coefficient, \(R_{zo}\) = local bulk Richardson number, and \(g\) = acceleration of gravity,
\(a\) and \(\delta\) = constants.

The integral description has been calibrated against a series of laboratory experiments, conducted under varying conditions of density difference and turbulence level. From the calibration the numerical constants has been determined to \(a = 1.35\) and \(\delta = 1.58\).

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