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by

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Behaviour of Dense Frederikshavn Sand During Cyclic Loading

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Abstract

This article investigates how Frederikshavn Sand behaves when subjected to cyclic loading with emphasis on the development of deformations and the number of cycles, which it can withstand before failure is reached. The investigation is done by performing a series of undrained cyclic triaxial tests, at the Geotechnical Laboratory at Aalborg University. Tests were conducted with a relative density of 80 % in order to simulate offshore conditions where relative densities are relatively high. The purpose is to develop design diagrams, which can be used in order to estimate the undrained cyclic bearing capacity of Frederikshavn Sand for an arbitrary stress level and cyclic loading condition. It is discovered that the governing parameters regarding the response is dependent on the stress path and insitu conditions; initial pore pressure, stress state and the combination of average and cyclic shear stresses.

1 Introduction

The Fatigue Limit State is very often the limiting design condition for offshore wind turbine foundations, which is due to the fact that these foundations are subjected to severe cyclic loading through current, wave and wind actions. During the lifetime of an offshore wind turbine foundation, cyclic loading will correspond to a drained situation since excess pore pressure is able to dissipate between storms. However, during a single storm the drainage path may be long compared to the permeability of the soil, and cyclic loading from a storm may therefore lead to an undrained situation. The tests in this article should imitate cyclic loading during a storm, hence the tests are conducted undrained.

When designing an offshore wind turbine foundation no common design regulation exists regarding cyclic loading. Hence, different approaches have been made as an attempt to include cyclic loading in the design procedure. One method is by the application of design graphs, which accounts for the stresses generated by cyclic loads and the deformations they lead to. These graphs are based on laboratory work in the form of cyclic triaxial or cyclic direct simple shear tests.

The design graphs resulting from the cyclic triaxial tests are to be applied in connection with the construction of an offshore wind turbine foundation in Frederikshavn, Denmark. The soil at this location is a marine sand defined as Frederikshavn Sand. This paper characterises the Frederikshavn Sand, which the cyclic triaxial tests have been conducted on, and the course of action regarding the execution of cyclic triaxial tests. Furthermore, it is described how Frederikshavn Sand reacts during undrained cyclic loading, which can be applied in a design diagram.

2 Characteristics of Cyclic Loading

Offshore cyclic loading is irregular, where both load period and amplitude changes over time. For laboratory work, however, the cyclic loads are simplified from irregular to regular with a constant period and amplitude. The cyclic load is defined by the cyclic shear stress, \( \tau_{cy} \), and the average shear stress, \( \tau_a \), with corresponding shear strain, \( \gamma_{cy} \) and \( \gamma_p \), which is illustrated in Figure 1. \( \tau_0 \) which is the the shear stress obtained from the

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Figure 1: Stress-strain behaviour under cyclic loading.
insitu condition, and $\Delta \tau_a = \tau_a - \tau_0$ which is the average shear stress from further loading. This can include the self-weight of a structure and the mean shear stress created by cyclic loading.

During a cyclic triaxial test the soil will experience the cyclic shear stress, $\tau_{cy}$, about the average shear stress, $\tau_a$, see Figure 2(a). The cyclic load depicted in Figure 2(a) gives rise to pore pressure build-up defined by a permanent pore pressure component, $u_p$, and a cyclic pore pressure component, $u_{cy}$, as seen in Figure 2(b). As the pore pressure components continue to increase over time it causes a decrease in effective stresses which results in larger and larger permanent shear strain, $\gamma_p$, and cyclic shear strains as well, $\gamma_{cy}$, see Figure 2(c) (Andersen, 2009). It should be noted, however, that the example given above is not always the case. There are situations where the pore pressure and shear strain evolution responds differently (Shajarati et al., 2012).

The undrained response of a sand is dependent on the relative density. A loose sand will try to compact, and positive pore pressure is generated, which reduces the effective stresses. A dense sand will try to dilate, which results in negative pore pressure. This entails that after initial undrained loading the effective stresses for a dense sand will be increased, and for a loose sand, it will decrease. This is decisive for how the initial stress path will look like. In Figure 2 an example of a loose sand is given. It is seen that the effective stresses decrease as cyclic loading continues, and the stress path will eventually intersect the failure envelope. For a dense sand the arc of the initial stress path will first go towards larger effective stresses, and thereafter the effective stresses will start to decrease as pore pressure builds up.

![Figure 2: Shear stress, pore pressure and shear strain as a function of time during undrained cyclic loading (Andersen 2009)](image)

**3 Soil and Test Specifications**

The Frederikshavn Sand has a minimum and maximum void ratio of $e_{min} = 0.64$ and $e_{max} = 1.05$. The preparation method for the triaxial sample was made by dry tamping to a relative density of $I_D = 80\%$, using undercompaction with 5 layers. When saturating the specimens, the stiffness of the soil skeleton, i.e. the bulk modulus, $K$, and the pore pressure level were taken into account (Amar, 1992). Through a consolidation test $K$ was determined to be 108 MPa and it was insured that the samples were at least 99.9 % saturated.

Drained preshearing of 400 cycles with an amplitude of 0.04 $\sigma'_{ec}$ was applied, at an effective mean stress level of 30 kPa, in order to remove any stress concentration from tamping and thereby creating a more homogeneous sample. The effective mean stress was afterwards raised to 60 kPa.

Through an earlier study made on Frederikshavn Sand by Hansson et al. (2005) an expression for the friction angle as a function of relative density, $I_D$, and confining pressure, $\sigma'_c$, was calibrated to

$$\varphi = 0.146 I_D + 41 \sigma'_{c, 0.0714} - 1.78^\circ$$

(1)

The expression has been validated by conducting three drained isotropic consolidated monotonic tests with an effective confining pressure in the range between 30 and 120 kPa. The deviation between the results and the expression is in the interval 1-5 %. From the monotonic tests the triaxial friction angle was found to be $\varphi = 39.6^\circ$ for an effective isotropic consolidation stress of 60 kPa. Thereby giving a $K_0$ value of 0.36. This produces an anisotropic consolidation with an effective vertical consolidation stress, $\sigma'_{vc}$, of 166.7 kPa and an effective horizontal consolidation stress, $\sigma'_{hc}$ of 60 kPa.

The test samples were cylindrical with an initial height, $H_0$, of 71 mm, and an initial diameter, $D_0$, of 70 mm, hence $H/D \approx 1$. At the cap and base, two rubber membranes with high vacuum grease in between were placed to make the cap and base frictionless. These initiatives were performed in order...
Table 1: Average and cyclic shear stress used in the test programme. Test No. 1 is a monotonic test, and test No. 2-17 is cyclic triaxial tests.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\tau_a$ [kPa]</th>
<th>$\tau_{cy}$ [kPa]</th>
<th>$u_0$ [kPa]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>400.2</td>
<td>0.0</td>
<td>110.7</td>
</tr>
<tr>
<td>2</td>
<td>209.8</td>
<td>185.2</td>
<td>105.8</td>
</tr>
<tr>
<td>3</td>
<td>260.2</td>
<td>100.8</td>
<td>109.2</td>
</tr>
<tr>
<td>4</td>
<td>166.9</td>
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<td>110.0</td>
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<td>99.9</td>
<td>100.0</td>
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<td>6</td>
<td>124.9</td>
<td>49.8</td>
<td>110.1</td>
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<tr>
<td>7</td>
<td>78.0</td>
<td>50.2</td>
<td>120.7</td>
</tr>
<tr>
<td>8</td>
<td>53.4</td>
<td>17.0</td>
<td>100.2</td>
</tr>
<tr>
<td>9</td>
<td>166.6</td>
<td>167.1</td>
<td>302.3</td>
</tr>
<tr>
<td>10</td>
<td>49.6</td>
<td>125.5</td>
<td>99.8</td>
</tr>
<tr>
<td>11</td>
<td>24.0</td>
<td>50.9</td>
<td>139.7</td>
</tr>
<tr>
<td>12</td>
<td>24.6</td>
<td>100.2</td>
<td>100.3</td>
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<td>24.8</td>
<td>100.4</td>
<td>299.7</td>
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<td>66.0</td>
<td>125.6</td>
<td>99.5</td>
</tr>
<tr>
<td>16</td>
<td>84.1</td>
<td>129.4</td>
<td>100.4</td>
</tr>
<tr>
<td>17</td>
<td>158.8</td>
<td>216.9</td>
<td>100.4</td>
</tr>
</tbody>
</table>

4 Cyclic Test Programme

A total of 17 undraind triaxial tests were conducted; 1 monotonic and 16 cyclic tests. 13 of the cyclic triaxial tests were performed with different combinations of average shear stress, $\tau_a$, and cyclic shear stress, $\tau_{cy}$. These tests are used for constructing the design graphs described in section 6. A complete list of the conducted tests are shown in Table 1.

4.1 Cyclic Triaxial Cell

In Figure 4 a sketch of the cyclic triaxial cell is shown. The principle of the system is that a cyclic load is applied via the hydraulic piston at the bottom of the cell.

In order to calculate the stresses and strains in the sample, the following parameters were measured from the triaxial apparatus:

- Axial deformations
- Cell pressure
- Pore pressure
- Piston force

4.2 Test Procedure

As mentioned, the dominating force on offshore wind turbine foundations is wave loads, which have a period of 10 to 20 seconds [Lesny 2010]. According to [Andersen 2009] the load period on sand seem to have no significant effect when a test is conducted undrained, and therefore the length of the period is only limited by the reaction time of the hydraulic piston. Based on this information the load period was kept as low as practically possible in the range from 10 to 100 seconds, to limit test duration.

To reflect the insitu conditions the sample was anisotropically consolidated. The process of consolidating the sample and conducting cyclic tests is illustrated in Figure 5 and described in the following. (a) isotropic consolidation 1: The sample was set up in the triaxial cell where it was exposed to an isotropic stress level of 30 kPa. (b) preshearing: The sample was presheared in order to remove stress concentrations originating from tamping when using the undercompaction method. Preshearing was performed at lower isotropic stress levels, than when the $K_0$ procedure was applied in order not to consolidate the soil too much during preshearing. (c) isotropic consolidation 2: After preshearing the confining pressure was increased to an isotropic stress level of 60 kPa in order to have horizontal stresses corresponding to insitu conditions. (d) Anisotropic consolidation: This step is the actual $K_0$ procedure where the vertical stress was increased so it corresponds to insitu conditions ($\sigma_{vc}' = 166.7$ kPa, $\sigma_{hc}' = 60$ kPa). (e) Cyclic loading: The processes up till cyclic loading were conducted drained. From the $K_0$ point the increase in average shear stress, $\Delta \tau_a$, and the cyclic shear stress, $\tau_{cy}$, was added undrained.
5 Cyclic Test Results

When analysing the cyclic test results it is observed that the failure modes can be separated into two main groups. One where cyclic shear strain, $\gamma_{cy}$, is dominating, and another where permanent shear strain, $\gamma_p$, is dominating. A common feature of the tests that fail with dominating $\gamma_p$ is that they are all subjected to one-way loading, i.e. $\tau_a > \tau_{cy}$. The opposite effect is observed when $\gamma_{cy}$ is dominating, i.e. $\tau_a < \tau_{cy}$. Another observation is that all one-way loaded tests fail by incremental collapse, while all two-way loaded tests fail by liquefaction. This will be outlined in the following sections.

For all the tests failure is defined as either $\gamma_p = 15\%$ or $\gamma_{cy} = 15\%$. A plot of the different tests with number of cycles to failure can be seen in Figure 6.

5.1 Liquefaction

The stress path in the $p' - q$ space for a two-way loaded cyclic test with $\tau_a = 25$ kPa and $\tau_{cy} = 100$ kPa is depicted in Figure 7. The sample is subjected to cyclic loading with an amplitude so large, that the excess pore pressure, $u$, exceeds the effective mean stresses, $p'$, which becomes zero, and thereby liquefaction occurs. The stress path has the characteristic butterfly shape as described in Randolph and Gouvernet (2011). At liquefaction the sample will start to dilate, which generates negative pore pressure, and effective stresses are again mobilised, and cyclic loading continues.

From Figure 6 it is observed that the initial pore pressure is equal to 300 kPa indicated by point (a), which means that the confining pressure is 360 kPa in order to keep effective mean stresses equal to 60 kPa. As the sample is exposed to more cycles the pore pressure will eventually increase to a value of 360 kPa, indicated by point (b), which is the point where liquefaction occurs.

When liquefaction occurs the soil has lost its bearing capacity, which produces large shear deformations as seen in Figure 8. During all the cyclic tests where liquefaction occurs, liquefaction is observed two times in each cycle; once in compression and once in extension. For each time liquefaction occurs, the shear strain increases as cyclic loading continues, as seen in Figure 9.
5.2 Incremental Collapse

Figure 10 shows a one-way loaded test with $\tau_a = 167$ kPa and $\tau_{cy} = 167$ kPa. The response shows that as cyclic loading is being applied $p'$ decreases, which is due to pore pressure build up. The pore pressure development is illustrated in Figure 11 and it is observed that initially the pore pressure decreases because the sample tries to dilate resulting in an increase in effective mean stresses. As cyclic loading continues the pore pressure starts to increase entailing a reduction in effective mean stresses.

Moreover, from Figure 10 it is observed that the inclination of the cycles becomes steeper as more cycles are applied, which is due to an increase in soil stiffness. This means that $\gamma_{cy}$ becomes smaller as $N$ increases.

In Figure 12 which shows a $\gamma$ - $q$ diagram, it is observed that the incremental shear strain decreases, but the total shear strain increases with number of cycles. This type of failure is also defined as incremental collapse by Peralta (2010).

Figure 13 also confirms the statement that the incremental shear strain decreases with increasing number of cycles, while the permanent shear strain increases and eventually resulting in failure at $\gamma_p = 15 \%$ for $N = 340$ cycles.

6 Design Graphs

When constructing diagrams which can be applied in practical design situations, the average and cyclic shear stress are often normalised with respect to a
certain stress value. When this normalisation is performed the average and cyclic shear stresses are defined as Average Load Ratio, ALR, and Cyclic Load Ratio, CLR.

Different authors have proposed various types of design graphs for cyclic loading, which all take the cyclic shear stress into account via the cyclic load ratio. Randolph and Gouverne (2011) made a strain contour diagram, shown in Figure 14 based on 1 undrained monotonic and 4 undrained cyclic simple shear tests on sand performed by Mao (2000). The diagram shows the strain contours as a function of the cyclic load ratio and number of cycles, and can thereby predict the shear strain from cyclic loading. However, during a literature study performed by Shajarati et al. (2012) and also by the conducted cyclic tests, it was found that both the cyclic and average shear stress level are very important, for the cyclic bearing capacity. Therefore strain contour diagrams, which only takes the cyclic load ratio into account, are insufficient for predicting the effects of cyclic loading.

Andersen and Berre (1999) made a study on the effects of cyclic loading, where both the cyclic load ratio and the average load ratio were taken into account. This produced the design graph in Figure 15, which was made for Baskarp sand with a relative density of 95 %. The normalisation in this diagram is performed with the effective vertical consolidation stress, \( \sigma_v' \), and it can be observed that failure is dependent on the combination of average and shear stresses. It should be noted that in this graph cyclic failure is defined as either 3% cyclic or permanent shear strain, and the tests were conducted with \( H/D = 2 \).

When cyclic soil testing is conducted on sand, the cyclic and average shear stress is most often normalised with respect to \( \sigma_v' \), as shown in Figure 15 (Andersen and Berre, 1999). This is useable under drained conditions since the drained failure envelope is only dependent on the friction angle and effective mean stress, and \( \sigma_v' \) can therefore be used as a normalisation parameter.

In the undrained case however, the undrained shear strength for a dilative sand is not only dependent on the friction angle and mean effective stresses, but also the amount of initial pore pressure (Ibsen and Lade, 1998b). This is due to the fact that the undrained shear strength is influenced by cavitation. Before a dense sand reaches failure (both in tension and extension) it tries to dilate, which generates negative excess pore pressure and thereby an
increase in effective stresses. At first this will "eat" the initial pore pressure and afterwards failure will occur at around $u = -95$ kPa, which will lead to failure. Even though this is a well known problem, the normalisation parameter for dense sand in the undrained state is still most often $\sigma'_{uc}$, as seen in Figure [15] which does not account for cavitation and the initial pore pressure.

6.1 Undrained Shear Strength

As mentioned $\sigma'_{uc}$ can in the drained case be related to the drained shear strength, $\tau_f$. The drained shear strength accounts for the friction angle, the effective mean stress and cohesion, and is given as

$$\tau_f = \frac{1}{2} \cdot \frac{6}{3 - \sin \varphi} \cdot (p' + c' \cot \varphi')$$  \hspace{1cm} (2)

where $c' = 0$ for cohesionless soils. Instead of using $\sigma'_{uc}$ as a normalisation parameter in the undrained case for sand, the undrained shear strength, $c_u$, is used. Therefore, the use of the above expression is extended to the undrained case by adding the initial pore pressure, $u_0$, and the pore pressure at which cavitation occurs $u_{cav}$, which results in equation (3). Furthermore, the used effective mean stress corresponds to failure in the drained case, $p'_{df}$.

$$c_u = \frac{1}{2} \cdot \frac{6}{3 - \sin \varphi} \cdot (p'_{df} + u_0 + u_{cav})$$  \hspace{1cm} (3)

The argument for using the above expression is that the undrained bearing capacity for a dense sand is governed by cavitation, as negative pore pressure develops during loading (Ibsen and Lade 1998b). It is therefore important to include the pore pressure when calculating $c_u$ in the undrained case for sand. The effect of adding the initial pore pressure, $u_0$, and the pore pressure at cavitation, $u_{cav}$, is illustrated in Figure [15]. The figure illustrates the effective stress paths for two examples with the same initial effective mean stress, $p'_0$. The two examples end up having a different undrained shear strength, because of differences in initial pore pressure. Following the total stress path will lead to drained failure in point (a), which is the point where $p'_{df}$ is measured. From this point the amount of initial pore pressure and the pore pressure at cavitation is added to $p'_{df}$. This means that a higher amount of initial pore pressure will lead to a higher value of the undrained shear strength before failure is reached, which is illustrated by point (b) and (c).

6.2 Modified Design Graph

Based on the expression for undrained shear strength in equation (3), a modified design diagram is constructed for the Frederikshavn Sand in the undrained case with $I_D = 80\%$. The modified design diagram is based on the 17 conducted tests and normalised with respect to $c_u$, as shown in Figure [17]. It is seen that the graph shares the same tendency as the design graph by Andersen and Berre (1999) in Figure [15]. However, an important feature of the modified design graph is that it accounts for the initial pore pressure, which is important when dealing with the undrained bearing capacity.

To illustrate the limitations of the design graph when normalising with $\sigma'_{uc}$ as proposed by Andersen and Berre (1999), a comparison between the two design graphs can be seen in Figure [18]. To make the comparison, three cyclic tests were conducted with the same average shear stress, $\tau_a = 25$ kPa, and same cyclic shear stress, $\tau_{cy} = 100$ kPa, but with different values of initial pore pressure, $u_0$, namely 100, 160 and 300 kPa. The calculated cyclic and average load ratios for the two design graphs can be seen in Table 2.

<table>
<thead>
<tr>
<th>$u_0$ (kPa)</th>
<th>Design Graph $\tau_a/\sigma'_{uc}$</th>
<th>$\tau_{cy}/\sigma'_{uc}$</th>
<th>Modified Design Graph $\tau_a/c_u$</th>
<th>$\tau_{cy}/c_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 kPa</td>
<td>0.15</td>
<td>0.6</td>
<td>0.07</td>
<td>0.29</td>
</tr>
<tr>
<td>160 kPa</td>
<td>0.15</td>
<td>0.6</td>
<td>0.06</td>
<td>0.26</td>
</tr>
<tr>
<td>300 kPa</td>
<td>0.15</td>
<td>0.6</td>
<td>0.05</td>
<td>0.20</td>
</tr>
</tbody>
</table>

Figure [18(a)] plots the three tests in the same point since they have the same ALR and CLR when normalising with $\sigma'_{uc}$. However, Figure [18(b)] normalises with $c_u$ and plots the three tests in different positions, because ALR and CLR is dependent on the initial pore pressure. The example given above illustrates that it is very important to construct a design diagram in a manner which represent the in-situ conditions as good as possible. Therefore, if the drained state is the design case it is sufficient to apply $\sigma'_{uc}$ as a normalisation parameter. On the other hand if the undrained state is the design case, the initial pore pressure should be taken into consideration, and therefore $c_u$ should be used when normalising the design diagram.
7 Conclusion

A modified design diagram is created for the Frederikshavn Sand in the undrained case for a relative density of $I_D = 80\%$. It can be used to estimate the number of cycles to failure for a given combination of pore pressure, average and cyclic load ratio.

When normalising cyclic and average shear stresses for use in design diagrams $\sigma'_{vc}$ is found insufficient to use as a normalisation parameter in the undrained case, as it does not take pore pressure into account. This is important, since the undrained shear strength for a dense sand is governed by cavitation. Therefore the undrained shear strength, $c_u$, is used as a normalisation parameter for the modified design graph and should be used for other design graphs in the undrained case.

When comparing Figure 15 and Figure 17, a considerable difference is observed at $ALR = 0$. The difference can be explained by a large difference in applied back pressure. In the tests performed by Andersen and Berre (1999), a backpressure in the range from 500 - 1800 kPa was applied. Compared to the tests conducted to make Figure 17 with a backpressure at around 100 kPa, the limit in $\tau_{cy}$ without reaching cavitation is raised significantly. This makes it possible to perform tests with a load ratio combination of $\tau_a/\sigma'_{vc} = 0$ and $\tau_{cy}/\sigma'_{vc} = 1.5$. If the same test is performed with a low back pressure, cavitation will occur, and the test will correspond to a monotonic test. This observation strengthens the argument, for choosing $c_u$ as the normalisation parameter for the undrained case.

![Figure 17: Modified design diagram for Frederikshavn Sand in the undrained case with $I_D = 80\%$. Red corresponds to two-way loading, while blue is one-way loading.](image)

![Figure 18: Comparison between design graphs. (a) Normal design graph ($\sigma'_{vc}$). (b) Modified design graph ($c_u$).](image)
References


