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Robust Parametric Fault Estimation in A Hopper System

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Abstract: The ability of diagnosis of the possible faults is a necessity for satellite launch vehicles during their mission. In this paper, a structural analysis method is employed to divide the complex propulsion system into simpler subsystems for fault diagnosis filter design. A robust fault diagnosis method, which is an optimization based approach, is applied to the subsystems of the propulsion system. The optimization problem has been solved within two different tools and the results are compared with two other optimization based approaches. The turbo-pump system is used to illustrate the employed methods and obtained results.

1. INTRODUCTION

Reliability is a highly desired topic in many industrial applications, particularly in aerospace. The mission objectives of a spacecraft may not be disrupted by any possible fault. A fault diagnosis system is able to monitor the system performance and alert the control system when a fault has occurred. In this regards, the problem of model-based fault diagnosis has been receiving increasing attention from the research communities (Willsky, 1996).

By the early 90's, the paradigm of the conventional methods for fault diagnosis problem, which included annihilating the matrices, was substituted by the methods based on norm minimization. This phenomenon opened the doors of the $\mathscr{H}_2$, $\mathscr{H}_\infty$, and other optimization approaches to the field of fault diagnosis (Frank and Ding, 1994; Mangoubi et al., 1995; Edelmayer et al., 1996; Edelmayer and Bokor, 2000). Most of those FD approaches (except parameter identification methods) have considered the models with additive fault input to the system. In the other words, they are modeled as exogenous perturbations to the system (Basseville, 1988; Chen and Patton, 1999; Frank, 1990).

In this paper, the fault is modeled as a parameter, since the nature of many faults are parametric. Indeed, an exogenous essentially bounded input cannot de-stabilize a linear system.; whereas a parameter change might do so. A fault diagnosis approach for systems with parametric fault which has been proposed by (Stoustrup et al., 1997; Niemann and Stoustrup, 1997) is used here. The optimization problem has been defined in the so-called standard set-up for robust control based on LFT. In this approach, the residual is in fact an estimation of the fault.

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A "Hopper", which is a horizontally launched and horizontally landing rocket-propelled launch vehicle comprising a non-disposable primary stage and one expendable upper stage, is under consideration as a reusable launch vehicle to replace the existing expandable launch vehicles in ESA (European Space Agency) in the future. The advantages include: reduction of transportation cost to orbit, return capability from orbit, and less environmental pollution.

A key element for the re-usability and maintainability is given by the health management system (HMS) being an integral part of the system design (Belau and Sommer, 2006). The HMS shall be able to diagnose faults of which the effect is hardly recognizable due to system uncertainties (unpredictable environmental conditions or system parameters) or sensor noise.

The main engine is a complex system with various subsystems. Designing a filter for this system, which is capable of determining faults in a reliable manner, is shown to be a nearly impossible task. To address and solve this problem, a structural analysis approach was employed. The structural analysis of the system leads to identifying subsystems with inherent redundant information required for designing appropriate filters.

The contributions of this paper are two-fold: 1- illustrating the advantage of combined utilization of qualitative as well as quantitative methods to design a fault diagnosis system. Structural analysis method, which is a qualitative method, is used to analyze the system and divide the system into manageable (and monitorable) parts; whereas, quantitative (here optimization based robust methods) are used for the detailed design. 2 - the application of the parametric fault diagnosis filter design based on the $\mathscr{H}_\infty$ as well as the μ synthesis, in the set-up presented in (Stoustrup and Niemann, 2002; Niemann and Stoustrup, 2000; Soltani et al., 2011). In addition, the main results of the designed filter has been compared with two other optimization based methods (Khosrowjerdi M.J., 2005; Zhong M., 2003).

This paper is organized as follows: In Section II, an structural analysis on the propulsion system is presented while the turbo-pump has been chosen as a subsystem. The fault diagnosis method has been presented in Section III and the fault estimator design procedure has been described. In Section IV, the results of fault estimation are illustrated and compared with two other methods, and eventually, the conclusions are brought in Section V.

2. STRUCTURAL ANALYSIS OF THE PROPULSION SYSTEM

2.1 Motivation

The overall nonlinear system of the considered engine model is translated to a model block diagram. The blocks in the diagram, which is shown in figure 1 on the following page, represent the functionalities of the engines main parts; valves, pumps, combustion chamber, and the generator. The considered plant has 14 independent inputs, 18 outputs, 14 intermittent (nonmeasurable) variables, and 6 dynamic/continuous states. 12 failure cases were considered in this system. There are 6 differential equations that describe the dynamic behavior of the valves and pumps (Soltani and Izadi-Zamanabadi, 2007). The number of the states in the system suggests that designing a model-based fault diagnosis algorithm should be a fairly manageable task. However, (Soltani and Izadi-Zamanabadi, 2007) shows a very limited success in detecting most of the chosen faults due to the level of system nonlinearity.

The complexity of this system appeals for a method that enables the design engineer(s) to break the system into small and manageable parts for which the detailed design can be carried out. In addition, it would be an advantage to be able to obtain additional knowledge about which parts of the system are monitorable and whether the selected faults can be detected and isolated.

2.2 Structural Analysis

Structural analysis is concerned with the properties of the system structural model, which is the abstraction of its behavior model in the sense that only the structure is considered. A case in point, only the existence of relations between variables and parameters is taken into account (Blanke et al., 2006). The links are represented by a bi-partite graph, which is independent of the of the values of the variables and the parameters. Hence, the structural model is a qualitative, very low level, easy to obtain, model of the system behavior. The structural analysis provides the following information:

- the subset of the components, in which the faults can be detected and isolated, are identified, i.e., monitorable subsets of the system.
- the possibility of designing residuals to meet specific requirements.
- the existence of reconfiguration possibilities.

To demonstrate the use of the structural analysis, we have taken the results for the liquid oxygen LOX turbo-pump. The structural model of the LOX pump is shown in table 1. The constraints are $\mathscr{C} = \{c_1, c_2, c_3, c_4, c_5, c_6\}$, the unknown variables and (intermittent) parameters are $\mathscr{X} = \{R_o, p_1, p_3, p_8, p_{13}\}$, and the known (measured) variables are $\mathscr{M} = \{m_1, m_4, m_6, m_{14}, m_{15}, m_{16}\}$.

	R_o	p_1	p_3	p_8	p_{13}	m_1	m_4	m_6	m_{14}	m_{15}	m_{16}
c_1	×				①			1	1	1	1
c_2	①					1					
c_3			①				1		1	1	
c_4			1	①					1		
c_5		①	1	1		1	1		1	1	
c_6		1	1	1	1		1		1	1	1

Table 1. Structural model of the LOX pump.

A *matching* between an unknown variable/parameter and a constraint, denoted by a ① in the cross section between the variable column and the constraint row, indicates that the matched variable can be calculated/computed through the corresponding constraint. For instance, p_{13} can be calculated through c_1 when the values of R_o and m_6 are known. The constraint $c_1(R_o, p_{13}, m_6) = 0$ represents the following dynamical behavior:

$$\dot{R}_o = ap_{13} - a(\alpha(\max(0, y_6))^2 + \beta max(0, y_6)R_o - \gamma R_o^2)$$

where a, α, β, γ are known parameters. The × in the table indicates that the value of corresponding variable can not be uniquely calculated through the corresponding constraint, hence can not be matched. The table shows that all unknown variables/parameters are matched. On the other hand the constraint c_6 is not matched. However, c_6 contains unknown variables that are already matched (and hence can be calculated uniquely). Therefore, c_6 can be used to derive a relation that contains only known variables. The obtained relation is hence a redundancy relation. From the fault diagnosis viewpoint, the subsystem that is represented by constraints $c_1, c_2, c_3, c_4, c_5, c_6$ is observable (i.e. monitorable) and since it includes dynamical behavior. Therefore, it is suited for detailed model-based fault diagnosis design.

The nonlinear version of the LOX pump's system dynamic is written in a compact form as:

$$\begin{aligned} \dot{R}_o &= \frac{a_o Q_o^2}{R_{oh}} + b_o T_o + c_o Q_o R_o + d_o R_{oh} R_0^2 \\ y_1 &= R_o \end{aligned} \tag{1}$$

where a_o, b_o, c_o, and d_o are constant coefficients depending on the design of the turbo-pump and T_o is the LOX turbine torque. The pump speed is represented by R_o, the pump flow by Q_o, and the mixture ratio by R_{oh}.

2.3 The fault augmented model

Due to the fact that efficiency loss δ has been considered as a parametric fault for LOX turbo-pump, this fault affects the pump shaft speed. The dynamic equation is satisfied only for no fault case ($\delta = 0$). The fault augmented model is

$$\begin{aligned} \dot{R}_o &= \big(\frac{a_o Q_o^2}{R_{oh}} + c_o Q_o R_o + d_o R_{oh} R_0^2\big)(1 - p(\delta)) + b_o T_o \\ y &= R_o. \end{aligned} \tag{2}$$

3. FAULT ESTIMATION METHOD

3.1 Robust Parametric FDI in A Standard Set-up

A general concept of parametric fault detection architecture in a robust standard set-up is proposed in (Stoustrup and Niemann,

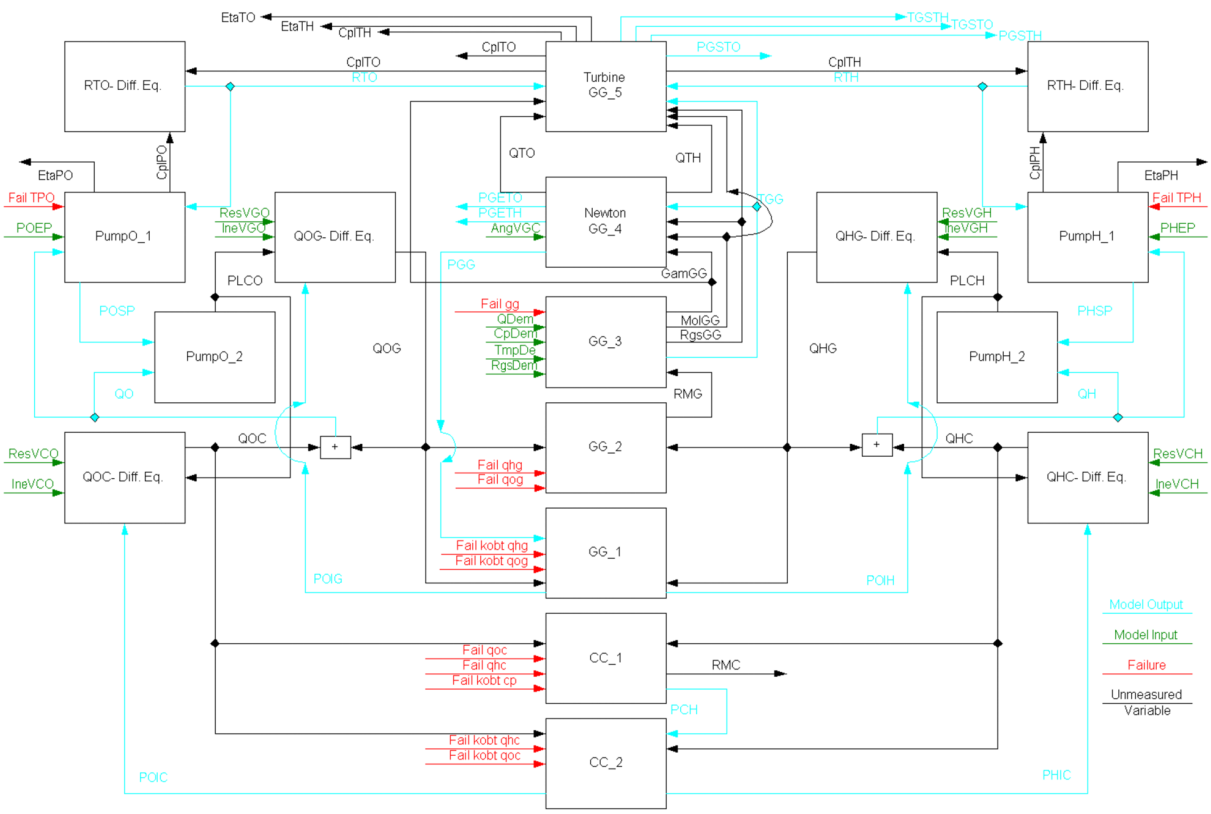

Fig. 1. Modular decomposition of engine system. (Known) Inputs to each block are shown by green color, faults have red color, Blue color represents the measured outputs, and black color represents internal variables which are not known (not measured).

2002). The approach is to model a potentially faulty component as a nominal component in parallel with a (fictitious) error component. Subsequently, the optimization procedure suggested here estimates the ingoing and outgoing signals from the error component. This works only well in cases where the component is reasonably well excited, but on the other hand, if the component is not active at all, there is absolutely no way to detect whether it is faulty. The considered plant is described by the model

$$\begin{cases} \dot{x} = A_\Delta x + B_u u \\ y = C_y x + D_{yu} u \end{cases} \tag{3}$$

where A_Δ is the deviated matrix from the nominal value A by a dependency to the fault where the dependency can be nonlinear. The possibly nonlinear parameter dependency of A_Δ is approximated with a polynomial. Therefore, $A_\Delta = A + p(\delta)A$, where p is a polynomial function of the parameter δ satisfying $p(0) = 0$ (the non-faulty operation mode). Finally, the model (3) is written in linear fractional transformation form. As a result we get a system of the form

$$\begin{bmatrix} \dot{x} \\ z \\ y \end{bmatrix} = \left[\begin{array}{c|cc} A & B_f & B_u \\ \hline C_f & D_{zf} & 0 \\ C_y & 0 & D_{yu} \end{array}\right] \begin{bmatrix} x \\ f \\ u \end{bmatrix} \tag{4}$$

where z is the external output, f is the fault input signal, the matrix D_{zf} is well-posed (LFT's are normally used), and the connection between z and f is given by

$$f = \Delta_{par} z, \tag{5}$$

$$\tag{6}$$

where Δ_{par} is a diagonal matrix $\Delta_{par} = \delta I$.

The next step in setting up the fault estimation problem as a standard problem is to introduce two fault estimation errors e_f ad e_z as

$$\begin{aligned} e_f &= f - \hat{f} \\ e_z &= z - \hat{z} \end{aligned}, \tag{7}$$

where $\hat{f}$ and $\hat{z}$ are the estimation of f and z to be generated by the filter respectively. Fig. 2 shows the setup for this approach. To design a filter F such that applying F to u and y provides the two desired estimates $\hat{f}$ and $\hat{z}$, one additional step is required.To this end, we introduce a fictitious performance block Δ_{perf} ; suggesting that the input u was generated as a feedback Δ_{perf} from the outputs $\begin{bmatrix} e_f \\ e_z \end{bmatrix}$

$$u = \Delta_{perf} \begin{bmatrix} e_f \\ e_z \end{bmatrix}. \tag{8}$$

Therefore, two filters ($W_f(s)$ and $W_z(s)$) are introduced to make sure that the norm of $\frac{\|e_f\|}{\|f\|}$ is minimized in the frequency area of interest. (For incipient faults a low frequency filter is used.) In fact, we introduce these filters to handle the high excitation level of the inputs. Finally we introduce

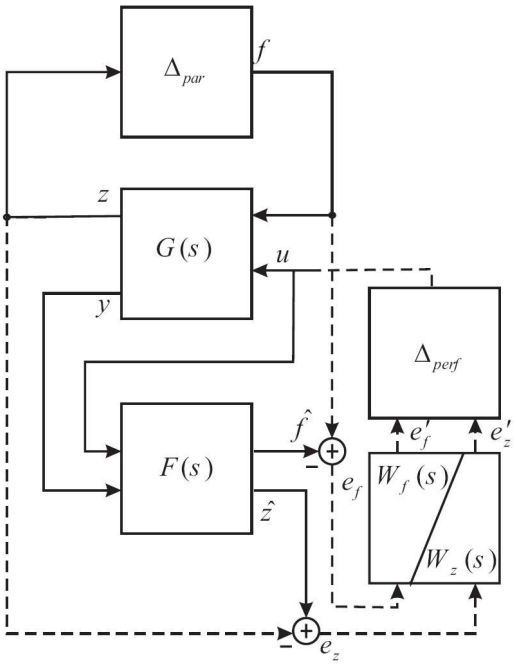

Fig. 2. Standard problem set-up for parametric fault detection combined with fictitious performance block (The dashed lines are the connections which are artificially assumed only for the design and they do not exist in implementation).

$$\Delta = \begin{bmatrix} \Delta_{par} & 0 \\ 0 & \Delta_{perf} \end{bmatrix}. \tag{9}$$

The significance of the Δ_{perf} block is the following. By the small gain theorem, the $\mathscr{H}_\infty$ norm of the transfer function from u to $\begin{bmatrix} \acute{e}_f \\ \acute{e}_z \end{bmatrix}$ is bounded by γ if and only if the system in Fig. 2 is stable for all Δ_{perf}, $\| \Delta_{perf} \|_\infty < \gamma$. Hence, the problem of making the norm of the fault estimation error bounded by some quantity has been transformed to a stability problem. Eventually, the main result for FDI problem with parametric fault is provided by the following (Stoustrup and Niemann, 2002):

Theorem 1. Let $F(s)$ be a linear filter applied to the system in Fig. 2

$$\begin{bmatrix} \hat{f} \\ \hat{z} \end{bmatrix} = F \begin{bmatrix} u \\ y \end{bmatrix},$$

and assume that $F(s)$ satisfies:

$$\| \mathscr{F}_l(G_{\tilde{z}\tilde{w}}, F) \|_\infty < \gamma, \tag{10}$$

where $\tilde{z} = \begin{bmatrix} z \\ \acute{e}_f \\ \acute{e}_z \end{bmatrix}$, $\tilde{w} = \begin{bmatrix} f \\ u \end{bmatrix}$, and $\mathscr{F}_l(.)$ is the lower Linear Matrix Transformation (LFT) representation of the two connected blocks (Zhou et al., 1995). Then the resulting fault estimation error is bounded by

$$\| \begin{bmatrix} \acute{e}_f \\ \acute{e}_z \end{bmatrix} \|_\infty < \gamma N \tag{11}$$

where N is the excitation level of the system i.e., $\| u \|_\infty = N$.

3.2 Design of The Fault Detector for Turbopump

As an example of the fault estimation method, we brought one of the subsystems in the propulsion system (Soltani et al., 2008). The Oxygen turbopump subsystem is actually the combination of the RTO and PUMP O-1 blocks in Figure 1. The dynamic model of this block is written as following

$$\dot{x} = (-ax - cQ_o)(1 - p(\delta)) + bT_o, \tag{12}$$

where a, b, and c are constants from the linearization, x is the shaft speed , Q_o is the pump flow , and T_o is the turbine torque and

$$p(\delta) = \lambda\delta^3 + \delta^2 - \lambda\delta \tag{13}$$

is the parametric fault model with some constant λ.

The system is formulated in a standard form as

$$\begin{aligned}
\dot{x} &= -ax - x_u + bT_o + \lambda f_1 + f_2 - \lambda f_3 \\
\dot{x}_u &= -Wx_u + WcQ_o \\
\dot{x}_{ef} &= A_{ef}x_{ef} + B_{ef}(\lambda f_1 + f_2 - \lambda f_3 - \hat{f}) \\
\dot{x}_{ez} &= A_{ez}x_{ez} + B_{ez}(\lambda z_1 + z_2 - \lambda z_3 - \hat{z}) \\
z_1 &= ax + x_u \\
z_2 &= f_1 \\
z_3 &= f_2 \\
\acute{e}_f &= C_{ef}x_{ef} + D_{ef}e_f \\
\acute{e}_z &= C_{ez}x_{ez} + D_{ez}e_z \\
y_1 &= x \\
y_2 &= T_o \\
y_3 &= Q_o
\end{aligned} \tag{14}$$

where W is a relatively big constant used to include the actuator fault in the state variables. The standard model (with $D_{ef} = D_{ez} = 0$) becomes

$$\begin{bmatrix} \dot{x} \\ \dot{x}_u \\ \dot{x}_{ef} \\ \dot{x}_{ez} \\ \cdots \\ z_3 \\ z_2 \\ z_1 \\ \acute{e}_f \\ \acute{e}_z \\ \cdots \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} A_1 & \vdots & B_1 & B_f & \vdots & B_2 \\ \cdots & & \cdots & \cdots & \cdots & \cdots \\ C_1 & \vdots & D_{11} & D_{1f} & \vdots & D_{12} \\ \cdots & & \cdots & \cdots & \cdots & \cdots \\ C_2 & \vdots & D_{21} & D_{2f} & \vdots & D_{22} \end{bmatrix} \begin{bmatrix} x \\ x_u \\ x_{ef} \\ x_{ez} \\ \cdots \\ T_o \\ Q_o \\ f_1 \\ f_2 \\ f_3 \\ \cdots \\ \hat{z}_1 \\ \hat{f}_1 \end{bmatrix}, \tag{15}$$

where the matrix values can be found in Appendix 5. Finally, a $\mathscr{H}_\infty$ filter F, which estimates $\hat{f}$ and $\hat{z}$ and takes u and y as inputs, is designed using *hinfsyn* in MATLAB. This filter results in e_f and e_z vanishing to zero as time goes to infinity.

4. RESULTS

4.1 Comparable fault estimation/diagnosis algorithms

To evaluate the described algorithm in the previous section, three other suggested optimization based robust methods are

employed. These methods are described in the following subsections.

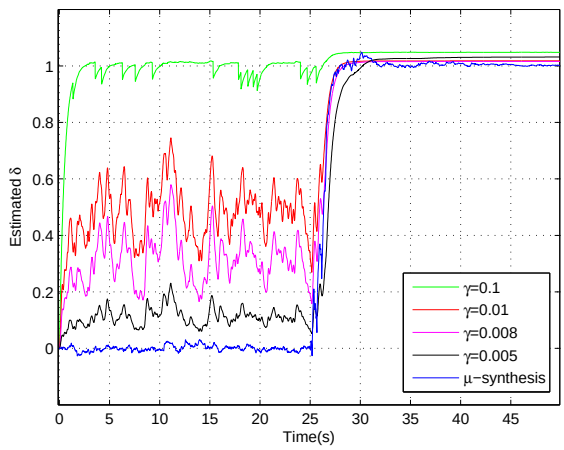

Fig. 3. The result of the $\mathscr{H}_\infty$ (with different γ values) and μ-synthesis method to the injected step fault.

μ-Synthesis The optimization problem in Theorem 1 can be solved using D-K iteration as a numerical method for μ-synthesis. The set-up, should be formulated in a way so that δ is augmented in the set-up and considered to be in the unit circle of the complex plane.

$$\begin{aligned}
\dot{x} &= -ax - x_u + bT_o + (ax+x_u)\lambda\delta^3 + (ax+x_u)\delta^2 - (ax+x_u)\lambda\delta \\
\dot{x}_u &= -Wx_u + WcQ_o \\
\dot{x}_{ef} &= A_{ef}x_{ef} + B_{ef}((ax+x_u)\lambda\delta^3 + (ax+x_u)\delta^2 - (ax+x_u)\lambda\delta - \hat{f}) \\
\dot{x}_{ez} &= A_{ez}x_{ez} + B_{ez}((ax+x_u)\lambda\delta^2 + (ax+x_u)\delta - (ax+x_u)\lambda - \hat{z}) \\
\acute{e}_f &= C_{ef}x_{ef} + D_{ef}e_f \\
\acute{e}_z &= C_{ez}x_{ez} + D_{ez}e_z \\
y_1 &= x \\
y_2 &= T_o \\
y_3 &= Q_o,
\end{aligned} \tag{16}$$

Mixed $\mathscr{H}_2$ / $\mathscr{H}_\infty$ Fault Diagnosis In (Khosrowjerdi M.J., 2005), the residual for the system

$$\begin{aligned}
\dot{x} &= Ax + Bu + B_f f_a + B_d d_a \\
y &= Cx + Du + D_f f_s + D_d d_s
\end{aligned} \tag{17}$$

is given by

$$\begin{aligned}
\dot{\hat{x}} &= (A-KC)\hat{x} + [B-KD \;\; K][u \;\; y_m]^T \\
\hat{\delta} &= -Cx + [-D \;\; 1][u \;\; y_m]^T,
\end{aligned} \tag{18}$$

where the gain K is obtained trough solving the convex optimization problem in (Khosrowjerdi M.J., 2005) and $\hat{\delta}$ is the estimated residual here.

Mixed $\mathscr{H}_\infty$ / LMI Fault Diagnosis In (Zhong M., 2003), a model-matching problem is solved by minimizing the $\mathscr{H}_\infty$ norm of the difference between the residual reference model and the real residual. In this method, the residual for the system 17 is given by

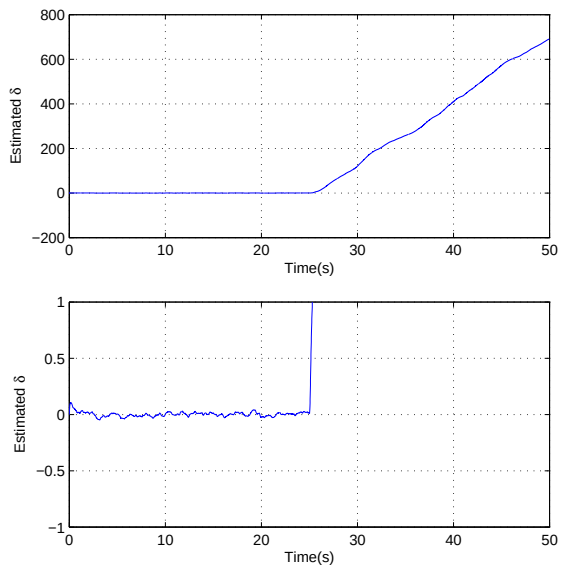

Fig. 4. The result of the mixed $\mathscr{H}_2$ / $\mathscr{H}_\infty$ method to the injected step fault (The upper graph is zoomed and illustrated in the lower graph).

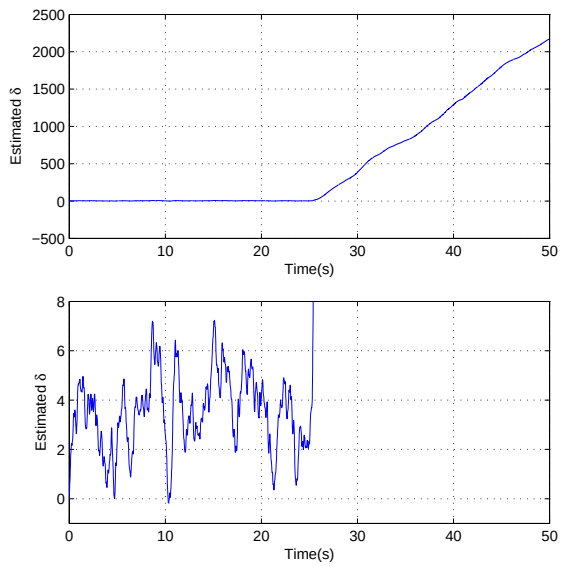

Fig. 5. The result of the mixed $\mathscr{H}_\infty$ / LMI method to the injected step fault (The upper graph is zoomed and illustrated in the lower graph).

$$\begin{aligned}
\dot{\hat{x}} &= (A-HC)\hat{x} + [B-HD \;\; H][u \;\; y_m]^T \\
\hat{y}_m &= C\hat{x} + Du \\
\hat{\delta} &= V[y_m - \hat{y}_m],
\end{aligned} \tag{19}$$

where $\hat{x}$ and $\hat{y}_m$ are the estimates of the state and measurement output vectors and the filter gains H and V are designed according to theorem 2 in (Zhong M., 2003).

4.2 Comparison of Estimation Results

Figure 3 shows the comparison of the different designs for $\mathscr{H}_\infty$ and μ synthesis. By reducing the γ of $\mathscr{H}_\infty$ optimization the estimation becomes more robust to the disturbances. The comparison of the $\mathscr{H}_\infty$ with μ-synthesis shows that in the no-fault interval (0-25s), the estimation has lower amount of

fluctuations and is more robust. However, in the fault interval (25s-50s), the residual generated by $\mathscr{H}_\infty$ design is more robust to the disturbances.

Figure 4 shows the output of the estimated residual generated by the mixed $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design method. As the change in the parameter results in instability of the system, the estimated residual is also unbounded. Consequently, the estimated residual does not estimate to the injected fault, but it determines the existence of a fault.

In figure 5, a similar type of output (unbounded) is observed. The residual is the result of the mixed $\mathscr{H}_\infty$ / LMI design method which does not represent the estimation of the fault, though it is less robust to the disturbances compared to $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design method.

In figures 6, 7, 8, 9, 10, 11, 12, and 13, the output of all four different FD filters are illustrated for different injected faults δ. These results show that the filter designed through μ-synthesis approach gives the best estimation of the injected fault in different scenarios.

4.3 Structural Analysis Results

The structural analysis, carried out on the propulsion engine model, identified 11 independent subsystems with inherent redundant information. Hence it is possible to derive 11 different and linearly independent residual expressions. 6 of these subsystems exhibit dynamic behavior while the other 5 are of algebraic nature. 12 different faults were considered in this system. A preliminary analysis of the fault impacts on each subsystem (represented by a corresponding residual) suggested that all faults were detectable. In addition, 7 faults were isolable while the other faults were group-wise isolable, i.e. a group of 2 faults and a group of 3 faults were isolable, but with no possibility of isolating the faults from each other in each group (Soltani and Izadi-Zamanabadi, 2007). Detailed design of fault diagnosis algorithms for each subsystem (in particular those with dynamic behavior) were carried out, and the results showed an exact match between the detected/isolated faults and the detectable/isolable faults determined in the structural analysis. Despite being a simple qualitative method the structural analysis showed to be an extremely powerful tool for developing health monitoring systems in complex dynamical systems.

5. CONCLUSION

The structural analysis approach was applied to identify the monitorable parts/subsystems of a complex propulsion system and provide information about the possibility of detecting and isolating the considered faults in the system. In this paper, the process of using the structural analysis was briefly illustrated by applying it on a turbo-pump subsystem.
The obtained filter was based on the parametric fault diagnosis filter design approach based on the $\mathscr{H}_\infty$ as well as the μ synthesis, where the chosen turbo-pump subsystem was used as the benchmark. Eventually, the results of the designed fault estimator have been compared with that of two other optimization based methods.

APPENDIX

Matrices Values

$$A_1 = \begin{bmatrix} -a & -1 & 0 & 0 \\ 0 & -W & 0 & 0 \\ 0 & 0 & A_{ef} & 0 \\ a\lambda B_{ez} & \lambda B_{ez} & 0 & A_{ez} \end{bmatrix},$$

$$B_1 = \begin{bmatrix} T_o & 0 & 0 & 0 \\ 0 & Wc & 0 & 0 \end{bmatrix}^T,$$

$$B_f = \begin{bmatrix} \lambda & 1 & -\lambda \\ 0 & 0 & 0 \\ \lambda & 1 & -\lambda \\ 0 & B_{ez} & -\lambda B_{ez} \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & -B_{ef} \\ -B_{ez} & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} a & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C_{ef} & 0 \\ 0 & 0 & 0 & C_{ez} \end{bmatrix},$$

$$D_{11} = 0_{5\times 2},$$

$$D_{1f} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$D_{12} = 0_{5\times 2},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_{2f} = 0_{3\times 3},$$

and $D_{22} = 0_{3\times 2}$.

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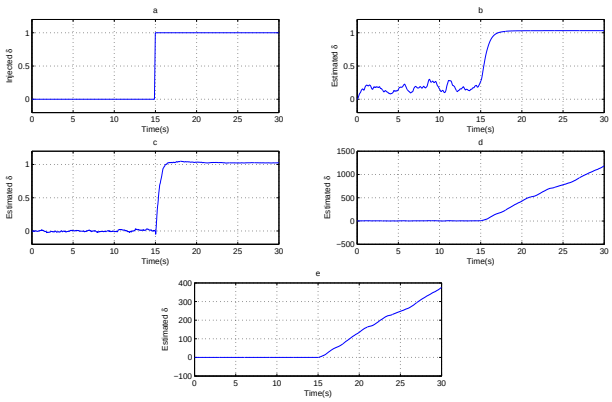

Fig. 6. Fault injected as a step (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$LMI design.

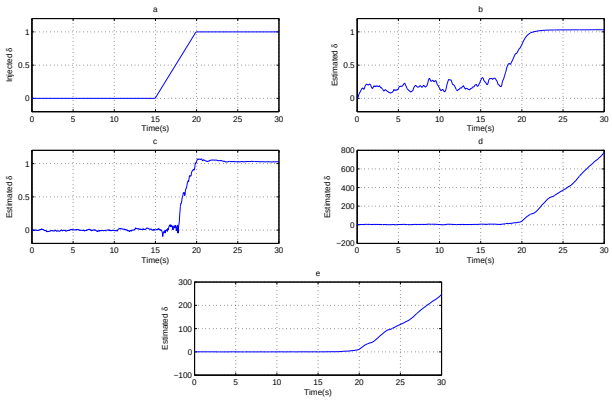

Fig. 7. Fault injected as a fast ramped-raising step (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

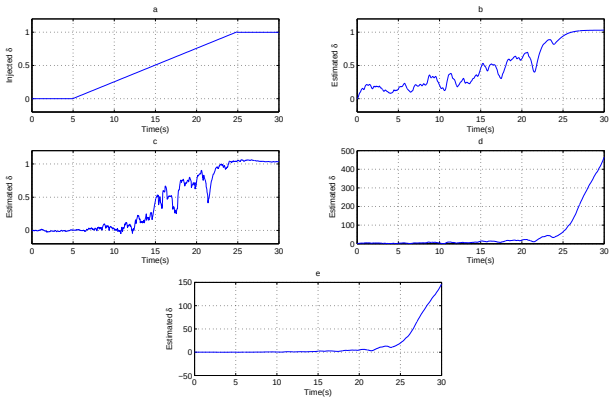

Fig. 8. Fault injected as a slow ramped-raising step (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

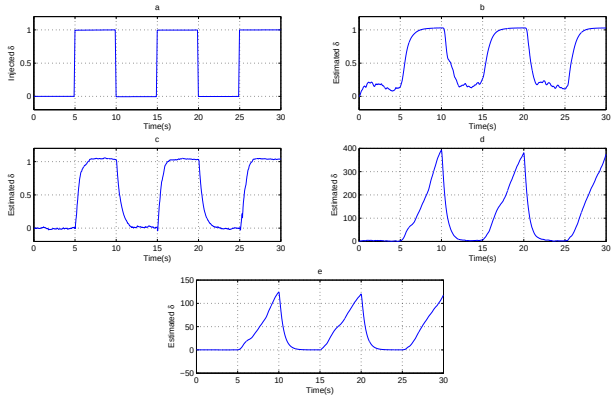

Fig. 9. Fault injected as rectangular pulses (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

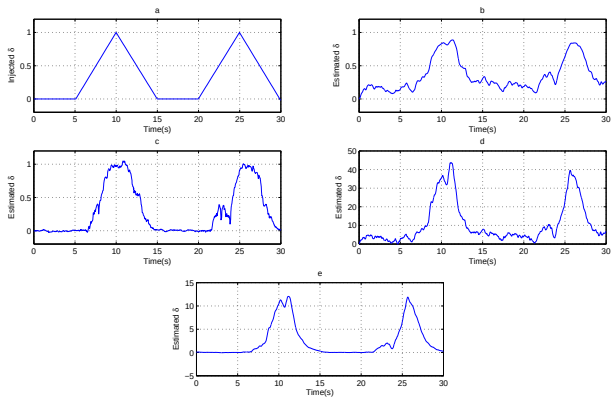

Fig. 10. Fault injected as triangular pulses (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

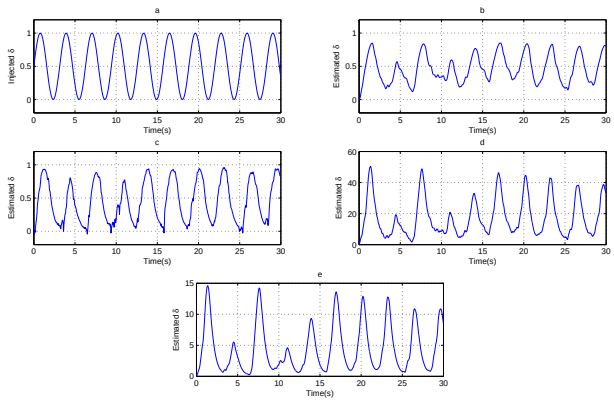

Fig. 11. Fault injected as sine with the frequency of $2\frac{rad}{s}$ (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

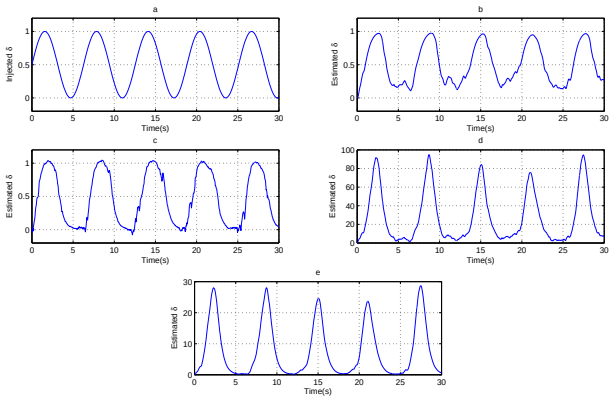

Fig. 12. Fault injected as sine with the frequency of $1\frac{rad}{s}$ (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

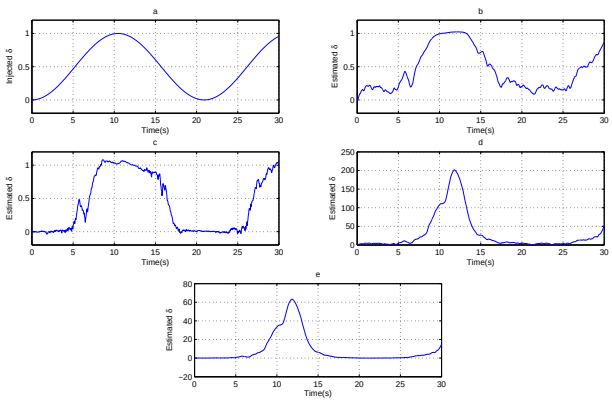

Fig. 13. Fault injected as sine with the frequency of $0.3\frac{rad}{s}$ (a) the injected δ, (b) the residual provided by $\mathscr{H}_\infty$ design, (c) the residual provided by μ design, (d) the residual provided by $\mathscr{H}_2$ / $\mathscr{H}_\infty$ design, and (e) the residual provided by $\mathscr{H}_\infty$ / LMI design.

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