Grid-Connected of Photovoltaic Module Using Nonlinear Control

H. El Fadil, F. Giri, Josep M. Guerrero

Abstract—The problem of controlling single-phase grid connected photovoltaic (PV) system is considered. The control objective is fourfold: (i) asymptotic stability of the closed loop system, (ii) maximum power point tracking (MPPT) of PV module (iii) tight regulation of the DC bus voltage, and (iv) unity power factor (PF) in the grid. A nonlinear controller is developed using the backstepping design technique based on an averaged nonlinear model of the whole controlled system. The model accounts, on the one hand, for the nonlinear dynamics of the underlying boost converter and inverter and, on the other, for the nonlinear characteristic of PV panel. It is formally shown, through theoretical analysis and simulation results, that the proposed controller does achieve its objectives.

I. INTRODUCTION

With the deregulation of electricity markets and thrust to reduce greenhouse gas emissions from the traditional electric power generation systems, renewable energy resources such as wind turbines, photovoltaic panels, gas turbines and fuel cells, has gained a significant opportunity as new means of power generation to meet the growing demand for electric energy. Solar energy is considered to be one of the most useful natural energy sources because it is free, abundant, pollution-free, and most widely distributed. It can be used either at remote regions as standalone apparatus or in urban applications as grid interactive power source [1]. On the other hand, a significant technology progress has been achieved over the few past years increasing efficiency of solar cells. The rapid growth of solar industry has expanded the importance of PV systems making them more reliable and efficient, especially for utility power in distributed generation (DG) (see Fig. 1) at medium and low voltages power systems. Implementing distributed energy resources (DER), into interconnected grids could be part of the solution to meet the rising electricity demand ([2], [3]). DG technologies are currently being improved through several research projects toward the development of smart grids.

PV energy applications are divided into two categories: stand-alone systems and grid-connected systems. Stand-alone systems require a battery bank to store the PV energy; this is suitable for low-power systems. On the other hand, grid-connected PV systems do not require battery banks; they are resorted generally in high power applications. The main purpose of the grid-connected system is to extract the maximal quantity possible of solar array energy and restitute it to grid with a unity power factor, despite changing atmospheric conditions (temperature and radiation).

PV grid-connected systems represent the most important field applications of solar energy ([4], [5], [6]). In general, a photovoltaic grid-connected system can be seen as a two-stage grid-connected inverter (Fig.1). The first stage is a dc-dc converter controlled so that the photovoltaic system operates in optimal condition i.e. seeking maximum power point tracking (MPPT). The second stage is a dc-ac converter that controlled in a way that allows a grid connection with a unity power factor (PF). To this end, the output current (entering the grid) must be sinusoidal and in phase with the grid voltage. The dc-dc and dc-ac converters operate independently making easier the whole system control.

The present paper is focusing on the problem of controlling single phase grid-connected PV power generation systems (Fig. 1). The control objective is fourfold: (i) global asymptotic stability of the whole closed-loop control system; (ii) achievement of MPPT of the PV array; (iii) ensuring a grid connection with unity PF; and (iv) ensuring a tight regulation of the dc-bus voltage. These objectives must be met despite changes of the climatic variables (temperature and radiation).
To this end, a nonlinear controller is developed using the backstepping design technique based on large-signal nonlinear model of the whole system. A theoretical analysis is developed to show that the controller actually meets its objectives a fact that is confirmed by simulation.

The paper is organized as follows: the single phase grid connected PV system is described and modeled in Section II. Section III is devoted to controller design and analysis. The controller tracking performances are illustrated by numerical simulation in Section IV.

II. PRESENTATION AND MODELING OF GRID CONNECTED PV SYSTEM

A typical configuration of a single-phase grid connected photovoltaic system is shown in Fig. 2. It consists of a solar array, an input capacitor $C_i$, a boost dc-dc converter which is used for boosting the array voltage and achieving MPPT for PV array, a DC link capacitor $C_{dc}$, a single-phase full-bridge inverter (including four power semiconductors) that is based upon to ensure a DC-AC power conversion a unity PF, a filter inductor $L_k$, and an isolation transformer.

![Fig.2: Single phase grid connected PV system](image)

**A. PV array model**

Typical $(I_p - V_p)$ characteristics of solar cells arranged in $N_p$-parallel and $N_s$-series are analytically defined by the following standard equation:

$$I_p = N_p I_{ph} - N_p I_0 \left\{ \exp \left[ A \left( V_p + \frac{N_s I_p R_s}{N_p} \right) \right] - 1 \right\} - \frac{N_p}{R_{sh}} \left[ \frac{V_p}{N_s} + I_p R_s \right] - \frac{I_p}{N_p}$$

(1)

The meaning and typical values of the parameters in (1) can be found in many places (see e.g. [7], [8], [12]). To fix idea, the PV array module considered in this paper is the NU-183E1. The corresponding electrical characteristics are listed in Table I. The associated power-voltage (P-V) characteristics under changing climatic conditions (temperature and radiation) are shown in Figs. 3 and 4. These emphasize the maximum power points (MPP) M1 to M4, the coordinates of which are shown in Table II. The data in Table I and Table II will be used latter for simulation purpose.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Power</td>
<td>$P_m$</td>
<td>183.1W</td>
</tr>
<tr>
<td>Short circuit current</td>
<td>$I_{scr}$</td>
<td>8.48 A</td>
</tr>
<tr>
<td>Open circuit voltage</td>
<td>$V_{oc}$</td>
<td>30.1V</td>
</tr>
<tr>
<td>Maximum power voltage</td>
<td>$V_m$</td>
<td>23.9 V</td>
</tr>
<tr>
<td>Maximum power current</td>
<td>$I_m$</td>
<td>7.66A</td>
</tr>
<tr>
<td>Number of parallel modules</td>
<td>$N_p$</td>
<td>1</td>
</tr>
<tr>
<td>Number of series modules</td>
<td>$N_s$</td>
<td>48</td>
</tr>
</tbody>
</table>

![Fig.3: (P-V) characteristics of The PV module NU-183E1 with constant temperature and varying radiation](image)

![Fig.4: (P-V) characteristics of The PV module NU-183E1 with constant radiation and varying temperature](image)

**TABLE II: MAXIMUM POWER POINTS (MPP) IN FIG.3 AND FIG.4**

<table>
<thead>
<tr>
<th>MPP</th>
<th>$V_m$</th>
<th>$P_m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1</td>
<td>23.82 V</td>
<td>183.1 W</td>
</tr>
<tr>
<td>M2</td>
<td>22.94 V</td>
<td>70.8 W</td>
</tr>
<tr>
<td>M3</td>
<td>23.82 V</td>
<td>183.1 W</td>
</tr>
<tr>
<td>M4</td>
<td>20.38 V</td>
<td>153.4 W</td>
</tr>
</tbody>
</table>

**B. Overall system model**

The control input $u_1$ and $u_2$ of the boost converter and for the inverter, respectively, are a PWM signals taking values in the set $\{0,1\}$. Applying the Kirchoff’s laws, successively with $u_1 = 1$ and $u_1 = 0$, to the boost converter circuit and with $u_2 = 1$ and $u_2 = 0$, to the inverter circuit of of Fig. 2 one obtains the following averaged model [9]:

![Table I: ELECTRICAL SPECIFICATIONS FOR THE SOLAR MODULE NU-183E1](image)
\[
\frac{dx_1}{dt} = \frac{1}{C_i} (i_p - x_2) \tag{2a}
\]
\[
\frac{dx_2}{dt} = -(1 - \mu_1) \frac{x_3}{L_i} - \frac{R_i}{L_i} x_2 + \frac{x_1}{L_i} \tag{2b}
\]
\[
\frac{dx_3}{dt} = (1 - \mu_1) \frac{x_2}{C_{dc}} - (2\mu_2 - 1) \frac{x_4}{C_{dc}} \tag{2c}
\]
\[
\frac{dx_4}{dt} = (2\mu_2 - 1) \frac{x_3}{L_g} - \frac{R_g}{L_g} x_4 - \frac{e_s}{L_g} \tag{2d}
\]
where \( R_i \) is the equivalent series resistance (ESR) of input inductance \( L_i \).

and where \( x_1, x_2, x_3, x_4, i_p, \mu_1 \) and \( \mu_2 \) denote the average values of the system variable (Table III). All variables averaging are performed over switching periods. Consequently, the quantities \( \mu_1 \) and \( \mu_2 \), commonly called duty ratios, vary continuously in the interval \([0,1]\) and act as the input control signals.

<table>
<thead>
<tr>
<th>Physical variable</th>
<th>Its Averaged Version</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV voltage ( v_p )</td>
<td>( x_1 )</td>
</tr>
<tr>
<td>inductor current ( i_{Li} )</td>
<td>( x_2 )</td>
</tr>
<tr>
<td>dc bus voltage ( v_{dc} )</td>
<td>( x_3 )</td>
</tr>
<tr>
<td>grid current ( i_{Lg} )</td>
<td>( x_4 )</td>
</tr>
<tr>
<td>PV current ( i_p )</td>
<td>( i_p )</td>
</tr>
<tr>
<td>boost converter binary control input ( u_1 )</td>
<td>( \mu_1 )</td>
</tr>
<tr>
<td>inverter binary control input ( u_2 )</td>
<td>( \mu_2 )</td>
</tr>
</tbody>
</table>

III. CONTROLLER DESIGN

In this section, we aim at designing a controller that will be able to ensure: (i) a global stability of closed loop system, (ii) a perfect MPPT (whatever the position of the PV panel).

Specifically, the controller must enforce the voltage \( x_1 \) to track, as accurately as possible, the unknown (and slowly varying) voltage \( V_m \). Note that \( V_m \) is unknown because it depends on the temperature \( T \) and solar variation \( \lambda \) and these are not supposed to be measured; (iii) a unity PF in the grid; and (iv) a tight regulation of the dc bus voltage \( x_3 \).

A. Controlling the boost converter to meet MPPT

Recall that the control objective is to enforce the voltage \( x_1 \) to track the optimal point \( V_m \). To this end, the backstepping design principles are invoked [10].

**Design Step 1.** Let us introduce the following tracking error:
\[
z_1 = x_1 - V_m \tag{3}
\]
Achieving the tracking objective amounts to enforcing the error \( z_1 \) to vanish. To this end, the dynamics of \( z_1 \) have to be clearly defined. Deriving (3), it follows from (2a) that
\[
\dot{z}_1 = \frac{1}{C_i} (i_p - x_2) - \dot{V}_m \tag{4}
\]

In the above equation, the quantity \( x_2 / C_i \) stands as a virtual control variable. Let us consider the following Lyapunov function
\[
V_i = 0.5z_1^2 \tag{5}
\]
The time-derivative of \( V_i \) along the trajectory of (4) is
\[
\dot{V}_i = z_1 \left( \frac{-x_2}{C_i} + \frac{i_p}{C_i} - \dot{V}_m \right) \tag{6}
\]
Equation (6) shows that the tracking error \( z_1 \) can be regulated to zero if \( \dot{z}_1 = \frac{x_2}{C_i} - \alpha_1 \) where \( \alpha_1 \) is a stabilizing function defined by
\[
\alpha_1 = \frac{i_p}{C_i} + c_1 z_1 - \dot{V}_m \tag{7}
\]
where \( c_1 > 0 \) is a design parameter. Since \( x_2 / C_i \) is not the actual control input, one can only seek the convergence of the error \( x_2 / C_i - \alpha_1 \) to zero. We then define the following second error variable:
\[
z_2 = \frac{x_2}{C_i} - \alpha_1 \tag{8}
\]
The next step is to determine a variation law for control signal \( \mu_1 \) so that the set of errors \( z_1 \) and \( z_2 \) vanish asymptotically. But, let us first establish some useful equations. Equation (4) becomes, using (8)
\[
\dot{z}_1 = -c_1 z_1 - z_2 \tag{9}
\]
Also, the derivative (6) of the Lyapunov function is rewritten
\[
\dot{V}_i = -c_1 z_1^2 - z_1 z_2 \tag{10}
\]

**Design Step 2.** The objective now is to enforce the error variables \( (z_1, z_2) \) to vanish. To this end, let us first determine the dynamics of \( z_2 \). Deriving (8) and using (2b), and (9), one obtains
\[
\dot{z}_2 = -(1 - \mu_1) \frac{x_3}{L_i C_i} - \frac{R_i}{L_i C_i} x_2 + \frac{x_1}{L_i C_i} - \frac{1}{C_i} \frac{di_p}{dt} + c_1 z_1 + c_1 z_2 + \dot{V}_m \tag{11}
\]
We are finally in a position to make a convenient choice of the control signal \( \mu_1 \) to stabilize the whole system with state vector is \( (z_1, z_2) \). Consider the augmented Lyapunov function candidate
\[
V = V_i + 0.5z_2^2 = 0.5z_1^2 + 0.5z_2^2 \tag{12}
\]
Our goal is to make \( \dot{V} \) non-positive definite
\[
\dot{V} = \dot{V}_i + z_2 \dot{z}_2 = -c_1 z_1^2 - c_2 z_2^2 + z_2 \left[ -z_1 + c_2 z_2 + \dot{z}_2 \right] \tag{13}
\]
where \( c_2 > 0 \) is a design parameter. Equation (13) shows that the equilibrium \( (z_1, z_2) = (0,0) \) is globally asymptotically stable if
\[ \dot{z}_2 = -c_2 z_2 + z_1 \]  
Combining (11) and (14) one gets the following control law 
\[ \mu_1 = 1 - \frac{1}{x_3} \left[ L_g C_i \left( c_1^2 - 1 \right) z_1 + \left( c_1 + c_2 \right) z_2 + \dot{V}_m \right] \]
\[ + x_1 - R_i x_2 - L_A i_p \]  
\[ (15) \]

B. Generation of the optimal voltage \( V_m \)

The MPP \((V_m, P_m)\) is reached when:
\[ \frac{\partial P}{\partial V_p} |_{v_p = V_m} = 0 \]  
\[ (16) \]

with \( P = v_p i_p \) being the PV power. That is, the controller must keep \( dP/dv_p \) equals zero, whatever the radiation \( \lambda \) and the temperature \( T \), by acting on the duty cycle \( \mu_1 \). Then, the optimal voltage \( V_m \) can be generated using a PI regulator as shown in Fig. 5. This regulator (denoted PI-1 in Fig. 5) is defined as follows:
\[ V_m = G_1(s) e_p \]  
\[ (17a) \]
where 
\[ G_1(s) = k_1 \frac{1 + \tau_1 s}{\tau_1 s} \]  
\[ (17b) \]
\[ e_p = \frac{dP}{dv_p} \]  
\[ (17c) \]
where \( s \) denotes the Laplace variable.

![Optimal voltage generation diagram](image)

\[ \text{Fig.5: Optimal voltage generation diagram} \]

C. Controlling the inverter to meet unity PF in the grid and regulation of the dc bus voltage objectives

1) Unity PF objective

The unity PF objective means that the grid current \( i_{L_g} \) should be sinusoidal and in phase with the AC grid voltage \( e_g \). We therefore seek a regulator that enforces the current \( x_4 \) to track a reference signal \( x_4^* \) of the form:
\[ x_4^* = \beta e_g \]  
\[ (18) \]
At this point \( \beta \) is any real positive parameter that is allowed to be time-varying. The regulator will now be designed using the backstepping technique [10]. Let us introduce the following current error:
\[ z_3 = x_4 - x_4^* \]  
\[ (19) \]
In view of (2d), the above error undergoes the following dynamics:
\[ \dot{z}_3 = \dot{x}_4 - \dot{x}_4^* = (2\mu_2 - 1) \frac{x_3}{L_g} - \frac{R_g}{L_g} x_4 - \frac{e_g}{L_g} - \dot{x}_4^* \]  
\[ (20) \]

To get a stabilizing control law for this first-order system, consider the quadratic Lyapunov function:
\[ V_3 = 0.5 z_3^2 \]  
\[ (21) \]
It can be easily checked that the time-derivative \( \dot{V}_3 \) is a negative definite function of \( z_3 \) if the control input \( \mu_2 \) is chosen to be:
\[ \mu_2 = \frac{1}{2} + \frac{1}{2 x_3} \left( R_g x_4 + e_g + L_g \left( c_3 z_3 + x_4^* \right) \right) \]  
\[ (22) \]
where \( c_3 > 0 \) is a design parameter. Indeed, with this choice one has
\[ z_3 = -c_3 z_3 \]  
\[ (23) \]
\[ \dot{V}_3 = -c_3 z_3^2 \]  
\[ (24) \]
This means, in particular, that the equilibrium \((z_3 = 0)\) is globally asymptotically stable and therefore the unity PF is asymptotically achieved.

2) DC bus voltage regulation objective

Now, the aim is to design a variation law for the ratio \( \beta \) in (18) so that the inverter dc input voltage \( x_3 = v_{dc} \) is steered to a given constant reference \( V_d > 0 \). To this end, the following PI control law (called PI-2) is used:
\[ \beta = G_2(s) e_{dc} \]  
\[ (25a) \]
where 
\[ G_2(s) = k_2 \frac{1 + \tau_2 s}{\tau_2 s} \]  
\[ (25b) \]
\[ e_{dc} = x_3 - V_d \]  
\[ (25c) \]
The performances of the controller, consisting of the control laws (25a-c), (22) and (15) are described in the following theorem.

**Theorem 1 (main result).** Consider the single-phase grid-connected PV system shown in Fig. 2, represented by its average model (2a-d), together with the controller consisting of the control laws (15), (22) and (25a-c). Then, one has the following results:

i) The closed loop system is globally asymptotically stable.

ii) The tracking error \( z_3 = x_3 - V_m \) vanishes exponentially, implying MPPT achievement (because \( V_m \) is the optimal PV voltage corresponding to maximum power i.e. \( dP/dv_p = 0 \)). Interestingly, \( V_m \) is computed online according to the law (18a-c).

iii) The error \( z_3 = x_4 - x_4^* \) converges to zero ensuring a unity PF (because \( x_4^* = \beta e_g \)).

iv) The tracking error \( e_{dc} = x_3 - V_d \) converges to zero guaranteeing a tight regulation of the dc bus voltage.
IV. SIMULATION RESULTS

The theoretical performances, described by Theorem 1, of the proposed nonlinear controller are now illustrated by simulation. The experimental setup, described by Fig. 6, is simulated using MATLAB. The characteristics of the controlled system are listed in Table IV. Let us emphasize that the controlled system is simulated using its instantaneous model. The averaged model (2a-d) is in effect used only in controller design. The control design parameters are given values of Table V which proved to be convenient. The resulting closed-loop control performances are illustrated by Fig 7 to Fig 10.

![Diagram Fig.5](image)

**TABLE IV: CHARACTERISTICS OF CONTROLLED SYSTEM**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PV array</td>
<td>PV power</td>
<td>183.1W</td>
</tr>
<tr>
<td></td>
<td>PV model</td>
<td>NU-183E1</td>
</tr>
<tr>
<td>Boost converter</td>
<td>C</td>
<td>4700µF</td>
</tr>
<tr>
<td></td>
<td>L</td>
<td>1mH</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>0.65Ω</td>
</tr>
<tr>
<td>DC link capacitor</td>
<td>Cdc</td>
<td>6800µF</td>
</tr>
<tr>
<td>Grid filter inductor</td>
<td>Le</td>
<td>2.2mH</td>
</tr>
<tr>
<td>PWM</td>
<td>Transformer frequency</td>
<td>25kHz</td>
</tr>
<tr>
<td>Grid</td>
<td>Transformer ratio</td>
<td>22:220</td>
</tr>
<tr>
<td></td>
<td>AC source</td>
<td>220V</td>
</tr>
<tr>
<td></td>
<td>Line frequency</td>
<td>50Hz</td>
</tr>
</tbody>
</table>

**TABLE V: CONTROLLER PARAMETERS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design parameters</td>
<td>c1</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>c2</td>
<td>10^4</td>
</tr>
<tr>
<td></td>
<td>c3</td>
<td>10^4</td>
</tr>
<tr>
<td>PI regulator (PI-1)</td>
<td>k1</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>r1</td>
<td>10ms</td>
</tr>
<tr>
<td>PI regulator (PI-2)</td>
<td>k2</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>r2</td>
<td>30ms</td>
</tr>
<tr>
<td>Desired DC bus voltage</td>
<td>Vdc</td>
<td>48V</td>
</tr>
</tbody>
</table>

400 W/m^2 and 1000 W/m^2, meanwhile the temperature is kept constant, equal to 298.15K (i.e. 25°C). The figure shows that the captured PV power varies between 70.8 W and 183.1 W. These values correspond (see Fig. 3) to the maximum points (M2 and M1) of the curves associated to the considered radiations, respectively. The figure, also shows that the DC bus voltage Vdc is regulated to its desired value Vd=48V. Fig. 8 illustrates the grid current iLg and the grid voltage eg. This figure clearly shows that the current iLg is sinusoidal and in phase with the voltage eg, proving unity PF achievement.

B. Temperature variation effect

Fig. 9 illustrates the controller behavior when facing temperature changes. Specifically, the temperature T varies between 298.15K and 333.15 K (i.e. between 25°C and 60°C), while the radiation λ is constant equal to 1000 W/m^2. It is seen that the controller keeps the whole system at the optimal operation conditions. Indeed, the captured PV power P achieves the values 183.1W or 153.4W corresponding (on the power curves of Fig. 4) to the maximum points (M3 and M4) associated to the temperatures 333.15K and 298.15K, respectively. The figure also shows that the DC bus voltage Vdc is regulated to its desired value Vd=48V. Fig. 10 illustrates the grid current iLg and the grid voltage eg. This figure also shows that the current iLg is sinusoidal and in phase with the voltage eg, which proves the unity PF achievement.

V. CONCLUSION

The problem of controlling a single-phase grid connected PV system has been considered. The controller is obtained from the nonlinear average model (3) using nonlinear control technique. Using both a theoretical analysis and simulation, it is proved that the controller does meet the performances for which it was designed, namely:

(i) global asymptotic stability of the closed-loop system,
(ii) perfect maximum power point tracking of PV array,
(iii) good unity power factor in the grid,
(iv) tight regulation of the DC bus voltage.

![PV Voltage Vp(V)](image)
![PV Power P(W)](image)
![Radiation flux J (W/m^2)](image)
![DC bus voltage Vdc(V)](image)

Fig.7: MPPT and DC bus voltage behavior in presence of radiation changes.
Fig. 8: Unity PF behavior in presence of radiation changes

Fig. 9: MPPT and DC bus voltage behavior in presence of temperature changes.

Fig. 10: Unity PF behavior in presence of temperature changes

REFERENCES