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Congestion Management in a Smart Grid via Shadow Prices ⋆

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Abstract: We consider a distribution grid interconnecting a number of consumers with flexible power consumption. Each consumer is under the jurisdiction of exactly one balancing responsible party (BRP), who buys energy at a day-ahead electricity market on behalf of the consumer. We illustrate how BRPs can utilize the flexibility of the consumers to minimize the imbalance between the consumed and the purchased energy thereby avoiding trading balancing energy at unfavorable prices. Further we show how shadow prices on the distribution lines can be used to resolve grid congestion without information sharing between the BRPs.

Keywords: Predictive control, Smart power applications, Dual composition control.

1. INTRODUCTION

Due to the increasing focus on renewable energy and the rising fossil fuel prices, the penetration of renewable energy is likely to increase in the foreseeable future throughout the developed world (Department of Energy (2008)). In Denmark, the wind penetration has increased from close to zero in the 1980s to around 20 % in 2009 (Danish Energy Agency (2009)), while the Danish Government Platform states that the wind penetration should be 50 % by 2020 (Danish Government (2011)). This increase of renewable non-dispatchable production causes a balancing problem between production and demand (Banakar et al. (2008)) which is typically solved at the production side (Venkat et al. (2006, 2008); Edlund et al. (2011))

In a *smart grid*, not only the production side is active; both producers and consumers participate in the balancing efforts. The consumer side can contribute by *moving loads* in time, *e.g.* by allowing local devices with large time constants to store more or less energy at convenient times, thereby adjusting the momentary consumption, see *e.g.* (Hiskens (2006), Moslehi and Kumar (2010) and Trangbaek et al. (2010)). One obvious method to do so is by exploiting large thermal time constants in deep freezers, refrigerators, local heat pumps etc. (Pedersen et al. (2011)). Consumers with this ability to move load in time will be referred to as *flexible consumers* in the sequel.

The control of such flexible consumers in a grid of limited capacity is described in (Biegel et al. (2012)). That work treats the problem at an overall level where the energy market is not taken into consideration: both optimization and congestion management relies on all information being centrally available. However, due to the deregulation of the European power market (The European Parliament (2003)), the congestion management should be handled via markets and not by regulations. In this paper we therefore take the current electricity market as starting point: energy is bought and sold at a day-ahead market while balancing energy is traded after the hour of operation to ensure financial balance. We show how balancing responsible parties (BRPs) can utilize flexible consumers to move load in time, thereby minimizing imbalance between the energy purchased at the day-ahead market and the actual consumption. This allows the BRPs to buy energy at the day-ahead market in the hours where the energy is cheap, *e.g.* in the hours of high renewable energy generation or at night. It also minimizes the amount of balancing energy the BRP has to trade at unfavorable prices. We further show how the distribution grid constraints can be honored based on the shadow prices at distribution line capacities; in this way grid congestion can be resolved via a market and not by regulations.

The outline of the rest of the paper is as follows. First, in Sec. 2 we describe the congestion management problem under consideration. Next, in Sec. 3 we design a distributed receding horizon controller for imbalance reduction using shadow prices. Section 4 describes how to implement this structure with the current players in the electrical market, while Sec. 5 illustrates the methods with a numerical example. Finally, Sec. 6 sums up the work.

2. MODELING

We consider a number of consumers and a number of BRPs: each consumer has entered an agreement with exactly one BRP who buys energy at the energy market on behalf of the consumer. In this work we consider the future scenario where each BRP is allowed to control some flexible consumption of the consumers under their jurisdiction based on a contract between the consumer and the BRP. This flexible consumption might be a refrigerated warehouse allowing BRP to control the refrigerator temperature within some band or it could be a private household, allowing the BRP to control the exact charging pattern of the batteries of an electric vehicle. Each BRP will benefit from this by utilizing the flexibility to optimize

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the energy purchase while the consumer will benefit from the contract by some payment from the BRP.

The active control of the consumers is likely to cause congestion on the distribution grid as the BRPs often will activate the flexible consumption at the same hours of operation, namely when favorable energy prices occur. It is therefore necessary for the BRPs to consult the distribution grid operator (DSO) before activating flexible consumption, such that congestion is avoided. In the following we show how this congestion management can be settled through shadow prices.

In the following we consider a star topology distribution grid (no loops) consisting of n_L distribution lines of limited capacity. A total of n_B BRPs are active in the distribution grid and BRP number i is responsible for m_i consumers. The setup is illustrated in Fig. 1 and discussed in detail in the sequel.

In the following modeling of the system, we describe the dynamics by discrete time equations. We use k to indicate sample number and use a sample time of 1 hour to ease the notation.

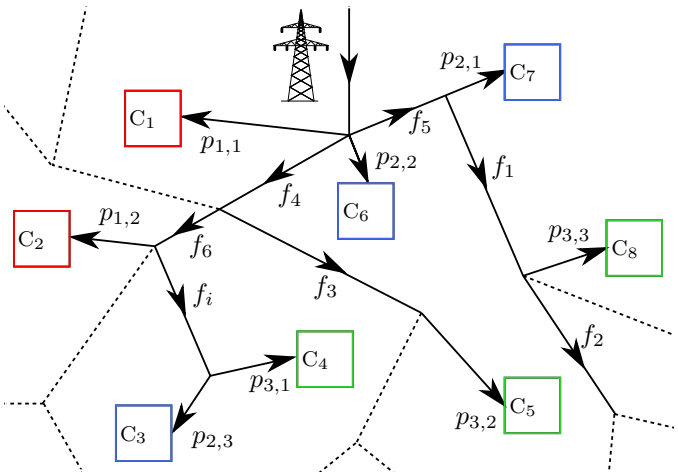

Fig. 1. Interconnected consumers under the jurisdiction of different BRPs sharing the same distribution grid (dotted lines indicate that only a small part of the total grid is shown). Note that the consumers are connected in a star-like topology, *i.e.*, there are no loops in the grid structure.

2.1 Dynamics and Constraints

The m_i consumers under BRP i are characterized by hourly energy consumptions $p_i = (p_{i,1}, \ldots, p_{i,m_i}) \in \mathbf{R}^{m_i}$ consisting of a controllable part $\tilde{p}_i \in \mathbf{R}^{m_i}$ and an uncontrollable part $\overline{p}_i \in \mathbf{R}^{m_i}$:

$$p_i(k) = \overline{p}_i(k) + \tilde{p}_i(k) \tag{1}$$

subject to hourly energy constraints

$$p_i^{\min} \preceq \tilde{p}_i(k) \preceq p_i^{\max} \tag{2}$$

where $p_i^{\min}, p_i^{\max} \in \mathbf{R}^{m_i}$ are the lower and upper limits, respectively and where $\preceq$ represents componentwise inequality. Note that with this notation, non-dispatchable producers (such as wind and solar) can be included in the model as negative consumers.

The stored energy is denoted $e_i = (e_{i,1}, \ldots, e_{i,m_i}) \in \mathbf{R}^{m_i}$; this may be energy stored as either heat, cold, energy in a battery, or similar. It depends on the controllable consumption

$$e_i(k+1) = D_i e_i(k) + \tilde{p}_i(k) \tag{3}$$

where $D_i \in \mathbf{R}^{m_i \times m_i}$ is diagonal with diagonal elements describing the proportional drain loss of each energy storage. The storages are limited in size as described by

$$0 \preceq e_i(k) \preceq e_i^{\max} \tag{4}$$

where $e_i^{\max} \in \mathbf{R}^{m_i}$ is the capacity limit of the storages under BRP i.

This setup is presented in Fig. 2 for the consumers under BRP i: the uncontrollable consumption (load) $\overline{p}_i$ is independent on the energy storage while the drainage depends on the energy level e_i.

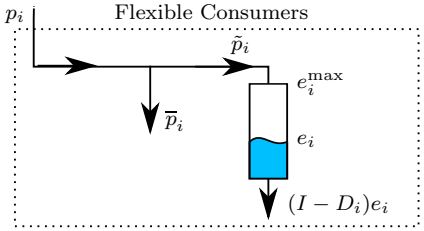

Fig. 2. Model of the intelligent consumers under BRP i (see *e.g.* Heussen et al. (2011)).

The consumers are powered through the distribution grid, as illustrated in Fig. 1. Each BRP will contribute to the loading of the distribution lines. Let $t_i \in \mathbf{R}_+^{n_L}$ denote the partial flow caused by BRP i to the n_L distribution lines. By flow conservation, *i.e.* no transmission losses, and by assuming a star topology, the partial flow caused by the consumers under BRP i is given by

$$t_i(k) = R_i p_i(k) \tag{5}$$

where $R_i \in \mathbf{R}^{n_L \times m_i}$ is given by

$$(R_i)_{mn} = \begin{cases} 1 & \text{if consumer } n \text{ is supplied through link } m, \\ 0 & \text{otherwise.} \end{cases}$$

A meshed grid topology can be modeled by reformulating (5), see Biegel et al. (2012).

The total flows $f = (f_1, \ldots, f_{n_L}) \in \mathbf{R}_+^{n_L}$ over the distribution lines are therefore given by

$$f(k) = \sum_{i=1}^{n_B} t_i(k) \tag{6}$$

where f_j is the flow through line j. The distribution grid is protected from overcurrents by electrical fuses; hence, the distribution line flows are subject to constraints

$$f(k) \preceq f^{\max} \tag{7}$$

where $f^{\max} \in \mathbf{R}_+^{n_L}$ denotes the limits of the fuses.

2.2 Objectives

The BRPs buy energy at a day-ahead spot market for each hour of the following day. We denote the energy bought by BRP i at the day-ahead spot market $q_{\text{spot},i} \in \mathbf{R}$: this means that BRP i has bought the energy $q_{\text{spot},i}(k)$ for the time interval from hour k to $k+1$.

During operation, the consumers under BRP i will consume the energy they need leading to a total hourly energy consumption $\mathbf{1}^T p_i(k)$ for the consumers under BRP i, where $\mathbf{1}$ is a vector of all ones. If this energy consumption does not match the energy bought at the day-ahead market, the BRP must settle the economic imbalance between the bought and consumed energy. This *balancing energy* is by definition traded with the transmission system operator (TSO): if the BRP has bought more energy than is consumed, he has per definition sold the excess energy to the TSO and vice versa. We denote the balancing energy $q_{\text{bal},i}$ and use the sign convention

$$q_{\text{bal},i}(k) = \mathbf{1}^T p_i(k) - q_{\text{spot},i}(k) \tag{8}$$

meaning that the regulating energy $q_{\text{bal},i}$ is positive when BRP i buys energy from the TSO and negative when the BRP sells energy to the TSO.

Trading balancing energy with the TSO is often disadvantageous for a BRP due to the prices on balancing energy. One strategy for the BRPs is therefore to minimize q_{bal} thereby avoiding trading balancing energy. This minimization of q_{bal} is currently done by estimating the future energy consumption and buying accordingly at the day-ahead spot market. Introducing flexible consumers, however, allows the BRPs to actively minimize the balancing energy during the hour of operation by utilizing the flexible consumers accordingly.

3. CONTROLLER SYNTHESIS

In this section, a controller is designed to utilize the flexible consumers under each BRP such that the imbalance is minimized. It is natural to design a receding horizon controller, as this allows us handle the constraints of the flexible consumers and to incorporate predictions of the future energy consumption (Maciejowski (2002)) which are available due to the very competitive nature of the energy market. We assume that good predictions exist $K-1$ hours into the future, and use this as a basis for the controller design in the following.

We assume that the strategy of each BRP is to minimize the balancing energy. Based on this, we describe the objective function of BRP i as a convex function of the balancing energy which we denote $\ell_i(q_{\text{bal},i}(k)) : \mathbf{R} \to \mathbf{R}_+$.

3.1 Compact Representation

To ease the notation when deriving the controller, we stack the variables introduced in the previous section: upper case variables denote the stacked version of the lower case variables, *e.g.* for $P_i(k)$ we have

$$P_i(k) = \left(p_i^T(k), \dots, p_i^T(k+K-1)\right)^T \in \mathbf{R}^{n_B K}$$

and similarly for $E_i, \tilde{P}_i, \overline{P}_i, T_i, F, F^{\max}, Q_{\text{bal},i}$ and $Q_{\text{spot},i}$.

Using this notation, we can describe the dynamics of the consumers under the jurisdiction of BRP i for time $k, \dots, k+K-1$ as follows.

$$\begin{aligned} E_i(k+1) &= \Omega_i E_i(k) + \tilde{P}_i(k) \\ Q_{\text{bal},i}(k) &= \Upsilon_i(\overline{P}_i(k) + \tilde{P}_i(k)) - Q_{\text{spot},i}(k) \\ T_i(k) &= \Psi_i(\overline{P}_i(k) + \tilde{P}_i(k)) \end{aligned} \tag{9}$$

where

$$\begin{aligned} \Omega_i &= \mathbf{diag}\,(D_i, \dots, D_i) \in \mathbf{R}^{m_i K \times m_i K} \\ \Upsilon_i &= \mathbf{diag}\left(\mathbf{1}^T, \dots, \mathbf{1}^T\right) \in \mathbf{R}^{K \times m_i K} \\ \Psi_i &= \mathbf{diag}\,(R_i, \dots, R_i) \in \mathbf{R}^{n_L K \times m_i K} \end{aligned}$$

where $\mathbf{diag}(X, Y, \dots)$ denotes a block diagonal matrix with diagonal blocks $X, Y, \dots$. We express the energy capacity constraint and rate constraints as

$$\begin{aligned} \mathcal{E}_i &= \{x \in \mathbf{R}^{m_i K} | 0 \preceq x \preceq E_i^{\max}\} \\ \mathcal{P}_i &= \{x \in \mathbf{R}^{m_i K} | P_i^{\min} \preceq x \preceq P_i^{\max}\}. \end{aligned}$$

Further, we describe the distribution line constraints as

$$F(k) = \sum_{i=1}^{n_B} T_i(k) \preceq F^{\max}. \tag{10}$$

We stack the variables

$$\begin{aligned} \eta(k) &= (\eta_1(k)^T, \dots, \eta_{n_B}(k)^T, F^T)^T \in \mathbf{R}^v \\ \eta_i(k) &= (\tilde{P}_i^T(k), E_i^T(k+1), Q_{\text{bal},i}^T(k), T_i^T(k))^T \in \mathbf{R}^{v_i} \end{aligned}$$

where $v_i = K(2m_i + n_L)$, $v = n_L + \sum_{i=1}^{n_B} v_i$ such that η_i describes the variables local to BRP i while η describes all variables. Based on this, we represent the cost of BRP i as

$$\Phi_i(\eta_i(k)) = \sum_{\kappa=k}^{k+K-1} \ell_i(q_{\text{bal},i}(\kappa))$$

and the total cost as

$$\Phi(\eta(k)) = \sum_{i=1}^{n_B} \Phi_i(\eta_i(k)).$$

3.2 Centralized Controller

Using the compact representation presented above, we can design a receding horizon controller. At time k we look $K-1$ steps ahead and solve the optimization problem

$$\begin{array}{ll} \text{minimize} & \Phi(\eta(k)) \\ \text{subject to} & E_i(k) \in \mathcal{E}_i,\ \tilde{P}_i(k) \in \mathcal{P}_i \\ & F(k) \preceq F^{\max} \end{array} \tag{11}$$

for $i = 1, \dots, n_B$ where the optimization variables are $\eta(k)$. The solution $\tilde{P}_i^\star(k)$ is the planned action for the following K steps. In a receding horizon manner, we apply the first of the planned actions $\tilde{p}_i^\star(k)$ and then redo the optimization at next time step.

Problem (11) is a convex optimization problem and thus readily solvable (Boyd and Vandenberghe (2004)). But this centralized controller has a huge disadvantage: all data must be centralized to solve the problem. In practice this means that each BRP would have to provide their cost functions, the states of all their flexible consumers, their consumption predictions, etc., to the central unit solving the problem. Due to the competitive nature of the energy market such information sharing is highly unlikely and we therefore decompose the optimization.

3.3 Distributed Controller

In the following we show how we can distribute the controller problem (11) to avoid sharing of local information among the BRPs. The centralized problem is coupled by the distribution line capacity constraints $F(k) \preceq F^{\max}$. As these are affine constraints, the problem is separable by dual decomposition (see, *e.g.*, Boyd et al. (2008), Samar et al. (2008)). By introducing Lagrange multipliers for

the coupling inequality constraints we obtain the partial Lagrangian of problem (11)

$$L(\eta(k), \Lambda(k)) = \Phi(\eta(k)) + \Lambda^T(k)(F(k) - F^{\max})$$

where $\Lambda(k) \in \mathbf{R}_+^{n_L K}$ is the Lagrange multiplier, or shadow price, associated with the inequality $F(k) \preceq F^{\max}$ (see, *e.g.*, Boyd and Vandenberghe (2004), Kelly et al. (1998)). The dual function is given by

$$g\left(\Lambda(k)\right) = \inf_{\eta(k)} \left(\Phi(\eta(k)) + \Lambda^T(k)(F(k) - F^{\max})\right).$$

A subgradient of the negative dual is given by

$$S(k) \in \partial(-g)(\Lambda(k))$$

where $\partial(-g)(\Lambda(k))$ is the subdifferential of $-g$ at $\Lambda(k)$ and where $S(k) = \overline{F}(k) - F^{\max} \in \mathbf{R}^{n_L K}$ with $\overline{F}(k)$ being the solution to the optimization problem

$$\begin{array}{ll} \text{minimize} & \Phi(\eta(k)) + \Lambda^T(k) F(k) \\ \text{subject to} & E_i(k) \in \mathcal{E}_i,\ \tilde{P}_i(k) \in \mathcal{P}_i \end{array} \tag{12}$$

for $i = 1, \ldots n_B$ (Boyd et al. (2008)) where the optimization variables are $\eta(k)$. This optimization is completely separable between the n_B BRPs, and can therefore be solved distributedly. For BRP i the optimization problem becomes

$$\begin{array}{ll} \text{minimize} & \Phi_i(\eta_i(k)) + \Lambda^T(k) T_i(k) \\ \text{subject to} & E_i(k) \in \mathcal{E}_i,\ \tilde{P}_i(k) \in \mathcal{P}_i \end{array} \tag{13}$$

where the optimization variables are $\eta_i(k)$. Solving problem (13) for $i = 1, \ldots, n_B$ gives flows $\overline{T}_i(k)$ that can be used to find a subgradient

$$S(k) = \sum_{i=1}^{n_B} \overline{T}_i(k) - F^{\max}. \tag{14}$$

3.4 Subgradient Algorithm

The centralized problem (11) is solved distributedly by the following algorithm where we use the subgradient method.

(1) Initialize dual variable $\Lambda(k) := \Lambda_0(k) \succeq 0$, *e.g.* using $\Lambda_0(k) = 0$ or $\Lambda_0(k) = \Lambda(k-1)$.
(2) **loop**
- Optimize flows using the dual variable $\Lambda(k)$ by locally solving problem (13).
- Determine capacity margins $S(k)$ based on the solutions $\overline{T}_i(k)$ to the subproblems using (14).
- Update dual variables $\Lambda(k) := (\Lambda(k) + \alpha_k S(k))_+$.

(3) Terminate by providing flows limits $T_i^{\max}(k)$ to each BRP base on the final solutions $\overline{T}_i(k)$.
(4) Increase k by one and go to step 1.

In the algorithm, $\alpha_k \in \mathbf{R}_+$ denotes the step size and can be chosen any standard way, *e.g.* square summable but not summable

$$\sum_{k=1}^{\infty} \alpha_k^2 \leq \infty, \quad \sum_{k=1}^{\infty} \alpha_k = \infty$$

such that convergence is guaranteed (Boyd et al. (2008)).

To ensure feasibility when the loop (step 2) is terminated, maximum partial flow limits $T_i^{\max}$ are provided to the BRPs (step 3) based on the final solutions $\overline{T}_i(k)$:

$$T_i^{\max}(k) = A\overline{T}_i(k)$$

where $A \in \mathbf{R}^{n_L K \times n_L K}$ is diagonal with entries $A_{jj} = F_j^{\max} / \left(\sum_{i=1}^{n_B} \overline{T}_i\right)_j$. This assures feasibility using backtracking. Each BRP must then ensure that their partial flow honor $T_i(k) \preceq T_i^{\max}(k)$.

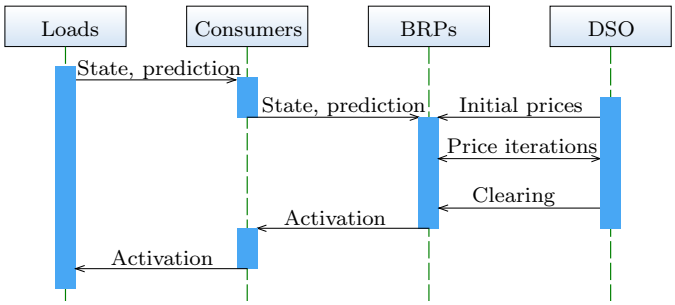

Fig. 3. Interaction between consumers, BRPs and the DSO resolving congestion in a distributed manner.

It is important to notice that the problem of finding dual variables is a simple summation and therefore is scalable even to a large number of BRPs.

4. MARKET IMPLEMENTATION

In this section, we describe how the distributed algorithm can be understood in an electrical market setting.

4.1 Interplay between BRPs and DSO

The interacting players are the BRPs, who utilize the distribution grid, and the distribution system operator (DSO), who is responsible for safe grid. At hour k, each distribution line is initially associated with non-negative prices $\Lambda_0(k)$. Based on these prices and based on state information aggregated from the loads of the consumers, each BRP locally optimizes their own portfolio, see Fig. 3. The BRPs then inform the DSO of their partial flows $\overline{T}_i(k)$ under the initial prices.

By summing all the partial flows, the DSO determines if the distribution grid is overloaded or underloaded; for an overloaded line the price is increased, for an underloaded line the price is decreased, according to the presented algorithm (illustrated by the price iteration double-arrow in Fig. 3). The prices will eventually converge to the shadow prices of the centralized problem (11): the distribution line prices will equal the marginal prices that a BRP is willing to pay for an additional unit flow in each distribution line and the BRPs will reach the global optimum (within the horizon) without information sharing.

When the duality gap is sufficiently small, or after a fixed number of iterations, the DSO stops the iterations by sending final partial flow constraints $T_i^{\max}$ to BRP i and by publishing the final distribution line prices $\Lambda^*(k)$. The BRPs can now activate the flexible consumption as desired under the constraint $T_i \preceq T_i^{\max}$, see Fig. (3).

4.2 Settlement

The BRPs pay tariffs to the DSO for utilizing the distribution grid. Let $t_i^{\text{tariff}} \in \mathbf{R}^{n_L}$ denote the capacity of each line in the distribution grid, which BRP i has paid for through the tariffs, *e.g.* based on yearly tariff averages. Further, let $\sum_{i=1}^{n_B} t_i^{\text{tariff}} = f^{\max}$, such that the total capacity is divided among the BRPs. Based on this, the additional cost $c_i(k)$ of BRP i at time k is given by

$$c_i(k) = \lambda^{*T}(k) \left(t_i^{\max}(k) - t_i^{\text{tariff}}\right) \tag{15}$$

where $\lambda^*(k)$ are the final distribution line prices at time k.

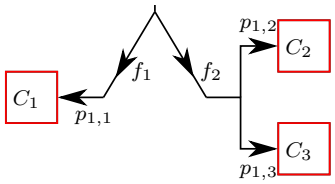

Fig. 4. Three interconnected consumers sharing the same distribution grid.

The interpretation of the suggested settlement is straightforward: if $c_i(k) > 0$, BRP i has utilized the distribution grid more than paid for via tariffs in an hour of congestion and will have to pay the amount $c_i(k)$. If $c_i(k) < 0$, BRP i used less capacity than paid for through tariffs in an hour of congestion to the advantage of other BRPs and will be paid the amount $-c_i(k)$. Finally, if $c_i(k) = 0$, there is no congestion on the grid or BRP i has used the exact grid capacity paid for through tariffs. Note that $\sum_{i=1}^{n_B} c_i(k) = 0$, meaning that this settlement is internally between the BRPs; the DSO will only earn money through the tariffs, not the shadow prices.

5. NUMERICAL EXAMPLES

In this section we first illustrate how the BRPs can benefit from utilizing the flexible consumers and secondly how grid congestion can be alleviated via shadow prices. We keep the examples at a conceptual level with a low number of consumers to make examples easy to follow.

5.1 Utilization of Flexible Consumers

We consider a simple case with a single BRP with three consumers C_1, C_2, C_3 under its jurisdiction, see Fig. 4. The characteristics of the consumers and the grid are

$$p_1^{\max} = -p_1^{\min} = (0,\ 30,\ 30)^T, \quad f_1^{\max} = (200,\ 90)^T$$
$$e_1^{\max} = (0,\ 200,\ 200)^T, \quad D_1 = \mathbf{diag}(0,\ 0.80,\ 0.99)$$

while the cost function is chosen to be

$$\ell_1(q_{\text{bal},1}(k)) = \|q_{\text{bal},1}(k)\|_2^2.$$

The characteristics show that C_1 is not controllable while C_2, C_3 are controllable with identical capacity and rate limits, but with higher storage quality in C_3 than C_2. The line capacity constraints lead to congestion on distribution line 2, but no congestion on line 1.

The top of Fig. 5 shows the predicted consumptions of C_1, C_2 and C_3; the total area thus corresponds to $\overline{p}_1(k)$. The red dashed line illustrates the energy bought at the day-ahead spot market $q_{\text{spot},1}(k)$. As is seen from the plot, not enough energy is bought in the hours of high consumption, while excess energy is bought in the hours of low consumption. This could represent a BRP buying cheap energy at night thereby being able to buy less energy in the expensive peak hours.

The lower plot of Fig. 5 shows how the controller uses the flexible consumption of C_2 and C_3 to alter the consumption pattern by solving problem (11). The corresponding utilization of the storages e_1 is illustrated in the top plot of Fig. 6 where the solid green line shows the storage utilization of C_3 and the blue dashed line shows that of C_2. The figure shows that the flexible consumers fill their energy reserves in the first hours, where excess energy is

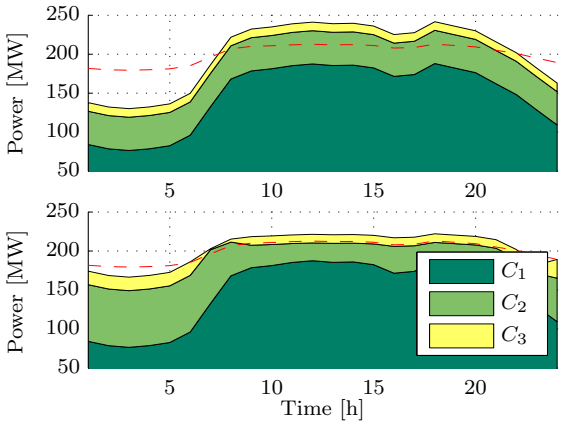

Fig. 5. Power consumption predictions of the three consumers (shaded areas) compared to the energy bought at the day-ahead spot market (red, dashed)

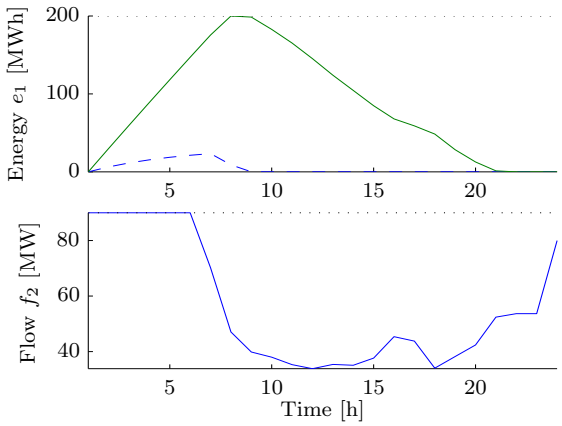

Fig. 6. Top: Energy levels of the flexible consumers C_2 (blue, dashed) and C_3 (green, solid). Bottom: Power flow in distribution line 2. The capacity constraints are shown in both plots (black, dotted).

bought at the day-ahead market, and empty their storages in the hours of missing energy. This utilization of the flexible consumers causes congestion on distribution line 2, which is illustrated in the lower plot of Fig. 6. Due to the congestion, the flexible consumers cannot both be fully utilized: as seen from the top plot, only the good storage of C_3 is fully utilized reaching both the capacity limit and the rate limit, while the storage capacity C_2 is only slightly utilized. Finally we note that the storage of C_2 discharges as soon as energy is needed (around $k = 7$), while the storage of C_3 does not discharge until later, again due to the fact that storage 3 is of higher quality than storage 2.

5.2 Distribution Grid Prices

We consider the case where C_1 and C_2 is under the jurisdiction of BRP 1 while C_3 is under the jurisdiction of BRP 2. Conflicting objectives cause congestion on the shared distribution line 2, see Fig. 7. Both BRP 1 and 2 desire to increase the controllable consumption in the first hours, and decrease the consumption in the later hours, as in the previous example. If no action is taken, this will violate the capacity constraint $f_2 \leq f_2^{\max}$.

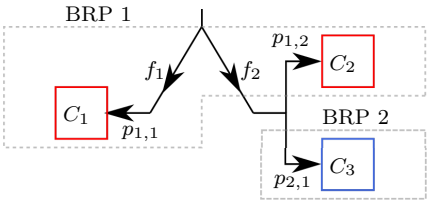

Fig. 7. Three consumers under the jurisdiction of two different BRPs sharing the same distribution grid.

To remedy the problem without information sharing, shadow prices are introduced by following the suggested algorithm. The DSO starts by publishing the initial prices $\Lambda(1) = 0$ where after the two BRPs report back to the DSO how they then plan to utilize the distribution grid, by respectively sending $T_1(1)$ and $T_2(1)$, to the DSO. The DSO discovers that congestion will occur with the initial prices and therefore updates the prices $\Lambda(1) := \Lambda(1) + \alpha S(1)$. The top plot of Fig. 8 shows the price adjustments, converging to the shadow prices $\Lambda^\star(k)$, optimally resolving the congestion (within the given horizon).

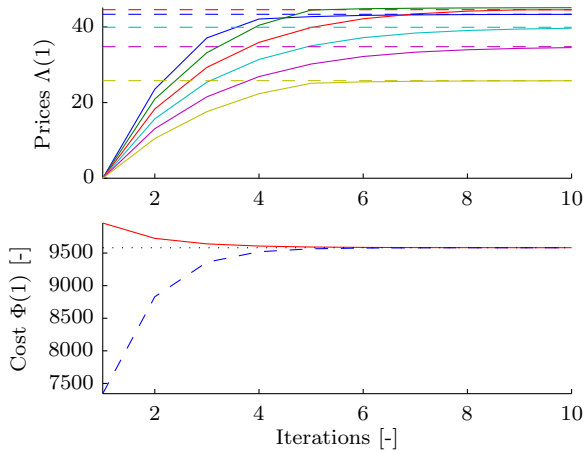

Fig. 8. Top: Convergence of $\lambda_2(1), \ldots, \lambda_2(6)$ (solid) towards the shadow prices (dashed). Bottom: Primal (red, solid) and dual (blue, dashed) objective.

Further, we observe the convergence of the optimization by looking at the primal and dual objective at each iteration. This is illustrated in the lower plot of Fig. 8. The solid red line shows the primal objective when using feasible flows while the blue dashed line is the dual objective and the black dotted line is the optimal value within the control horizon.

6. CONCLUSION

In this paper, a receding horizon control approach was proposed for the control of flexible consumers under the jurisdiction of a BRP allowing the net consumption to be moved in time. We further showed how different BRPs sharing the same distribution grid could obtain the global optimum via the shadow prices at the distribution grid capacities thereby avoiding sharing local information. Finally we suggested how this approach could be implemented in an energy market by an appropriate communication pattern between the BRPs and the DSO.

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