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Research, Practice and Responsibility

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Research, Practice and Responsibility

by
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Abstract. Three issues concerning the relationship between research and practice are addressed. (1) A certain ‘prototype mathematics classroom’ seems to dominate the research field, which in many cases seems selective with respect to what practices to address. I suggest challenging the dominance of the discourse created around the prototype mathematics classroom. (2) I find it important to broaden the school-centred discourse on mathematics education and to address the very different out-of-school practices that include mathematics. Many of these practices are relevant for interpreting what is taking place in a school context. That brings us to (3) socio-political issues of mathematics education. When the different school-sites for learning mathematics as well as the many different practices that include mathematics are related, we enter the socio-political dimension of mathematics education.

On the one hand we must consider questions like: Could socio-political discrimination be acted out through mathematics education? Could mathematics education exercise a regimentation and disciplining of students? Could it include discrimination in terms for language? Could it include sexism and racism? On the other hand: Could mathematics education bring about competencies which can be described as empowering, and as supporting the development of mathematical literacy or a ‘mathemacy’, important for the development of critical citizenship?

However, there is no hope for identifying a one-way route to mathemacy. More generally: There is no simple way of identifying the socio-political functions of mathematics education. Mathematics education has to face uncertainty.

In order to make a start we ignore the fact that both ‘research’ and ‘practice’ are highly complex terms to which no specific meaning can be assigned.¹ They are open; they are contested; they are what Discourse Theory have called ‘undecidable’.² Their meanings can be acted out in many and also radically different ways. I do not think of practice as first of all the teachers’ practice, as for instance, suggested by Malara and Zan (2002: 555) when they refer to the distance between theory and practice as a distance between “a corpus of knowledge on mathematics education in the hands of researchers” and “the actual teaching carried out by teachers”. Other practices are related to mathematics education: the

¹ A practice could be considered a cultural practice or, in other words, a socio-political practice. This could be emphasised by talking about ‘praxis’ instead of ‘practice’. I will, however, keep using ‘practice’.

² See, for instance, Torfing (1999).

students' practices in the classroom, the students' practices outside the classroom, daily-life practices, and professional practices.

It is possible to consider research, as expressed in theory, as being ahead of practice. If so, the question becomes to what extent it is possible for research to influence practice. If we have mathematics education in mind, a key-question becomes how research results, which may have clarified aspects of learning mathematics, could be *implemented* in practice. We might, however, also consider the inverse situation. A celebration of practice is also a possibility, and then the question becomes: How might different practices be *reflected* in research and theoretical insight?

No doubt the relationship between theory and practice is much more complex than that. Neither research nor practice need to be straightforward signifiers of progress. The point could be how a dynamic relationship between research and practice is acted out. However, terms like 'improvement' and 'progress' are also highly problematic when we talk about changes in a field like mathematics education. Such terms might only have well-defined meanings within the horizon of, say, Modernity or Enlightenment, while beyond such a horizon their meanings may be dissolved in the acid bath of relativism.³ 'Progress' is a term that finds applications only within a framework enveloped in assumptions. What is even more doubtful is to ask for the meaning of 'progress' within a specific field like mathematics education.⁴

Let me now comment on three issues concerning the relationship between research and practice within the field of mathematics education. (1) A certain 'prototype mathematics classroom' seems to have dominated the research field, which in many cases seems to have been rather selective with respect to what practices to address. I suggest challenging the dominance of the discourse created around the prototype mathematics classroom. In this way I suggest that attention is paid to the variety of sites for learning mathematics, i.e. to the very many different practices that include a learning of mathematics. (2) I find it important to broaden the school-centred discourse on mathematics education and to address the very different out-of-school practices that include mathematics. Furthermore, many of these practices are relevant for interpreting

³ In his classic study Bury (1955) states that the idea of progress emerged with Modernity and was unfolded during the Enlightenment. The idea might be condensed into the assumption that if we ensure scientific development, we ensure progress on a grander scale. Scientific progress can be seen as the 'engine' for progress in general. See also Nisbet (1980) for a careful discussion of the idea of progress.

⁴ I use 'mathematics education' in different ways. Sometimes it refers to processes of learning mathematics, or to what in German is called *Mathematikunterricht*. Sometimes it refers to considerations and studies of processes of learning mathematics, or to what in German is called *Didaktik der Mathematik*. I hope that the context in which I use 'mathematics education' might help to clarify its meaning.

what is taking place in a school context. That brings us to (3) socio-political issues of mathematics education. When the different school-sites for learning mathematics as well as the many different practices that includes mathematics are related, for instance by considering possible transitions between such practices, we enter the socio-political dimension of mathematics education. The principal issues are: Could a socio-political discrimination be acted out through mathematics education? How could mathematics education ensure an empowerment?

It could make good sense to explore the relationship between research and practice within both the community of researchers and the community of practitioners. However, when, in what follows, I discuss this relationship, I do so first of all with reference to the community of the researchers, as I look at how the relationship has been acted out according to research documentation. My approach is asymmetrical. And there are further limitations: Certainly, I look at things from a particular perspective, both geographically and theoretically. Not to mention the general limitation of my own knowledge regarding the whole field. Anyway, I try to look around the world as best as I can.⁵

Let me make a few comments about my perspective on mathematics education. First, I find mathematics education to be a *significant* social system to the extent that it has socio-political and economic impact. Thus, mathematics education could be of interest for a globalising learning economy.⁶ It could be of importance for productivity. Mathematics education could provoke both exclusion and suppression; as observed by Valerie Walkerdine (1988, 1989), girls could be ‘counted out’. Mathematics education may operate as a secret weapon of Western imperialism, as indicated by Alan Bishop (1990), or as part of cultural colonisation, as observed by Ubiratan D’Ambrosio (2001) and Arthur Powell (2002). It could also be that mathematics education, when organised in an adequate form, ensures empowerment and provides a basis for critical citizenship, as proposed by Marilyn Frankenstein (1989, 1995). To me all such observations indicate that mathematics education is significant. It could make a difference. As mathematics education can be acted out in many different ways, it could produce ‘wonder’ as well as ‘horrors’.⁷ It could generate discrimination as well as empowerment. In this sense, I regard mathematics education as being not only significant but also *undetermined*. It has no a priori ‘essence’, which implies that, basically speaking, it becomes what it is doing. Its ‘essence’ is

⁵ Research in mathematics education is globalised, as discussed by Atweh and Clarkson (2001); and Atweh, Clarkson and Nebres (2003). This globalisation seems also to include a dominance of the English language, implying that an educational idea only exists when it is presented in English. Although I am aware of this problem, I am afraid that this very presentation also suffers from this distortion and limitation.

⁶ See, for instance, Archibugi and Lundvall (Eds.) (2001). See also Woodrow (2003).

⁷ See D’Ambrosio (1994) for a similar remark about mathematics.

produced as it is being acted out. For me the significance and the indeterminism of mathematics education bring about an *uncertainty*, which effects both research and practice as well as their relationship. To me this uncertainty makes socio-political awareness important, as there is no predetermined function of mathematics education. Furthermore, I find that the notions of uncertainty and responsibility are connected, to which I shall return later.

1. The variety of sites for learning mathematics

I find that the research discourse in mathematics education has been dominated by the *prototype mathematics classroom*. In this classroom students are willing to learn, and the classroom is well equipped. This makes it possible through research to develop conceptions of learning mathematics which are not receptive to the many conflicts that often form part of the context of learning. Instead we have to deal with theoretical constructs that reflect the ‘format’ of the prototype mathematics classroom. This could make the developed interpretations less sensitive to the reality of the many other scenes for learning mathematics. Therefore, I find it important to look beyond the prototype mathematics classroom. Much research has already done so, and I will come back to this. My worry concerns the extent to which research grounded in the proto-type mathematics classroom becomes a defining element of the research paradigm. However, when we address the very different sites of learning mathematics, the possible ‘horrors’ and ‘wonders’ of mathematics education can be better identified.

In 1.1, I want to broaden the perspective on mathematics education by commenting on globalisation and ghettoising. In 1.2, I try to point out some of the many sites for learning mathematics beyond the prototype mathematics classroom. In 1.3, I present some example of activities and considerations that address such other sites.

1.1 Globalisation and Ghettoising

Globalisation has some attractive connotations. It refers to the opening up of borders, and it can include a sense of being together and sharing concerns for each other. However, globalisation may simply refer to the fact that new connections are established between previously unconnected social entities; and that what is happening and being done by one group of people may affect, for good or for bad, a completely different group of people, even those unaware of the nature of the effect. The concept of globalisation contains both positive and negative connotations: “For some, ‘globalization’ is what we are bound to do

if we wish to be happy; for others ‘globalization’ is the cause of our unhappiness. For everybody, though, ‘globalization’ is the intractable fate of the world, an irreversible process ...” (Bauman, 1998: 1). Furthermore: “Globalization divides as much as it unites; it divides as it unites – the causes of division being identical with those which promote the uniformity of the globe.” (Bauman, 1998: 2)

In the *End of Millennium*, Manuel Castells defines social exclusion, “as the process by which certain individuals and groups are systematically barred from access to positions that would enable them to an autonomous livelihood within the social standards framed by institutions and values in a given context” (Castells, 1998: 73). Next, Castells makes the following observation which causes him to consider the notion of the ‘Fourth World’:

“This widespread, multiform process of social exclusion leads to the constitution of what I call [...] the black holes of informational capitalism. These are the regions of society from which, statistically speaking, there is no escape from the pain and destruction inflicted on the human condition of those who, in one way or another, enter these social landscapes.” (Castells, 1998: 162) The Fourth World makes up the black holes of informational capitalism. It comprises large areas of the globe, and Castells mentions regions of Sub-Saharan Africa, and impoverished rural areas of Latin America and Asia. Then he adds: “But it is also present in literally every country, and every city, in this new geography of social exclusion. It is formed of American inner-city ghettos, Spanish enclaves of mass youth unemployment, French banlieues, warehousing North Africans, Japanese Yoseba quarters, and Asian mega-cities’ shanty towns. And it is populated by millions of homeless, incarcerated, prostituted, criminalized, brutalized, stigmatized, sick and illiterate persons. They are the majority in some areas, the minority in others, and a tiny minority in a few privileged contexts. But, everywhere, they are growing in number, and increasingly in visibility, as the selective triage of informational capitalism, and the political breakdown of the welfare state, intensify social exclusion. In the current historical context, the rise of the Fourth World is inseparable from the rise of informational, global capitalism.” (Castells, 1998: 164-165)

Knowledge production, the administration of knowledge, knowledge development, learning and educational planning contribute to the processes of globalisation, but also of its accompanying process, that of ghettoising. Economy and sociological studies have recognised knowledge development (and therefore learning, and information management) as economic factors. As mentioned, we can talk about a globalising learning economy. This also applies to mathematics education, which can help to include (some) people in the world-wide informational economy,⁸ but which might also serve as a mechanism of

⁸ I prefer to use the term ‘informational economy’ and not ‘information economy’. This is due to observations made by Castells (1999), who, in order not to assume some simplicities associated with the notion ‘information economy’, chooses to elaborate on ‘informational economy’.

exclusion of (some) people. It forms part of the universal processes of inclusion and exclusion, including the production of the Fourth World.

1.2 Beyond the prototype mathematics classroom

Following the classification in UNESCO (2000), the world can be roughly grouped in: (1) North America, Western Europe, Australia, Japan, New Zealand; (2) Sub-Saharan Africa, Latin America and the Caribbean, Eastern Asia and the Pacific, South and Western Asia, Arab States and North Africa; and (3) Central Asia and Central and Eastern Europe.⁹ According to the statistical documentation, the distribution in 1998 of 6 to 11 years old children for the three different regions was: (1) 10 %; (2) 86 %; and (3) 4 %. Furthermore it was shown that 16% of the world's children were not going to school.

Naturally, far from all the 10% of the children belonging to the region (1) find themselves in prototype classrooms, and certainly there are plenty of prototype classrooms also for many of the children belonging to the other two regions. But the numbers may anyway indicate that what has been characterised as a prototype mathematics classroom seems to belong to a small minority of the sites for learning mathematics. The well-equipped classrooms from countries ranking high on the world's welfare scale make a minor site for learning mathematics. And, it should be noted that the total number of children from region (1) and (3) is smaller than number of the world's children not in school.

Reconsidering the notion of the Fourth World, we should not be surprised to find very different sites for learning mathematics existing side by side. All groups of children, from those living in the streets to those who frequent extremely well-resourced schools, can be found in the same metropolis. The same geographical unit, however, might be criss-crossed by lines, bars, walls, separating those who have too much from those who have too little.

Naturally, the extent to which a particular research addresses one or another situation is not well-defined. Nevertheless, the research discourse in mathematics education appears biased as, to a large extent, it has been addressing well-to-do sites for mathematics teaching and learning. As already indicated, in such classrooms there are no students who lack interest in learning mathematics and disturb the classroom peace. There are no hungry children. There is no violence. No police are stationed on the school-premises. There are seldom students with a cultural background foreign to those born in the country (who, for brevity, I refer to as immigrant students). The prototype classroom is

⁹ According to statistical and economic stereotyping these regions are referred to as (1) more developed regions; (2) less developed regions; and (3) countries in transition.

homogeneous. It is well-equipped. Furthermore, it appears that the prototype mathematics classroom can only be found in certain countries. If research, however, addresses ‘other sites’ for learning mathematics, this will often be explicitly stated in the title of the research paper, like ‘Chinese children’s conception of proportionality’, while in the report of a study from say, an English or even a Danish classroom, the title may have been ‘Children’s conception for proportionality’. Considering their number, however, Chinese children should represent the world’s children pretty well. If we skip the classroom with violence, poverty, cultural conflicts, etc. we might skip a vast majority of the world’s classrooms.¹⁰

What we could call the prototype-bias of research in mathematics education concerns a whole set of underlying paradigmatic priorities and presumptions. Not only does the prototype classroom represent a minority of classrooms, it also signifies a discursively constructed classroom, where the students could be expected to behave in a particular way. Paola Valero (2002) has characterised such students as ‘schizo-mathematics-learners’, who do not seem to express any real-life interest, but appear like a stereotype of the ‘epistemic subject’ as characterised by Jean Piaget in his elaboration of a genetic epistemology (see Beth and Piaget, 1966). When such an idealised learner has been conceptualised, it becomes possible to sort out what data may count as relevant, and what not, for a further understanding of learning mathematics. And if we consider the total number of transcripts which over time have been presented to the research community (in all kinds of reviewed research publications), then it appears that a high degree of (self-)censorship must have operated. Only a few cases give voice to clearly obstructive or violent students. Strong paradigmatic criteria of relevance must have existed, reflecting preoccupation with the prototype mathematics classroom. If we try to move beyond this paradigmatic preoccupation, then we face issues like the ‘disruption for data’ and the notion of generalisability and validity have to be rethought.¹¹

We can easily provide suggestions for explanations of this prototype-bias. If we consider that research in mathematics education is a costly affair, we should expect that it addresses issues that can obtain funding. We can expect progress in research that produces results that can be transformed into learning materials, textbooks, computer programmes, etc. Research in mathematics education is sensitive to the system of demand and supply scheme.¹² Reconsidering the economic perspective, there is nothing surprising (nor much praiseworthy) in the bias of mathematics education. Let me finally emphasise that I am

¹⁰ A preliminary empirical underpinning of this point can be developed along the same lines as the survey presented in Skovsmose and Valero (2002b).

¹¹ See Vithal and Valero (2003); and Vithal (2000).

¹² For comments on the political economy of mathematics education, see Skovsmose and Valero (2002a).

talking about a paradigmatic bias that has to with priorities acted out by the research community. Thus, I find it important that *also* the prototype mathematics classroom is researched (and much of my own research relates to this prototype). This is an important site for learning mathematics, although only one site amongst many others.

1.3 Examples

A non-prototype classroom could have an overwhelming number of students, it could be located in a poor neighbourhood, it could be infected by violence. The non-prototype classroom could be located in cultural settings, which according to paradigmatic norms in mathematics education could be counted as ‘foreign’. Insecurity could dominate a non-prototype classroom. We find classrooms located in almost war-like situations. Jeanne Albert and Intisar Natsheh (2002) refer to the Middle East Children Association (MECA) that was established in 1996 jointly by Israeli and Palestinian educators to promote the peace process. In 2001 MECA started a group for mathematics teachers in elementary schools. Albert and Natsheh refer to practices so different from what we normally address in research: “Israeli teachers who work in schools whose neighbourhood is being shelled reported that the children quickly became accustomed to the situation, and the lessons progressed as before.” (2002: 127). This observation surprises me. Furthermore, they observed that the Palestinian teachers found that the students’ motivation was low: “The students complained of the lack of connection between mathematics and their ‘real life’.” (2002: 75). This observation, however, does certainly not surprise me.

Let me, however, address a couple of examples more carefully – and less extreme in being non-prototypical. Eric Gutstein (2003b) describes and analyses mathematics education for social justice in an urban, Latino school in the USA.¹³ This exemplifies what a close interaction, if not a unity, could mean between research and practice. He taught a seventh grade mathematics class from November 1997 to the end of the school year, and then he moved with the class to eighth grade in 1998-1999. The school, referred to as the Diego Rivera School (pseudonym), is located in a working class, Mexican and Mexican-American community in Chicago.

The Diego Rivera School is, in many ways, a typical Chicago public school, and the students must meet all the required tests. Students wear uniforms, which reduces the economic pressure on families, and it also helps to keep the school in ‘neutral territory’ with respect to gangs. That is, in Chicago, certain clothing styles and colours are associated with particular street gangs that are present in the Rivera neighbourhood.

¹³ Gutstein has provided me with supplementary information for the present description. See also Gutstein (2001, 2003a, in press).

Uniforms minimize “representation” of gangs in the school. Most students live in the neighbourhood and attend the neighbourhood high school but there is a dropout rate of over 50%. The class with 26 students that Gutstein was teaching was ‘demographically representative’ for the school. All students were from Latino, immigrant, working-class families. About half were born in the USA, and the rest in Mexico except for one student from the Dominican Republic and another whose family was from Puerto Rico. Spanish was the first language of all students, and all but one was fluent in English as well.

I consider the class to exemplify a non-prototype classroom for several reasons, and let me mention three. First, the school is located in a poor area (although not in an extremely poor one), which has implications for the resources that are available in the school, for instance in terms of computers and access to the internet. The poverty of the neighbourhood could have implications for the students’ possibilities of doing homework and for getting support at home. An important issue with Rivera families is that many parents either do not speak English well enough to help their children with homework or were themselves denied educational opportunities in Mexico, and thus have little formal schooling. When a community organisation surveyed the neighbourhood some years back, they found that two thirds of the adults did not have a high school education, extremely high for a United States community. However, Gutstein also found that there were many culturally specific ways that Rivera families supported the education of their children. Although the neighbourhood need might not be part of the Fourth World, as described by Castells, it might be located in such close vicinity to it, that there are implications for the life opportunities available to the students. Second, I consider the classroom to be non-prototypical because it contains immigrant students. In the Diego Rivera School, the immigrant students are a majority within the school and within the neighbourhood, while being a minority within society as a whole. Naturally, there are many cases, in Denmark for instance, where immigrant students make up a minority in both school and neighbourhood. In both cases I would, however, talk about non-prototype mathematics classrooms. One particular issue for immigrant students has to do with the opportunities they experience in the socio-political context as being their realistic opportunities in future life. They might experience a restricted set of opportunities, even when they belong to a local majority as is the case of students from the Diego Rivera School. They might have access only to what could be called ‘immigrant-opportunities’, which might be very restricted compared to the opportunities experienced by non-immigrant students. Third, I consider the classroom from the Diego Rivera School to be non-prototypical, as the experience of violence is part of the daily reference for many students. I do not simply think of violence as necessarily being experienced directly by the students, but as being so present in their environment that it becomes part of the way students makes sense of

what they are doing and learning in school. Thus students at the Diego Rivera School are quite familiar with stories of undocumented relatives (those without legal immigration papers) ‘crossing over’ (i.e., sneaking in) to the USA, or of ‘la migra’, the USA Immigration and Naturalization Service, conducting raids on factories or farms to look for, and round up, undocumented workers who are known to toil in the sweatshop, non-unionised conditions of semi-slavery. These forms of violence permeate the consciousness of Rivera students.

Naturally, research on a non-prototype mathematics classroom can focus on very many issues: the violence, poverty, immigration and discrimination in general. However, in his presentation of Diego Rivera School, Gutstein describes first of all in what sense it is possible through mathematics education to develop a ‘conscientização’ that the students can experience an empowerment and what a learning for social justice could mean. Paulo Freire (1972) has introduced the notion of ‘conscientização’, referring to the power of reading the world as being open to change. Reading the world drawing on mathematical resources means, according to Gutstein, to use mathematics to “understand relations of power, resource inequities, and disparate opportunities between different social groups and to understand explicit discrimination based on race, class, gender, language, and other differences. Further, it means to dissect and deconstruct media and other forms of representation and to use mathematics to examine these various phenomena in one’s immediate life and in the broader social world and to identify relationships and make connections between them.” (Gutstein, 2003b, p. 45) This effort can be expressed as a concern for supporting the students’ development of ‘mathematical literacy’ or of ‘mathemacy’. In Section 3.3, I will return to a discussion of these notions. However, it is already clear that we cannot talk about ‘mathematical literacy’ in a uniform way, as such literacy might take very different forms depending on the contexts and opportunities available to the students.

Let us move from Chicago to Barcelona. When a student with a foreign cultural background enters a classroom, it appears from general observation that some difficulties do occur. One broadly accepted explanation for this is that immigrant students bring along norms that do not harmonise with the prevailing norms of the classroom. Thus, it could be assumed that cultural conflicts become revived. This interpretation, however, has been seriously questioned in the studies of Núria Gorgorió and Núria Planas (in preparation), who examine schools with many immigrants in neighbourhoods of Barcelona. They identify different norms that regulate classroom practices, as ‘in this class we work collaboratively and people must help each other’. Another norm is that ‘the contextualisation of the mathematics task should be considered seriously’. Such norms stimulate practices that are open to explore situated mathematics; they facilitate group work and inquiry cooperation. In their study, they investigated classrooms which, according to the teachers, were ruled by such norms.

Gorgorió and Planas (in preparation) concentrate on two immigrant students: Ramia and Esmilde. As an outcome of the investigated episodes both students decided to become non-participants. An exclusion had taken place. Ramia did not accept the classroom norms. She restricted her considerations, with respect to a certain task, to purely mathematical aspects of proportionality, which implies that she calculated that 6.66 eggs should be applied in a recipe, and that this was the proper result. Here the teacher and other students disagreed and referred to the norm of contextualisation: contextualisation has to be taken seriously, and it does not make sense to use 6.66 eggs when cooking food or baking. However, Esmilde, also an immigrant student, was well aware of the norm of contextualisation, and he explicitly referred to it when a mathematical task concerned the density of the population in a neighbourhood familiar to him. He found that his knowledge was relevant to the solution of the problem. However, in this case both teacher and the other students claimed that the task concerned proportionality, and that there was no need to make any more 'fuss' about this. The norm of contextualisation was abandoned for a while. The teachers were different in the two cases, but, as emphasised by Gorgorió and Planas, they were both open and supported the idea of situated learning in mathematics.

The interesting observation by Gorgorió and Planas is that the general pattern of explanation, as indicated above, does not seem to apply generally. This brings us forward to a more bitter interpretation of the observation. The point is not simply that immigrant students bring different norms to the classroom, and in this way cause conflicts. It might very well be that immigrant students struggle to become members of the classroom community by observing prevailing norms. Other forms of exclusion might be operating. These forms need not have much to do with the fact that immigrant students do not observe such norms. They may do it, or they may not: both cases could mean exclusion. It might happen that the teacher and students who are familiar with the norms choose to cancel the norms for a while in such a way that exclusion can be maintained. This might indicate that causes for exclusion are produced from within the established classroom community, and that they need not be sought in some divergent norms of the immigrant student. Reasons might be found in a stereotyping of 'immigrant', and this stereotyping might be exercised through particular ways of administering recognised norms. Exclusion might be caused by the 'appearance' of the immigrant, and not simply by any norms the immigrant student brings to the classroom.

However, Gorgorió and Planas do not leave us there. They suggest that negotiation of norms might be a useful strategy for coping with these situations. By a negotiation, norms and their justification can be made explicit. This is important, however, not only for the immigrant students (who in fact might have grasped them very well). The importance of negotiation also has to do with the teacher and the other students. Even when teacher

and students do not want to exclude, they might administer classroom norms in ways that provoke exclusion. To prevent this, a negotiation of norms may be useful.¹⁴

2. The variety of practices including mathematics

That mathematics education could play significant but undetermined socio-political roles, can be further explored when we address the sites for learning mathematics beyond the school contexts. School mathematics practices are not the only ones involving mathematics that is relevant to mathematics education. It is important to consider to what extent school mathematics practice is related to other practices. Mathematics forms part of many different work contexts, often in an implicit way. This means that what is happening in school can have effects on and be affected by many different practices that include mathematics. To address these different forms of interaction forms part of addressing the uncertainty related to mathematics education, i.e. to address its socio-political dimension.

In 2.1, I consider in what sense ‘mathematics could be everywhere’. In 2.2, I indicate that there are many sites operating with mathematics outside the school. In 2.3, I present some examples illustrating this.

2.1 Mathematics could be everywhere?

As mathematical thinking, theories and techniques are developing in all possible directions, the very notion of mathematics has developed as well. We cannot expect mathematics to represent any unity. It might be an idea to reconsider Ludwig Wittgenstein’s remarks in *Philosophical Investigations* about language and games. There are many different forms of games, and we shall not expect the existence of any common characteristic of ‘game’. We can only think of different family resemblances. And Wittgenstein continues: nor should we expect to find any unifying characteristic of ‘language’. Instead we should pay attention to the different jobs language performs. In a similar way it makes sense to assume that we cannot hope for a unifying characteristic of ‘mathematics’. The concept may refer to a variety of activities, as do ‘game’ and ‘language’.

¹⁴ Through these short ‘visits’ to Chicago and Barcelona, I have only addressed particular aspects of non-prototype classrooms. The vast majority of non-prototype classroom are still awaiting for exploration. This is not caused by my lack of intention, but simply by the limitation of my insight.

More than any other studies, the ethnomathematical approach has pointed out the plurality of ‘mathematics’. We can count numerous and very different activities as being mathematical. Examples are: The calculations of change at the baker’s shop; solving cubic equations for home work; searching for a more efficient algorithm of prime factorisation; investigating the functioning of a robot arm by using matrix calculus; doing algebra; reading statistical figures; estimating the risks connected to the construction of an atomic power plan; planning the cheapest route for going on holiday at the beach; estimating how much to leave in tips at a restaurant; constructing the roof of a hut; weaving baskets; knitting a pullover; carrying out the planning of bridge construction; making a time schedule for a conference.

We could find mathematics everywhere. Or could we? We could extend this list much further by activities like: Watching somebody doing knitting; eating bread from the baker’s shop; copying the solutions of cubic equation from a friend; forgetting to pay tips, etc. Such activities could only be called mathematical stretching the concept beyond reason. In a similar way: although ‘game’ can be applied in many different situations, it does not imply that it can be applied in any situation. Nevertheless, the mathematics-is-everywhere discourse opens new perspectives on interpreting daily activities. But calling an activity mathematical, it may be ‘colonised’ as a ‘domain’ for mathematics education. Paul Dowling (1998) has emphasised this in his critique of ethnomathematics.¹⁵

2.2 Practices of mathematics

In order to develop further a socio-political awareness in mathematics education, it is important to explore how the practices of school mathematics relate to other practices including mathematics and to explore these in detail. Power can be exercised through the mathematics curriculum through its priorities of those out-of-school practices, both those that are referred to and those that are ignored. In order of simplicity, I will make a grouping by referring to practices of (1) of constructors, (2) of operators, (3) of consumes, and (4) of the ‘disposable’. Any categorisation expresses a perspective, and this one might express the economic priorities of the informational economy, in particular in pointing out some practices as ‘fringe’ practices.

First, any area of technology seems to bring mathematics into operation. We cannot think of the emergence of informational technology without mathematics being in action. Systems of management operate with mathematics, so does systems for economic planning, technological design, traffic planning, etc. Certain groups of people are going to

¹⁵ Knijnik (1996) also addresses this issue.

maintain and develop further the knowledge, techniques, ICT systems, economic resources, management priorities, etc. by means of which technology, in the broadest interpretation of the term, is further developed. These groups I call *constructors*, and mathematics appears as one element included in their competencies. It is the task of universities and other institutions of further education to provide these competencies, and any education of engineers, economists, computer scientists, pharmacists, etc. includes mathematics. There are many mathematics-rich practices of construction.

The mathematics in operation in technical fields has been discussed in many studies. I could in particular refer to the many studies carried out by both students and teachers at IMFUFA, Roskilde University Centre. One of these addresses the Danish macro-economic model ADAM (Annual Danish Aggregate Model), which is used by the Danish Government as well as by other institutions.¹⁶ This is just one among many studies addressing the question of how mathematics makes part of technology, management, decision-making in different spheres of life. Leone Burton (1999, 2001, in print) has studied the practices of mathematicians. I find that further investigations of these numerous practices are important for mathematics education. This makes it possible to relate what is done in school with what might take place outside of school. More generally, I find that transition issues, to which I will return later, are important for clarifying how meaning in mathematics education is constructed by different groups of students.

Second, we see mathematics in operation, often in an implicit form, in many work-processes. Mathematics education functions also as a preparation for people who are not going to use what they learn in mathematics in secondary or upper secondary school for pursuing further mathematics-dense studies. They may face a working situation where they will operate with mathematics, often only implicitly. Mathematics might be available in 'packages' (easy to install), which it is important to be able to use, while the details of how the packages function need not be grasped by the person who operates it. Mathematics need not surface in the situation. Much mathematics education can be seen as preparation for people who are going to operate in contexts where mathematics is incorporated in their job situations, for instance as part of the tools and instruments with which they operate. These people will be involved in what we can call practices of *operators*.

Tine Wedege has made many observations about mathematics at work. As an example, she observed how the person responsible for loading an airplane has to take into account certain numbers which indicate how well-balanced the plane is before take off (Wedege, 2002a). The luggage has to be loaded into different compartments in such a way that the balancing factor is within a certain interval. The actual calculation of the balancing factor is done by the computer according to formulae that only the engineer may know. However, the person responsible for the loading has to provide inputs to the computer. The

¹⁶ See Dræby, Hansen and Jensen (1995).

luggage can be placed in different ways, and it has to be judged what could be done in case the balancing factor might be too close to the security limits. The person has to make estimations and judgments based on figures and numbers. Should the baggage be reloaded? Would that cause a delay? As Wedege (1999, 2002b) points out many adults do not think of themselves as doing any mathematics. They are just ‘using their common sense’. Elsa Fernandez (2002, 2004) has studied a relationship between a school mathematics practice and the mathematics of a practice not socially identified with mathematics, namely the mathematics included in metalworking. All such studies point towards the question stated in the title of the book written by Gail E. FitzSimon: *What Counts as Mathematics?* This question is essential when we consider the practice of the operators.¹⁷

Third, mathematics contributes to our daily life, for example when information and decisions are presented with reference to numbers and figures. Statements from experts are expressed each and every day on television and in newspapers. Number could refer to proposed investments. The ‘necessity’ of introducing some economic restrictions can be justified by certain calculations. Systems of salary and pensions reflect degrees of inflation and schemes of productivity. Taxes have to be paid, and the tax-level is a permanently discussed issue. It has an impact on how each and every one may manage their daily-life situation. It seems that the whole forum of democratic debate is deluged by numbers, figures and statistics. Experts can be interviewed, and they can express their opinion in public. All such practices, I refer to as practices of the *consumers*. A classic study is made by Jean Lave (1988), investigating mathematics at the supermarket, which appears to be an explicit example of mathematics in a practice of consumption.

Fourth, we see many groups of people who are living at the fringe area of the informational economy. Not everybody necessarily appears in the informational economy, as this is organised according to dominant economic interests and priorities. Some groups of people seem to be swallowed up by the “black holes of informational capitalism”. Given capitalist priorities, these people might appear *disposable*. People can be marginalised if not excluded. Anyway, they are involved in practices including mathematics, for example in selling and buying. Such practices also have relationships to what is taking place in school. Many students in school might experience these practices as belonging their potential future. Terezinha Nunes, Analucia Dias Schliemann and David William Carraher (1993) studied street mathematics (see also Vithal, 2003b). Madalena Santos and João Filipe Matos (2002) investigate the mathematical practices of the *ardinas*, who are described as the young boys between 12 and 17 that sell newspapers in the streets of Praia, the capital of the Republic of Cabo Verde.

Being aware of how mathematics operates in many different practices are important in order to understand meaning production. This then relates to processes of transition that

¹⁷ See also Bessot, A. and Ridgway, J. (Eds.) (2000); Hoyles, C., Noss, R. and Pozzi, S. (1999); and Wedege (2000).

are possible between practices. For some students in some classrooms, certain out-of-school practices might seem accessible, attractive, interesting. For other students, these very same out-of-school practices are devoid of interest. Let me finally emphasise that I do not claim that any particular categorisation of these practices is adequate, but just that the variety of research that has have tried to illuminate such practices is relevant also to understand and develop school practices.

2.3 Examples

The ethnomathematical programme, introduced world wide by Ubiratan D'Ambrosio's plenary at ICME-5 in Adelaide, put into focus the idea that mathematics operates in a variety of cultural settings. As already mentioned, the ethnomathematical programme broadens the concept of mathematics. Not only can we experience mathematics in textbooks and in mathematical research journals, but also in boats constructed by Indians for sailing at the Amazon River. Mathematics can be integrated in tools, craftwork, arts, routines. It can be part of a chair as well as of a computer. The concept of mathematics has been opened. D'Ambrosio has interpreted the notion of ethnomathematics by considering its three conceptual elements: ethno-mathema-tics. 'Ethno' refers to people; 'mathema' to understanding; while 'tics' refers to techniques.¹⁸ Thus, ethno-mathema-tics refers to culturally embedded techniques for understanding. It must be noted that the notion of 'mathema' is broader than 'mathematics' as we normally consider it, and that 'ethno' has to be understood as people/culture, and that it does not include any reference to 'ethnicity'.

The ethnomathematical research programme has proliferated worldwide. Thus we can see studies dealing with mathematics in sugar cane farming (Abreu, 1993; Regnier, 1994). Claudia Glavam Duarte (2003) addresses the 'world of construction', for instance the mixing of mortar (sand, cement, water) and, depending on the particular use, some crushed stones. Ieda Giongo (2001) analyses the practice of shoemakers. In Brazil researchers and practitioners have struggled with the problems of dealing with hybrid forms of knowledge that characterise the life conditions of many groups of Indians (see, for instance, Amancio, 1999; Scandiuzzi, 2000, 2004). Gelsa Knijnik (1999) addresses the education for landless people in Brazil. Recently the perspective of street children has been addressed by Mônica Mesquita (2004) by investigating the notion of space. Education of indigenous people in Brazil has been addressed by José Pedro Machado Ribeiro and

¹⁸ See, for instance, D'Ambrosio (2001). See Ribeiro, Domite and Ferriera (Eds.) (2004) for a recent contribution to the ethnomathematical research programme. See also Gerdes (1996); Powell and Frankenstein (1997); and Knijnik (1998, 2002a, 2002b, 2002c).

Rogério Ferreira (2004), while the overall ethnomathematical approach has been addressed by Bill Barton (2004).

According to the conceptual delineation of ethnomathematics, we could talk about the mathematics of bakers, carpenters, street children, vendors, bank assistance; we could talk about the mathematics of the Incas, as well as of tele-engineers, system developers, dentists, statisticians; not to forget that we could talk about the ethnomathematics of the pure mathematicians. However, it seems that the ethnomathematical programme itself has incorporated a priority in focussing on the ethnomathematics of only certain ‘ethnicity’ groups. Thus, there is not much research within the ethnomathematical programme that addresses, say, engineering mathematics.

Tine Wedege (2003, 2004) presents *sociomathematics* as concerning relationships between people, mathematics and society.¹⁹ Sociomathematical problems concern: (1) peoples’ relationships with mathematics and mathematics education in society, (2) the functions of mathematics and mathematics education in society as well as society’s influence on mathematics and mathematics education; and (3) learning, knowing and teaching mathematics in society. This conceptual suggestion might help to establish further relationships between the ethnomathematical programme, and those very many studies which share a number of the same concerns, but which might find it awkward to operate with the notion of ethnomathematics.

The broadening of the notion of mathematics brings *transition* into focus. I interpret transition broadly as referring to any issue relating insight, competence, understanding, notions, techniques, tools, etc. operating in one context to ‘similar’ insight, competence, understanding, notions, techniques, tools, etc operating in a different context. Briefly: transitions concern the bridging between different practices. And when the variety of in- and out-of-school practices that might include mathematics (in one or another interpretation of the notion) expands enormously, the number of transition issues also grows. How to express the relationship between these different practices? What use might we make of this bridging in an educational setting? Transition problems are addressed in *Transitions Between Contexts of Mathematical Practices* edited by Guide de Abreu, Alan J. Bishop and Norma C. Presmeg (2004). Transitions have to do with the production of meaning. When relationships are made visible between what is happening in the classroom and some practices outside school in which the students might become involved, then a resource for students’ meaning production has been established.

‘Classic’ questions of transition in mathematics education have been formulated with reference to school organisation. For instance: How to ensure a smooth transition between the mathematics of secondary school and the mathematics of the upper secondary school? Such a transition problem has to do with the relationship between different parts of an overall curriculum. However, this issue is only a minor one within a network of

¹⁹ The notion of sociomathematics has also been presented by Zaslavsky (1973).

transition issues. The students are, or in later life could become involved in many different activities and practices that include mathematics, and it becomes an educational challenge to consider what a transition might consist. Transition problems could concern students from a small city in Denmark, an African township, a metropolis. Such problems emerge everywhere, but their meaning depends on the context. Even when we address the ‘same’ mathematical issues, say solving quadratic equations, the transition problems might be very different for different groups of students. In particular, when we search outside the prototype mathematic classroom, we easily come to see how problems of transition point towards a huge variety of socio-political issues, the delineation of immigrant-opportunities being only one of them. In the following Section 3, I will be more explicit about the socio-political issues of mathematics education, and the transition problem is one that introduces this area.

One example of the transition issue was addressed by Marta Civil and Rosi Andrade (2002). The project they describe primarily took place in schools located in working class, Mexican-American communities. The children could be recent immigrants, or their parents could be immigrants. Some children were bilingual, some Spanish monolingual, some English monolingual. The project tried to relate out-of-school mathematics with in-school mathematics. Establishing such relationships may ensure meaning to activities in the classroom, it may serve an ‘empowerment’ of students. However, to establish such a relationship is problematic, which was also emphasised by Civil and Andrada. One element in the project was household visits. Here teachers could become aware of the children’s situation and learn in what tasks the children might be involved, when at home. Such tasks could contain elements of mathematics. A mathematical archaeology, consisting of analysing a situation or a practice for its mathematical elements, could be carried out. Such an archaeology could be helpful for reflecting on how to introduce a mathematical topic in school and what examples to use as illustrations. Certainly difficulties were experienced by the teachers: “The transformation of household knowledge into pedagogical knowledge for the classroom is not easy. As much as we enjoyed the wealth of information that comes out of these household visits, we find ourselves constantly wondering about connection to the teaching of mathematics in school.” (Civil and Andrade, 2003: 156)

The aim of the project was not to *bring* home-mathematics to school but to *bridge* home-mathematics and school-mathematics. Home mathematics is one out-of-school site for mathematics. Mathematics at the workplace is another site, and in the project the household visits were supplemented by interviews with a mechanic, a carpenter, a welder, a construction worker, and a seamstress. Could there, one way or another, be mathematics integrated in the practices in question? Could mathematics become excavated? Could some

of the activities be read mathematically? Again, the difficulty of carrying out a mathematical archaeology was experienced.²⁰

Reconsidering the very many different sites for learning mathematics (of which the prototype mathematics classroom is only one) as well as the very many sites of mathematics practices, we see that the transition problem can take very many forms. Clarifying possibilities for transition could be a sense-making activity for many students. Transitions could be explored analytically, for instance by means of a mathematical archaeology; but practices can also be bridged by identifying realistic (and attractive) opportunities for students to make use of. Transition could mean empowerment. However, transitions could also turn out to become socially impossible, which could become a learning obstacle. Lack of opportunities for transition could mean disempowerment. Transition concerns both inclusion and exclusion. This observation brings our discussion into the socio-political dimension of mathematics education.

3. Exploring the socio-political dimensions of mathematics education

We could think of learning mathematics in general as preparation for any social practice rich in mathematics, explicit or implicit, in a ‘functional’ or ‘accommodating’ way. Mathematics education could regiment and discipline students; it might support the development of an ‘ideology of certainty’. Mathematics education could ensure ‘accommodating’ forms of transition. Alternatively, mathematics education could bring about competencies which can be described in terms of ‘Bildung’, ‘Mündigkeit’, ‘empowerment’, ‘citizenship’, ‘conscientização’, ‘ubuntu’ and ‘bhora’.²¹ In short, mathematics education could mean empowerment.

²⁰ Any such archaeology runs the risk of presuming what is questioned: mathematics is found in the practice (often to the great surprise of those who are involved in the practice), and the mathematics found might well be the mathematics included in the perspective of those who interpret the practice.

²¹ The German notions ‘Bildung’ and ‘Mündigkeit’ have been used broadly in the educational discourse. ‘Bildung’ has no straightforward English translation, while ‘Mündigkeit’ has connotations similar to ‘empowerment’. The Portuguese notion ‘conscientização’ has been used by Freire to indicate a crucial aspect of literacy. The Zulu term ‘ubuntu’ and the Sotho term ‘bhora’, have both achieved educational significance through the work of Bopape (2002) who has shown how they encompass ideas of solidarity and ‘critical citizenship’.

As already mentioned, I find that mathematics education is significant as well as undetermined. It can be acted out in many different ways, and this could make a social difference. But it also brings about an uncertainty that opens up the socio-political dimensions of mathematics education. For me it is important that this uncertainty becomes explored, and this has to do with the following observation: Notions like *Mündigkeit*, empowerment, citizenship, etc., are relational. They refer to relations between different sites for learning mathematics, inside school or out of school, and they refer to different practices. Thus, the notion of empowerment may have one meaning when it refers to a group of Mexican immigrant students in the USA, and another when it refers to students in a middleclass school in Los Angeles.

In 3.1, I present some considerations about mathematics and mathematical rationality that question the intrinsic ‘goodness’ of mathematics education, i.e. the idea that an ultimate goal of teaching mathematics could be expressed with reference to mathematics itself. In 3.2, I emphasise the importance of including socio-political issues in the research-practice discourse. I refer to studies that point out forms of discrimination that might be associated with mathematics education, but also to studies that interpret certain forms of mathematics education as being empowering. In 3.3, I discuss the notions of ‘mathematical literacy’ and ‘mathemacy’.

3.1 Uncertainty about mathematics education

Michael Apple makes the following observation: “In the process of individualizing its view of students, it [mathematics education] has lost any serious sense of the social structures and race, gender and class relations that form these individuals. Furthermore, it is then unable to situate areas such as mathematics education in a wider, social context that includes larger programs for democratic education and a more democratic society.” (Apple, 1995: 331)²² Pierre Bourdieu makes a similar observation. Thus, in *The State Nobility*, he refers to an investigation which identifies ‘categories of perception’ and ‘forms of expression’ used by mathematics teachers to label differences in the students’ performances. These categories and forms of expressions enable the teachers “to suppress or repress the social dimension of both recorded and expected performances and to dismiss any questioning of the causes, both those causes that are beyond their [the teachers’] control, and are thus independent of them, and those that are entirely dependent upon them” (Bourdieu, 1996: 10-11). Bourdieu finds that mathematics education manifests an ignorance of social, political and cultural aspects of the life of the students.

²² Lerman (2001) also draws attention to this observation made by Apple.

In *Counting Girls Out*, as previously mentioned, Valarie Walkerdine provides an even more bleak picture of what mathematics education might be doing: “We have argued that modern governments works through apparatuses like school, hospitals, law courts, social work offices, which depend upon what Foucault has described as technologies of the social: scientific knowledge encoded in practices which defines the population to be managed – not through simple and overt coercion, but by techniques which naturalise the desired state in the bourgeois order: a rational citizen who rationally and freely accepts that order and obeys through ‘his own free will’, as it were. Those knowledges, apparatuses, practices, seek constantly to define and map processes which will naturally produce this subject. They constantly define girls and women as pathological, deviating from the norm and lacking, but they also define them as necessary to the procreation and rearing of democratic citizens.” (Walkerdine, 1989: 205) Following Michel Foucault’s suggestion in *Discipline and Punish*, both prisons and schools can be seen as technologies of the social, making sure that the population can be managed. And Walkerdine points towards mathematics education as one sublime such technique, which helps to ensure the functioning of the social order, not by overt coercion, but by ensuring that rational citizens accept the rational order by their own ‘free will’. The rational order becomes exercised through mathematics and mathematics education. In this sense, mathematics education could ensure ‘accommodating’ forms of transition.

Mathematics education cannot operate from a basis of certainty concerning its own intrinsic values. Much research in mathematics education seems, however, to assume the existence of such an intrinsic value in mathematics education. This essentialism assumes that there is a value in mathematics education guaranteed by the very fact that this education addresses mathematics. And it will ensure that mathematics educators can operate as ‘ambassadors’ of mathematics, with the certainty that they are acting on behalf of a good cause. Instead mathematics education must face an uncertainty, and this brings us to the open landscape of socio-political issues of mathematics education.²³

3.2 Addressing socio-political issues in mathematics education

Mathematics education makes part of the processes of globalisation which, according to Bauman, as quoted previously, “divides as much as it unites”. In fact, “it divides as it unites”. This is an essential reminder to mathematics education. When we see education as part of universal processes of globalisation, we should also see it as part of the universal processes of ghettoising, of exclusions as well as inclusions. Both the notion of

²³ Socio-political issues have recently been addressed in Keitel (2000, 2003); Adler (2001b); and Atweh, Forgasz and Nebres (Eds.) (2001).

discrimination and empowerment might concern mathematics education, which also might 'divide as it unite'. Let me point out some of the ways in which mathematics education is discriminatory. I will make remarks about (1) discrimination in terms of (lack of) resources, (2) racism, (3) sexism, (4) discrimination in terms of language, and (5) discrimination in terms of what is referred to as 'ability'.

First, mathematic education presupposes investment. Computers enter the classroom, and often they are celebrated as ensuring a new powerful learning environment. The computer with the proper software can engage students in mathematical activities. They can develop their creativity; they can make experiment and exploration; and they can construct mathematics knowledge. Computers can ensure motivation as well as 'learning efficiency'. So goes the celebration. What is seldom under discussion, however, is the implication of this observation for the majority of the world's children, who learn mathematics in classrooms without any computer in sight. Are they left behind? Do we have here to do with a new form of social exclusion?²⁴ A taken-for-granted perspective pervades the discussion of technology in mathematics education. Naturally, there is no problem that particular studies of the use of technology in mathematics education assume that this technology is available, but the technology-can-be-taken-for-granted perspective turns into a problem for the research paradigm, if the majority of studies assume the perspective as a natural given.²⁵ To make the story brief: Mathematics education and poverty is not a topic explored widely in research in mathematics education. But it is an essential topic as discrimination of learning opportunities are caused by an unequal distribution of resources.²⁶

Second, it is not difficult to find examples of sheer racism exercised through mathematics education, especially when we consider the role of education through the apartheid era in South Africa. Other, possibly more indirect expressions of racism, are revealed when, as suggested by Wenda Bauchspies (in print), we consider to what degree learning, and in particular the learning of mathematics, can mean colonisation. There are many studies that have emphasised that learning mathematics in a particular form could serve as a suppression of an existing form of thinking. Munir Fashed (1993, 1997) has talked about the occupation of the mind and related issues, and Herbert Khuzwayo (2000)

²⁴ For reflections on access and non-access to computers, see Borba and Penteadó (2001).

²⁵ To me the extensive discussion of responses in mathematics education to technological development in Bishop, Clements, Keitel, Kilpatrick, and Leung (Eds.) (2003) suffers from this limited perspective, so does the section 'Influences of Advanced Technologies' in English (Ed.) (2002).

²⁶ For a discussion of mathematics education and poverty see, for instance, Kitchen (2002, 2003).

has developed in further detail what this could mean for interpreting the operation of mathematics education during the apartheid era in South Africa.

Third, sexism, or the issue of gender, has been addressed in mathematics education for a longer period. Mathematics can be interpreted as a language giving access to power, to technology, to job opportunities. Statistics have documented the unequal distribution of men and women with respect to mathematics-dense studies and later jobs. It is apparent that mathematics education includes or 'materialises' discrimination. Gender issues are analysed by studies like that of Gilah, C. Leder, Helen J. Forgasz and Claudie Solar (1996). The issue has also been addressed by the International Organization of Woman and Mathematics Education (IOWME), which organised meetings at the ICME conferences.²⁷ The gender issue has been broadened in an important way by Gelsa Knijnik (1998) with reference to the Movimento Sem Terra (The Movement of Landless People) in Brazil. In that study, she focussed on a group of people suffering from poverty and social exclusion. Within such a group, the gender issue can assume definite shapes that could hardly be grasped by research set within a traditional framework.²⁸

Fourth, the language issue includes many socio-political controversies. According to formal regulations the language spoken in schools in Barcelona must be Catalan, while Spanish is forbidden. This might serve to counterbalance the dominance of Spanish. However, Catalan represents a middleclass culture, and students in poorer parts of Barcelona often come from other parts of Spain or from other Spanish speaking countries. Although these students' mother tongue is Spanish, the teacher has to address them in Catalan. To be forced to learn (mathematics also) in a language different from one's mother tongue could amount to suppression. At the same time it could make available new life opportunities. Thus, mastering Catalan could turn out to be very useful for the students later in life. The conflict between the cultural suppression provoked by having to learn in a language different to one's mother tongue, and the possible advantages of accepting to be taught in the 'dominant' language, has to be addressed with reference to the particular context.²⁹ The conflict is present in South Africa with 11 official languages. What is the

²⁷ The work from the meetings in 1988, 1992 and 1996 has been documented in Burton (Ed.) (1990); Rogers and Kaiser (Eds.) (1995); and Keitel (Ed.) (1998). See also Grevholm and Hanna (Eds.) (1995).

²⁸ See also Burton (Ed.) (2003).

²⁹ When different – maybe complementary, maybe contradictory – forms of knowledge come to co-exist many problems emerge. The chief Aritana of the Iawalapiti located in the Indigenous Xingu Park in the Brazil's Central region, critically commented on the statement that Indigenous people have to learn only in their own language: "How will we learn Portuguese? We want the two sides. ... We have been transmitting much of our knowledge. They need to transmit their knowledge to the indigenous." (See the article 'Língua camaiurá está em livro escolar' by M. R. Costa in *Folhinha*, São Paulo, 19 July 2003. Chateaubriand Amancio drew my attention to this remark.)

price and what is the gain of learning mathematics in English?³⁰ The language issue is also an important aspect of many ethnomathematical studies. Finally, I should mention that according to the dominant educational policy in Denmark, the approach to immigrant students is to teach them Danish as fast as possible. The issue of cultural suppression is simply ignored.

Fifth, discrimination in terms of ability could take the form of elitism in mathematics education. In many cases it has been argued that it is important to differentiate between students according to their so called ‘abilities’ (certainly the notion of ‘ability’ is a problematic one). Differentiation could turn into elitism when groups of students are treated differently according to their seemingly different capacities for learning mathematics and when the ‘best’ are best resourced. Elitism might be sought ‘justified’ in economic terms by claiming it profitable to invest in the seemingly better students. But if we consider education as human right, then this appeal to economic productivity as an underlying principle for an unequal distribution of learning possibilities appears absurd.

However, the political dimension of mathematics education not only has to be explored in terms of different forms of discrimination. It also has to be explored as a discussion of what ‘empowerment’ could mean in mathematics education. And in many cases the discrimination and the empowering elements are addresses in the same study. In his paradigmatic presentation, *The Political Dimension of Mathematics Education* from 1987, Stieg Mellin-Olsen opened a complex terrain for research. In this book he brought together many of his studies and observations, which had been expressed previously in Norwegian.³¹ Many studies consider both discrimination and empowerment which might be acted out through mathematics education, for instance by analysing what a mathematics education for social justice could mean as already exemplified in the study by Gutstein (2002). Peter Gates and Catherine Vistro-Yo (2003) discuss to what extent mathematic education is in fact ‘for all’, and what such a rhetoric must presuppose in order to be more than a rhetoric.

3.3 Mathemacy and mathematical literacy

³⁰ The complexities of such multilingual issues are addressed by Adler (2001a); Gorgorió and Planas (2000, 2001); Gorgorió, Planas and Vilella, X. (2002); Moschkovich (2002); Setati, Adler, Reed and Bapoo (2002); and Setati (in print).

³¹ In German and in the Scandinavian languages, the socio-political dimension of mathematics education has been intensively discussed during the 1970’s. An overview of the issues addressed are found in Volk (Ed.) (1979). See also Damerow, Elwitz, Keitel and Zimmer (1974) and Mellin-Olsen (1977).

As the socio-political preoccupation with mathematics education also concerns the very content (including the mathematical content) of the education, notions like ‘Mündigkeit’, ‘conscientização’ and ‘ubuntu’ also have to be understood in terms of content issues. The same is the case when we talk about mathematics education for citizenship, for critical citizenship, for empowerment, for democracy, for equity, for autonomy, etc.

Several notions have been coined in order to highlight such socio-political potential for mathematics education. Although *numeracy* in many cases has been elaborated as referring to competencies that could be expressed in mathematical terms, it has also been defined in terms of critical citizenship. D’Ambrosio has used the notion of *matheracy*, while I prefer to talk about *mathemacy* in order to signify a ‘critical’ content of mathematics education. In Alrø and Skovsmose (2002) *mathemacy* is related to the notions of dialogue, intention, reflection and critique.³²

A careful clarification of the notion of ‘mathematical literacy’ is presented by Eva Jablonka (2003), who points out different interpretations of the term (which also helped to show the variety of possible interpretations of *mathemacy*). (1) Mathematical literacy can be seen as a resource for developing human capital. Such interpretations often emphasise the capacity to ensure productivity by means of mathematical knowledge itself. The work force will be better qualified when they master mathematics, and the individual will get better opportunities in life, in particular in the labour market. Such a ‘numeracy-philosophy’ has guided many programmes in adult mathematics education. Thus, ‘mathematical literacy’ are interpreted as ‘literacy in mathematics’, i.e. as a literacy that can be described in mathematical terms – a literacy that appears straightforward to test and evaluate. This interpretation of mathematical literacy might, however, be rather narrow in opening for a socio-political perspective. (2) Mathematical literacy for cultural identity can be referred to ethnomathematical studies. As already mentioned, it has been emphasised that mathematics makes part of a variety of cultural traditions and that it is integrated in many different contexts and situations. Developing a mathematical literacy means, therefore, to develop a sensitivity and an awareness of culturally embedded competencies. (3) Mathematical literacy for social change emphasises a related although different issue. The point is not to learn mathematics in order to appreciate what might exist within a particular culture but to be able to interpret and to read a socio-political setting as open to change. Thus, a critical reading of statistical information could, as suggested by Frankenstein (1998), make possible a critical interpretation of social states of

³² See, for instance, Keitel (1997) for a discussion of numeracy and scientific and technological literacy; and Yasukawa (2002) for investigating the relationship between mathematics and technological literacy. Several other studies have addressed similar issues. See, for instance, Gellert, Jablonka and Keitel (2001); Johnston and Yasukawa (2001); Keitel (1993); Jablonka (1996); Yasukawa (1998). See also Niss (2003) and Niss and Jensen (Eds.) (2002) for a discussion of competencies.

affairs that need not be taken for granted. (4) Mathematical literacy can support an environmental awareness. More generally, we could think of mathematics as an important tool, in the broadest possible interpretation of tool, for addressing problematic issues, being environmental or not. (5) Mathematical literacy could include an evaluation of mathematics itself. Mathematics can be seen a tool, part of economic and technological superstructures, but also as a problematic tool as many uncertainties and questionable implications emerge from its applications. Therefore, mathematics has also be critically addressed within mathematics education.

Without doubt the list of possible interpretations could be continued, but the five dimensions pointed out by Jablonka appear wide enough to illustrate the different possible interpretations of mathematical literacy, and therefore of mathemacy. The notion of mathemacy (to stick to this concept) is complex. It contains tensions if not contradictions. It is a contested concept. The notion cannot be depict within a well-elaborated definition. As a consequence, there is no recipe waiting for how to organise a practice which might support the development of mathemacy. This is an important point to consider when attempts are made to develop a practice of mathematics education with a critical dimension.

Vithal (2003a) provides a careful elaboration of a ‘pedagogy of conflict and dialogue for mathematics education’, which also illuminates the complexity of outlining a practice for the development of mathemacy. This, however, does not mean that nothing can be done, but any practice will be subjected to uncertainty. A ‘pedagogy of dialogue and conflict’ takes its departure from the post-apartheid situation in South Africa. Vithal describes different projects in mathematics education, planned in close collaboration between teachers, student teachers and Vithal herself. Many things happen during these projects, but how to interpret the events? What do the students in fact get from their involvement? Do they develop any mathemacy? We have to do with a non-prototype classroom representing the initial step away from the apartheid structures that had dominated the school system, separating students according to racial classifications. The school is located in a predominately Indian neighbourhood frequented by a number of black students. The activities in the classroom are marked by cultural differences and conflicts, carefully presented in ‘crucial description’ i.e. descriptions that are so rich in detail that it might be possible, based on the very descriptions, to question the interpretation that Vithal herself proposes. I shall, however, not go into the actual descriptions, but present an important aspect of the categorical framework that Vithal pulls out of her analysis.

Vithal identifies five dual themes, each characterised by a pair of complementary concepts. She refers to the notion of complementarity, in particular as it was developed by Michael Otte (1994). The dual themes are the following: (1) freedom and structure, (2) democracy and authority, (3) context and mathematics, (4) equity and differentiation, (5) potentiality and actuality. In some sense complementary concepts contradict and exclude

each other, although the contradiction is not direct nor without possibilities for compromise, as complementary terms also are in need of each other. The notion of complementarity refers both to a contradiction and to a completion. By addressing classroom practices by means of complementary concepts, Vithal makes apparent that it is not possible to develop simple strategies for organising a mathematics education practice to promote a mathemacy. The complementary terms pull forward the contested nature of mathemacy.

Let me comment briefly on each of the five dual themes. (1) Structure refers to the fact that the mathematics classroom is governed by many rituals, some explicit, many implicit. At the same time learning presupposes a degree of freedom. The point, however, is that both structure and freedom are present in any educational situation, including situations which might support the development of a mathemacy. (2) Authority is present in the classroom. It is maintained by the textbook, the curriculum, and by the timetable. It is marked by the knowledge of the teacher, but also by the teacher's responsibility, in the classroom, for managing the communication, the behaviour of the children, etc. Still, democracy cannot be ignored. Learning for democracy must include the practice of democracy in the classroom. (3) Mathematics is a well-elaborated structure (even if we consider mathematics from outside a structuralist paradigm). At the same time mathematics can be contextualised, and in a classroom practice some contextualisations could almost obliterate mathematics. Children could be so involved in contextualised activities that they simply ignore that they do mathematics, and a particular contextualisation could turn into an obstacle for learning mathematics. (4) It is important that education pay special attention to individual students. Thus, the idea, inspired by constructivism, that a mathematics teacher should learn the mathematics of the students, seems to presuppose that he or she should learn quite a number of different mathematics. Differentiating is important. So is equity. From the perspective of equity there is no doubt that students have to be treated according to uniform standards. (5) Mathematics education takes place in certain contexts, in certain situation. It is a here-and-now activity. However, it cannot simply concentrate on what is actual, it must also address the potential of the students.

I take from Vithal's classification of the dual themes that if we want to discuss whether or not a certain educational practice might bring forward mathemacy or ensure the development of mathematical literacy, then we have to give up any 'simplicity' in our analysis. There is no 'programme' to be identified for bringing forward mathemacy. There is no one-way route to mathemacy. More generally: There is no simple way of discussing an educational practice from a socio-political perspective. When we want to address socio-political issues, we apply a conceptual framework which is complex, contradictory, at least complementary. A concern for the socio-political dimension of mathematics education cannot rely on any platform. It has to face uncertainty.

However, this does not mean that there are no possibilities. Thus, we could listen to the student Lupes (pseudonym) from Grade 8, who participated in the mathematical activities presented by Gutstein (2003b). Lupes states: “With every single thing about math that I learned came something else. Sometimes I learned more of other things instead of math. I learned to think of fairness, injustices and so forth everywhere I see numbers distorted in the world. Now my mind is opened to so many new things. I’m more independent and aware. I have learned to be strong in every way you can think of it.” (Lupe, quoted from Gutstein, 2003: 37). This is clear statement of an experienced empowerment.

4. Conclusions: Uncertainty and responsibility

The lack of essence in mathematics education, to which I referred in the introduction of this paper, produces uncertainty. This uncertainty reflects two aspects. First, that mathematics education is a significant social system, which means that it has social impact, both on society and on groups of learners. Second, that mathematics education is undetermined as its actual socio-political functions can be acted out in many different ways. I find that when non-prototype mathematics classrooms are addressed, and when the many different sites for mathematics practices are investigated, this uncertainty is exposed. Furthermore, this uncertainty is highlighted by the variety of those concepts, often contradicting each other, that seem applicable to mathematics education. This education can mean many different things: empowerment, ‘Mündigkeit’, suppression, colonisation, exclusion, inclusion, critical citizenship, adaptability, sexism, racism, equity. We can face wonders as well as horrors, and we might even lack any adequate perspective that makes it possible to distinguish properly between horrors and wonders. Uncertainty brings us into the socio-political dimension of mathematics education.

There is no particular strategy out of this uncertainty. We cannot hope to locate any ‘foundation’ from which to define a strategy. So, it becomes important to struggle with this uncertainty. The notion of *aporia* could be useful to symbolise the situation.³³ The word has a Greek origin. It refers to a paradox or to a situation where there seems to be no way out. Thus, Socrates, in particular in the dialogues written by the young Plato, turned the

³³ According to *Webster's Encyclopedic Unabridged Dictionary of the English Language* (New York: Gramercy Books, 1996) *aporia* could refer to “a difficulty encountered in establishing the theoretical truth of a proposition, created by the presence of evidence both for and against it”. This meaning of the word I try to put into a more general framework. See also Skovsmose (1998a, 2000a) and FitzSimon (2002) for explorations of *aporia*.

dialogue into a conceptual trap. When he tried to clarify a concept, one line of argument produced one insight into the concept, while a different line of seemingly just as sound argument brought about not only a different but also a contradictory insight. At the same time it was recognised there were no means of solving the dispute. The contradiction was left unresolved. An instance of *aporia* was recognised.

Although there is no direct way of solving an *aporia*, this does not mean that it should be left unaddressed. While the *aporia* pointed out by Socrates opened a route into philosophy, I find that the *aporia* with respect to mathematics education brings us into the socio-political dimension of that education. Hence my concern for establishing a socio-political sensitivity in mathematics education. I find that *responsibility* refers to an attitude of trying to address apparent contradictions involved in mathematics education. Naturally, responsibility can be interpreted as an individual undertaking, and I would not deny that this is one useful interpretation. However, I am primarily interested in seeing this notion as a possible characteristic of a research paradigm in mathematics education. Thus, I do not suggest that each and every research initiative should look beyond the prototype mathematics classroom, and certainly not that all research should go beyond the school sites for learning mathematics. My point is that the research community as a whole should demonstrate a broad scope of interests that the many different sites for learning mathematics are reflected properly in research. This is a way of becoming aware of the uncertainties with respect to possible functions of mathematics education. To me responsibility refers to how the community of mathematics education addresses the uncertainties, for instance with respect to how mathematics education might involve discrimination or empowerment, inclusion or exclusion, and in this way address the socio-political dimension of mathematics education.

The relationship between research and practices can be acted out in different ways. One observation, however, might be useful. The research-practice relationship could include a differentiation of roles. Thus, the practice could constitute an object of research, which then could provide suggestions for reorganising the practice. This line of thought could bring one to assume that researchers do research *on* teachers and *on* students. Alternatively, research could take place in a collaborative form, meaning that the researcher could do research *with* teachers and *with* students. Data could be produced in a collaborative way. (But when the principal aim of doing research is to provide explanations, which could be interpreted as providing predictions, then it becomes problematic to involve the 'objects' of the research in the research process.) I find it important also to consider possible relationships between research and practice from a methodological point of view. If we wish to address socio-political issues in a responsible way, then it might be an idea to look for forms of research in which nobody is turned into

an ‘object’ of the research.³⁴ However, it seems that any further step in this discussion would presuppose that we face the challenge, mentioned at the beginning of the paper, namely to try to provide some more adequate definitions of ‘research’ and ‘practice’.

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³⁴ For a discussion of such issues see, for instance, Breen (2003); Vithal and Valero (2003). See also Skovsmose and Borba (in print).

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