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Abstract—For centralized selection of communication relays, the necessary decision information needs to be collected from the mobile nodes by the access point (centralized decision point). In mobile scenarios, the required information collection and forwarding delays will affect the reliability of the collected information and hence will influence the performance of the delay selection method. This paper analyzes this influence in the decision process for the example of a mobile location-based delay selection approach using a continuous time Markov chain model. The model is used to obtain optimal relay policies via a heuristically reduced brute-force search. Numerical results show how forwarding delays affect these optimal policies.

I. INTRODUCTION

Two-hop relaying has been shown to improve throughput in WLANs. However, for measurement-based approaches as in [1], [2], [3], user movements cause link quality measurements to become outdated and inaccurate, leading to performance degradations. In [4] we considered a location based relay selection scheme that uses a path-loss model to estimate the achievable throughput of different relay path choices. The location based approach is promising due to lower signaling overhead and since it allows for movement prediction.

While the simulation models that are used to analyze the location-based relaying scheme in [4] are sufficient to evaluate individual relay policies, the search for optimal policies becomes computationally very expensive in such simulation approaches. Furthermore, there is a risk that the understanding of the impact of delayed location knowledge on the relaying process is obscured via other system features. As a consequence, this paper derives a continuous time Markov Chain (MC) model that allows to numerically calculate achievable performance of location based two-hop relaying while capturing the delay of the information forwarding. The location information is thereby periodically sent from the mobile nodes to the access point (AP). Such periodic information forwarding represents one of the three basic information access schemes that have been analyzed via analytic models in [5]. The analysis approach of the latter paper also inspired the model setup for the relay scenario in this paper. Location based relaying was considered in for example [6], and the impact of feedback delay on relaying in for example [7], however we believe that the proposed model is the first to account for both mobility and measurement collection delays in relay selection.

Section 2 introduces the general relay system, while the corresponding Markov model is described in Section 3. The approach to calculate optimal relay policies is given in Section 4. Finally, Section 5 analyzes these optimal policies in a case study and validates the model against simulation results based on the authors’ simulation framework presented in [4].

II. SYSTEM DESCRIPTION

We consider the three node system sketched in Fig. 1, consisting of a static access point (AP), mobile relay R, and static destination D. The candidate relay R has a position \((\hat{x}_R(t), \hat{y}_R(t))\) that changes over time. Note that the D is static, so dynamic changes result purely from R’s mobility. In a location-based relaying approach, the AP in this situation needs to take a decision whether – based on its inaccurate (delayed) knowledge of the position of the candidate relay – relaying is beneficial or not. The mapping of the estimated position of R to the relay decision is called a relay policy, here represented by \(\pi(\hat{x}_R, \hat{y}_R) \in \{R, D\}\), where R stands for ‘relay’ while D stands for a direct transmissions. In this paper, we assume that such relay decision is taken just before each individual data fragment transmission.

The positions of the static AP and of the destination D are assumed to be known at the AP, hence only the mobile relay node will periodically (with rate \(\tau\)) send position updates \((\hat{x}_R, \hat{y}_m)\) to the AP. It is assumed that such location information is available at R through, e.g., a GPS system. In order to investigate the impact of the forwarding delays of positioning information in mobile scenarios, it is assumed that the only cause of inaccurate information is the mobility. Therefore, the position information obtained at R is assumed to be exact.

The location measurements are transmitted from the relay to the AP, as sketched in Fig. 2: Node R obtains an exact
coordinate of its current location, wraps it into a WLAN packet and passes it to its WLAN interface. At the WLAN interfaces, there could be a queuing delay until the location message reaches the first position in the (finite) interface queue, followed by a subsequent MAC and transmission delay. The sum of MAC and transmission delays are assumed to show a distribution with mean $1/\mu$.

The AP’s estimate of Node R’s position is based on the last received location measurements. Since the relay is mobile its true position may differ from the AP’s estimate, depending on the stochastic mobility model of the candidate relay. For many mobility models, the older the most recent measurement becomes, the less accurate the AP’s view is expected to be.

Depending on the AP’s belief on the relay’s location it will choose to either make a relayed (R) or direct data transmission (D). The resulting achieved throughput will depend on this choice. Increased throughput can result due to the possible choice of a high bit rate coding scheme when the relay to node distances are shorter than the AP to destination distance. However, as we assume decode and forward relaying here, the relayed transmission requires two packets to be sent which will lead to additional channel occupation affecting throughput. For the numerical results later, we utilize the throughput model of [8], which is summarized in Section V-A.

For optimal performance, it is desirable to make the choice that maximizes the overall achieved throughput. In the considered scenario, this optimal choice depends on the node mobility models, on the dependent contextual propagation characteristics, and on the strategy (period of the location updates) and forwarding delays (queueing, MAC and transmission) of these location updates. The next section develops a Markov model for finding policies that maximize throughput.

III. MODEL FOR RELAYING PERFORMANCE ASSESSMENT

In order to investigate optimal policies when the location information is subject to the delays of the location update procedures, this section develops a Markov model. The main approach is to start from a product-space representation of two parts: 1) a continuous time Markov model for the spatial mobility of the candidate relay node (the ’true’ coordinates); 2) a model of location update procedures and of the resulting AP view. As these two parts are not completely independent, a pure product space approach however is not sufficient, but requires subsequent modifications as described in this section.

A. Markov Mobility Model

First element of the relaying Markov model is a continuous time Markov model that describes the candidate relay’s stochastic mobility. The geographic 2-dimensional space is discretized, for instance via a equidistant grid. The states then represent the current true position of the candidate relay within the grid. Transition rates between the states characterize the mobility. Fig. 3 shows the example mobility model used later in the numerical case study. Transitions are only allowed to the neighboring grid states and all states have the same overall state leaving rate $\mu_0$. As a consequence, the average movement speed of the candidate relay can be readily obtained as $\bar{v} = d/\mu_0$, where $d$ is the distance between neighboring grid-points. Note that more general mobility models can be utilized (see also [9]), in particular along the following lines: 1) states can be associated with any discretization of the geographic space (so equidistant is not needed); 2) transition rates and transition structure can be arbitrary (though for physical movement resemblance, typically transitions would only target neighboring states); 3) multiple Markov states can be utilized for each discrete coordinate in order to keep memory of directional information or to mimic non-exponential state-holding times via Phase-type distributions.

The example in Fig. 3 describes mobility in a quadratic space with equidistant grid-points using $M = N \cdot N$ states. The corresponding generator matrix will be denoted as $Q_{mob}$.

For the general case, a mapping function of state number $m$ to geographic coordinate is required: $c : (m) \rightarrow (x, y)$.

B. Relay Policy Representation

A relay policy is a function that maps geographic coordinates of the candidate relay node to the actual relaying
decision, which is either a direct (‘D’) or relayed (‘R’) transmission. Taking the discretization and the mobility model state space representation into account, we use as input instead a mapping of the state number. Hence, the policy is a function \( \pi : (m) \rightarrow \{R, D\} \), which is constant within all state-sets that represent the same geographic coordinate. Note that the policy is implemented at the AP, hence it uses the estimated relay position (state) as input. The modeling of this position estimate at the AP is described in the next section.

C. AP View and Information Forwarding

In order to model the AP view on the relaying input information and the location update process, the state-space at each coordinate (state of the mobility model) has to be extended as shown in Fig. 4. First, memory of the AP on the last received coordinate needs to be introduced. Instead of keeping track of coordinates, it is however sufficient to just keep track of the relay decision \( \pi(m) \) that corresponds to that last communicated coordinate. Hence the AP can be in state ‘D’ (last received location update was a position that corresponds according to the policy to a direct transmission; upper half of Fig. 4) or state ‘R’ (analogous for a relay transmission; lower half of Fig. 4). When the candidate device triggers a location update (with rate \( \tau \)), it is also not needed to encode in the state-space the actual coordinate of this location update, rather it is enough to encode the resulting decision \( \pi(m) \). If the current state of the mobility model (true coordinate) is marked with \( \pi(m) = 'D' \), then the update in progress is memorized as such (State 2 and 9 in Fig. 4; otherwise States 3 and 10). As soon as the update is received at the AP, the AP’s view is adapted accordingly, leading for instance to the transition from State 3 (AP view is ‘D’ but coordinate update leading to ‘R’ is in progress) to State 8 (AP view is R, no update in progress).

As queuing of state-updates at the WLAN interface queue is possible, additional states are needed. In the shown example, the max queue-size is set to 2 (one update in progress of being transmitted, while one can be in the buffer), corresponding to States 4-7 and 11-14. The state label describe the content of the location updates, where the first element refers to the location update in progress. Loss of update messages on the air interface is also modeled; they occur with probability \( P_{\text{loss}} \). Finally, measurements are transmitted according to the measurement transmission delay rate \( \mu \). In order to express the dependency on the geo-dependent relay policy, the \( \tau \) transitions in Fig. 4 are weighted by the binary variables \( w_D \) and \( w_R \), which are set to one, if the current true position is mapped in the relaying policy to the corresponding action.

The depicted 14 states exist for each grid point in the mobility model. Note that not all states are needed, as a coordinate associated with a relay decision cannot generate an update message with content ‘R’. However, the regular setup of the state space facilitates the creation. The transitions between grid points in the mobility model, shown for the considered example in Fig. 3, are independent of the states of the AP view and information forwarding; hence they do not lead to the corresponding state in Fig. 4.

D. Geographic Throughput Model

In order to calculate expected throughput of the relaying system, we here assume that this expected throughput is only influenced by variability due to mobility; statistical variations due to changing propagation environments are not considered. Hence, the model uses a mapping of node coordinate pairs to link throughput, respectively for the case of relaying, a mapping of node-coordinate triplets to throughput of the 2-hop link. As we here assume that AP and target nodes are static, we only require the candidate node’s position as input; motivated by the discretization of the geographic space, we utilize the state-number rather than the geographic position as input to these throughput functions. Hence, we use two functions \( T_D : (m) \rightarrow R_0^D \) for the direct transmission, and \( T_R : (m) \rightarrow R_0^R \) for the relayed transmission that represent the throughput. \( T_D \) will later be a constant, but in more general models, the position of the candidate relay node may also influence the direct transmission via interference or influences on the propagation model. If the mobility model uses multiple states for one geographic coordinate, then \( T_D \) and \( T_R \) have to be constant within these state sets.

The specific WLAN 802.11 throughput model from [8] is used in the case study; it will be summarized in Section V-A.
E. Calculation of Performance Metrics: Lost Throughput

Performance metrics are calculated from the steady-state solution of the MC defined by the generator matrix $Q$. The resulting $p$ is the $M \times 14$ elements stationary probability distribution vector. In the following, indexes $p_{m,s}$ refer to the $s$th state of the information state model in the $m$th grid point.

The lost throughput metric is the difference in throughput relative to a scheme that has ideal location information (i.e. zero delays and infinitely high update rate) as well as perfect knowledge of achievable throughput. In this case the ideal policy is simply given from the highest throughput in each grid point. This metric is therefore useful for comparing the impact of different scenario parameter settings.

$$S_{\text{lost}} = S_{\text{ideal}} - S_{\text{loc}}$$

For the simulation results, this can be calculated directly from the corresponding simulation results. For the MC model $S_{\text{ideal}}$ and $S_{\text{loc}}$ are calculated as follows:

$$S_{\text{ideal}} = \sum_{m=1}^{M} \sum_{s=1}^{7} \pi_m \cdot p_{m,s} \cdot T_D(m) + \sum_{s=8}^{14} p_{m,s} \cdot T_R(m)$$

$$S_{\text{loc}} = \sum_{m=1}^{M} \sum_{s=1}^{7} \pi_m \cdot p_{m,s} \cdot T_D(m) + \sum_{s=8}^{14} p_{m,s} \cdot T_R(m)$$

where $T_D(m)$ and $T_R(m)$ are the expected throughput with the relay being at the $m$th grid point for direct and relayed transmissions, respectively. For constructing the throughput functions $T_D(m)$ and $T_R(m)$, the throughput model that we presented earlier in [8] is used as described in Section V-A.

IV. OPTIMAL POLICY CALCULATION

We express the policies of using direct or relayed transmission as a binary sequence, representing the decisions to be taken for each state in our mobility model in Fig. 3 with 0 and 1 that refer to D and R transmission modes, respectively. Even for such binary representation, a huge space of $\Omega_\pi = 2^{N^2}$ unique policies exists. Therefore, to allow computation when having more than a few grid points, we need to smarten our search for optimal policies, rather than just brute force searching the policy space for the best policy.

Heuristic Policy Optimization

Assuming first that a policy (as exemplified in Fig. 5) is composed of four quadrants, our heuristic solution to search the policy space is based on 1) a monotonicity assumption: that a 0 in a row or column in a quadrant is never closer to the center than any 1; and 2) that the policy is symmetric in each quadrant (mirrored in both x and y-axes). Notice that while these assumptions allow us to limit the search space significantly, they may not apply in every possible scenario.

Assuming symmetry, we only consider the upper left quadrant. Let each row of the $K \times K$ quadrant ($K = \frac{1}{2}N$, see Fig. 3) be represented by a $K$ items binary sequence. Assuming monotonicity, the search space can be covered by the combinations of rows that each contain $\{k | k \in N_0 \leq K\}$ ones, followed by $K - k$ zeros. For example, the possibilities for a $K = 4$ row would be: 0000, 0100, 1100, 1110, and 1111. Since such a row is given as the binary interpretation of:

$$b = 2^k - 1,$$

(4)

each row is a set of $k$’s denoting the number of ones in a row:

$$R_K = [k_0 \ k_1 \ k_2 \cdots \ k_K].$$

(5)

When iterating over all combinations of such rows, the ones that do not fulfill that a D cannot be closer to the center than any R, can by skipped immediately, by ensuring that $k_0 \leq k_1 \leq k_2 \leq \cdots \leq k_K$. (6)

The optimal policy can now be determined by iterating over the significantly reduced set of candidate policies.

Our assumptions were confirmed by testing all possible policies for a small version ($K = 4$) of the scenario presented in the following case study. The assumptions hold since the case study uses a location dependent throughput model. For e.g. multi-path propagation it can be the case that they do not hold. If symmetry does not apply all combinations of quadrants need to be considered, which leads to a heavy increase of computational effort.

V. CASE STUDY AND NUMERICAL RESULTS

For demonstrating the application of the proposed model, we consider a case study that reflects the scenario in Fig. 1. AP and D Nodes are static, whereas the R Node moves according to the Markov mobility model presented in Section III-A. The Nodes are equipped with 802.11a based radios, but modified to support relaying as mentioned in [4]. Table I lists the used scenario and simulation parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensions</td>
<td>$80 \times 80 \text{ m}^2$</td>
</tr>
<tr>
<td>No. of grid points</td>
<td>$10 \times 10$</td>
</tr>
<tr>
<td>Grid spacing $d$</td>
<td>$8 \times 8 \text{ m}$</td>
</tr>
<tr>
<td>AP coordinate</td>
<td>$(20, 40)$</td>
</tr>
<tr>
<td>Destination coordinate</td>
<td>$(600, 40)$</td>
</tr>
<tr>
<td>Number of simulation runs</td>
<td>1000</td>
</tr>
<tr>
<td>Duration of simulation run</td>
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</tr>
<tr>
<td>Data transmission interval</td>
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</tr>
<tr>
<td>Measurement delay rate $\mu$</td>
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<tr>
<td>Wireless link error probability $p_{\text{loss}}$</td>
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<tr>
<td>Noise floor</td>
<td>$-95 \text{ dBm}$</td>
</tr>
<tr>
<td>Receive A</td>
<td>$b$</td>
</tr>
<tr>
<td>Data frame payload $D_{\text{MSDU}}$</td>
<td>1500 bytes</td>
</tr>
</tbody>
</table>

TABLE I

SCENARIO AND SIMULATION PARAMETERS.

A. Used Throughput Model

The throughput model of IEEE 802.11 that is used in this paper is based on previous work, further detailed in [8]. Here,
as the throughput is given by \( \frac{\text{Delivered data}}{\text{Transmission time}} \), the throughput can be calculated as:

\[
S_{\text{dir}} = \frac{P_{\text{suc}} \cdot B_{\text{MSDU}}}{E[T_{\text{tx}}]} \tag{7}
\]

where \( P_{\text{suc}} \) is the probability of a successful MAC layer frame delivery, \( E[T_{\text{tx}}] \) is the duration of a MAC frame delivery attempt, and \( B_{\text{MSDU}} \) is the MAC payload size given in octets. In the following, we use the indices 1 and 2 to indicate the AP-r and r-d transmissions. The throughput for the two-hop relaying algorithm is calculated as:

\[
S_{\text{rel}} = \frac{(P_{\text{suc,1}} + P_{\text{suc,2}}) \cdot B_{\text{MSDU}}}{E[T_{\text{tx,1}}] + E[T_{\text{tx,2}}]} \tag{8}
\]

The throughput model is used to estimate the transmission throughput functions \( T_D(m) \) and \( T_R(m) \) for each of the \( M \) grid points, indexed by \( m \). Fig. 5 shows the throughput for a \( 10 \times 10 \) grid realization of the considered scenario.

The nearly identical curves in 7(a) show that the impact of movement speed on lost throughput is accurately accounted for in the model. Further, since the optimal policy hardly brings a noticeable gain we can conclude that the standard policy (black dots in Fig. 5) is sufficient in this scenario. Fig. 7(b) shows the number of grid points in which relaying is preferred for the determined optimal policy \( \pi_{\text{opt}} \) and we see that direct transmissions are preferred more often as the movement speed increases. This result corresponds with the intuition that a wrongly chosen relay transmission may be more expensive than a wrong direct transmission, since the direct transmission at least gives a medium throughput whereas a wrongly chosen relay transmission may give close to zero throughput, c.f. Fig. 5.

### C. Optimized policies

In addition to Simulation and MC model we now introduce a result curve named MC model - opt. policy, which uses the heuristic algorithm from Section IV to determine the optimal policy, with lost throughput as the optimization criterion.

Fig. 7 shows results for varying the movement speed. The nearly identical curves in 7(a) show that the impact of movement speed on lost throughput is accurately accounted for in the model. Further, since the optimal policy hardly brings a noticeable gain we can conclude that the standard policy (black dots in Fig. 5) is sufficient in this scenario. Fig. 7(b) shows the number of grid points in which relaying is preferred for the determined optimal policy \( \pi_{\text{opt}} \) and we see that direct transmissions are preferred more often as the movement speed increases. This result corresponds with the intuition that a wrongly chosen relay transmission may be more expensive than a wrong direct transmission, since the direct transmission at least gives a medium throughput whereas a wrongly chosen relay transmission may give close to zero throughput, c.f. Fig. 5.

Fig. 7. Varying movement speed for relaxed scenario A.

In Fig. 8 we show similar results, however for the considered challenging scenario with longer measurement update intervals. Again the model and simulation results are nearly identical, emphasizing the good model fit. The heuristically optimized policy brings a slight improvement compared to
the standard policy, which is achieved by further limiting the number of relay transmissions as shown in Fig. 8(b).

![Figure 8](image)

Fig. 8. Varying movement speed, for challenging scenario B.

In Fig. 9 we show the impact of varying the measurement update interval (i.e., $1/\tau$) for the relaxed and challenging scenarios (higher movement speed). As for movement speed, the impact of varying the update interval is accurately accounted for in the proposed model.

![Figure 9](image)

Fig. 9. Varying measurement update interval.

D. Model-based Adaptive Update Rate

A possible application of the proposed model is to use it for adjusting parameters such as the measurement update frequency according to the scenario conditions. Since measurement updates generate signaling overhead, it is desirable to be able to determine the update rate to achieve a required level of performance. In Fig. 10 we have used the MC model to calculate the update rate $\tau$ that is required to achieve a certain level of lost throughput, for varying movement speeds. The figure shows that $\tau$ is linearly dependent on the average movement speed, however the slope depends on how much lost throughput can be tolerated.

![Figure 10](image)

Fig. 10. Required $\tau$ for different mobility speeds to achieve certain levels of lost throughput.

VI. CONCLUSIONS AND OUTLOOK

In this paper we have proposed a Markov Chain based model of information collection and mobility impact on location based two-hop relaying. The model has been validated using simulations from previous work in [10], [4], for a scenario with static AP and destination nodes, but mobile relay node. Results show that the model accurately accounts for relay mobility and information collection delays. We have further proposed a heuristic algorithm to determine the optimal policy for when to transmit directly or relayed, which uses monotonicity and symmetry assumptions to reduce the policy search space, and make the problem computationally feasible. We have shown that the optimal policy only brings very slight improvements in the considered scenarios, suggesting that the standard non-optimal policy is sufficient. Finally, we have demonstrated how the proposed model can be applied to adapt, e.g., the measurement update frequency to varying scenario conditions according to a given performance requirement.

Future work includes validating the model in a broader range of scenarios, e.g., with cross-traffic to provoke higher values of transmission delay and transmission loss, which affect $\mu$ and $\kappa$ model parameters. Also, an extension of the model to account for location inaccuracies is envisioned. Further work will also focus on extending the model to account for non-static AP and destination nodes, which is envisioned through rotation and scaling of the used coordinate system.

REFERENCES