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Data Driven Modelling of the Dynamic Wake Between Two Wind Turbines ⋆

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AbstractWind turbines in a wind farm, influence each other through the wind flow. Downwind turbines are in the wake of upwind turbines and the wind speed experienced at downwind turbines is hence a function of the wind speeds at upwind turbines but also the momentum extracted from the wind by the upwind turbine. This paper establishes flow models relating the wind speeds at turbines in a farm. So far, research in this area has been mainly based on first principles static models and the data driven modelling done has not included the loading of the upwind turbine and its impact on the wind speed downwind. This paper is the first where modern commercial mega watt turbines are used for data driven modelling including the upwind turbine loading by changing power reference. Obtaining the necessary data is difficult and data is therefore limited. A simple dynamic extension to the Jensen wake model is tested without much success. The best model turns out to be non linear with upwind turbine loading and wind speed as inputs. Using a transformation of these inputs it is possible to obtain a linear model and use well proven system identification methods. Finally it is shown that including the upwind wind direction to explain the wake improve the prediction performance.

Keywords: System identification; Turbines; Wind speeds; Prediction

1. INTRODUCTION

This work is part of the EU project *Distributed Control of Large-Scale Offshore Wind Farms* with the acronym *Aeolus*. The overall goal in Aeolus is to optimize power production and minimize fatigue loads in a wind turbine farm by exploiting that the wind turbines share a common flow field [Madjidian and Rantzer, 2011, Grunnet et al., 2010, Knudsen et al., 2009, Aeolus]. The wake behind a wind turbine is the area where the wind speed are decreased compared to what it would have been if the turbine was not there. The flow field can be changed by controlling the power set point on individual turbines. When the power set point for a turbine is decreased the wake behind the turbine also decreases. To use this for control of a farm it is crucial to know both the size and the dynamics of this wake effect. Moreover, prediction models for wind speed at a turbine are useful for control design methods such as e.g. LQG and MPC.

Static wake models based on first principles can be found in the literature [Jensen, 1983, Frandsen et al., 2006]. In Jensen [1983] the well known *Jensen* model is described. Also dynamical models have been suggested. In Larsen et al. [2008] a model is developed based on the assumption that the wake behind the turbine travels like a passive tracer in the flow field. However, this model is not verified experimentally. Considerable amounts of data from large commercial wind farms has been available to the Aeolus projects. Until 2011 all these data were from normal operation with out power set point changes. Based on these data models taking upwind wind speed as input and downwind wind speed as output has been developed [Knudsen et al., 2011]. In the summer of 2011 the first short data series became available where the upwind turbine was excited by changing power reference. To the authors knowledge these data is the first of its kind.

The contribution in this paper is the application of system identification to this particular wake modelling problem where data driven modelling for dynamic models has not been applied before.

The paper continues with a brief presentation of the experiment and data. Then different wake models from the literature, mainly based on physics, are discussed. Based on this, different linear and non linear model structures are developed, parameters are estimated and the results are discussed. Finally a conclusion is made.

2. WIND FARM DATA

Thanet wind farm is located approximately 12 km off Foreness Point, the most eastern part of Kent in England. It consist of 100 V90 3MW turbines. The Aeolus project has been so fortunate to get measurements from a number of turbines. So far only one measurement series have been obtained that is suitable for this investigation.

In this experiment there are measurements from two turbines placed in the corner of the wind farm. The measurements are started when the wind direction is along the first row and the average speed is close to rated wind speed. The upwind turbine faces the free undisturbed flow

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and the wake has on the average a direction towards the downwind turbine. This is illustrated by the met mast measurements in figure 1 where the wind direction measured by the met mast is nicely centered around the row direction of 318 degrees.

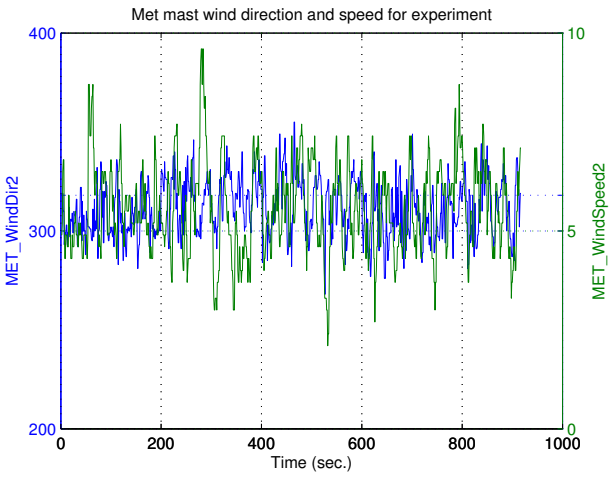

Figure 1. Met mast data for the experiment. The turbine row direction is also indicated with a extra dotted blue line.

The experiments last for approximately 15 min corresponding to 900 sec and samples are collected with a sampling frequency of 1 Hz. The excitation is only at the upwind turbine and it consists of a derating of the power reference which occurs three times and last for 1 min, 1 min and 5 min respectively. Figure 2–3 shows relevant signals for the upwind and downwind turbine. Statistics for these signals are summarized in table 1.

In table 1 it is clearly seen that the average wind speed as expected is lower at the downwind turbine compared to the upwind turbine. Also the turbulence is higher downwind. The met mast wind direction is 312 compared to the turbine row direction which is 318 degrees. All these observations shows that the downwind turbine is in wake on the average. On the other hand the wake is known to meander [Larsen et al., 2008] and the wind direction upwind in figure 2 is not constantly 318 degrees so the downwind turbine will experience a wake that covers more or less the turbine rotor. The power reference excitation is seen in figure 2 as periods with constant power reference which is lower than the available power which causes the turbine to control rotational speed by pitching. In between these periods the turbine is in normal operation where the nominal generator power is higher than the available power and then the rotational speed is controlled by generator torque i.e. power. As seen in figure 3 the downwind turbine is in normal operation all the time. The careful reader may notice the smaller met mast average wind speed, which is most likely a result of the mast being placed in the middle of the farm where there is a lot of wakes.

3. WAKE MODELS

After having defined the experimental conditions above the next step is to chose inputs and outputs and the rest of the model structure.

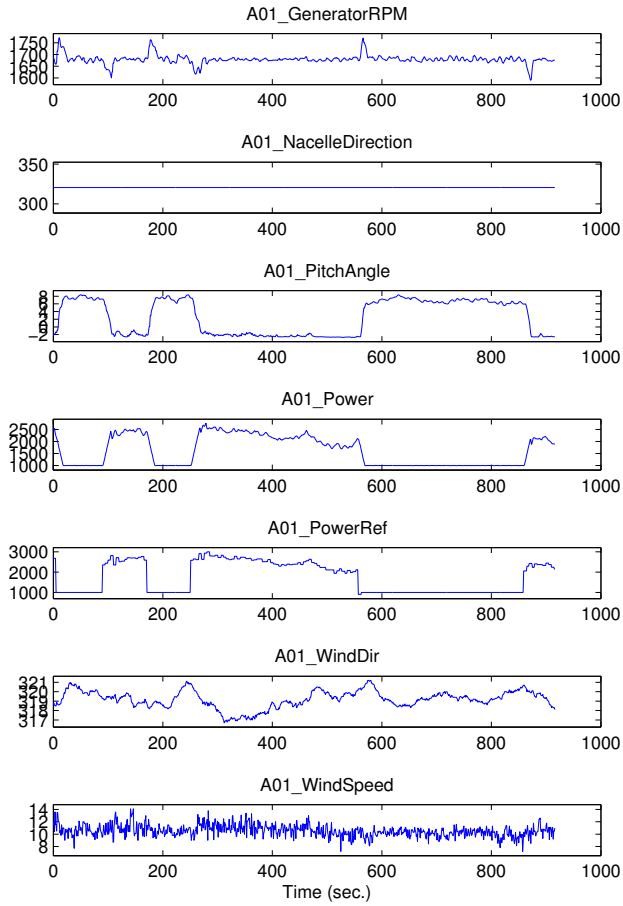

Figure 2. Upwind turbine with power reference excitation.

Stat	Gen. rpm	Nac. dir.	Pitch angle	Power	Power ref.	Wind dir.	Wind speed
			Upwind turbine				
Mean	1679	320.5	2.58	1601	1727	319	10.5
Std	18.9	0	4.55	619	762	0.919	0.971
			Downwind turbine				
Mean	1571	302.8	-2.18	1267	1531	300	8.92
Std	132	2.81	1.07	366	369	3.22	1.65
			Met mast				
Mean						312	5.57
Std						14.5	1.15

Table 1. Statistics for the experiment

3.1 Input and output

The purpose of the model described in this paper is to predict the relevant wind speed at a downwind turbine from measurements from an upwind turbine. The output is the *relevant wind speed* at the downwind turbine. The wind speeds used in the model are not the wind speed measured directly at the nacelle as they represent wind speeds in one point in the rotor area and changes with the pitching and rotor speed even when the free stream wind speed is constant. The relevant wind speed is the so called *effective wind speed* (EWS). In reality the turbine is exposed to a wind field that changes in both time and space. The corresponding EWS is the wind speed that gives a similar behavior when applied to the whole rotor. This similar behavior of course only covers the main structural dynamics as e.g. rotor speed, tower movements

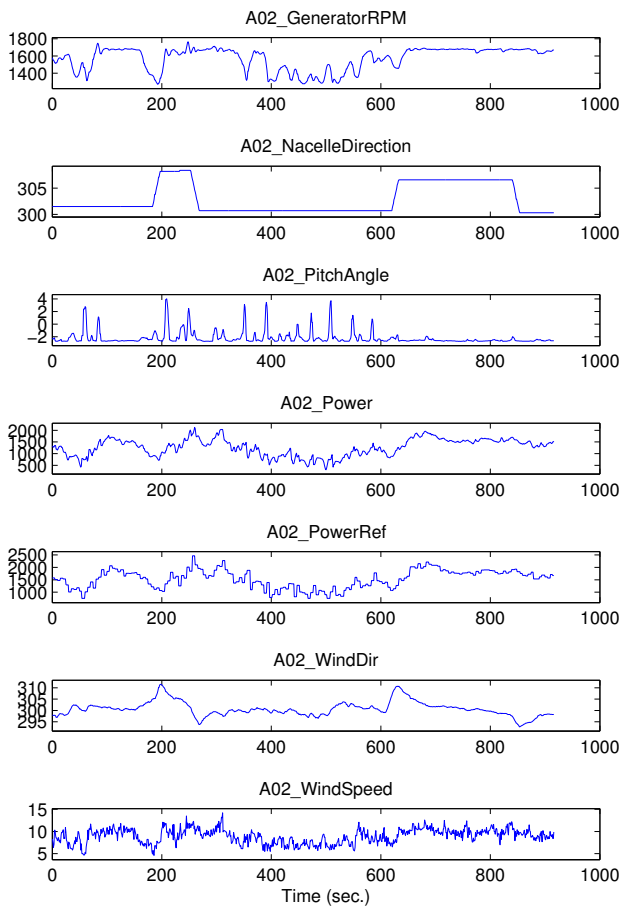

Figure 3. Downwind turbine in normal operation.

and produced power whereas individual blade movements are not covered. EWS is estimated with an extended Kalman filter from standard turbine signals as described in Knudsen et al. [2011]. The resulting EWS for the two turbines is shown in figure 4 where it is clear that the EWS is larger for the upwind turbine compared to the downwind.

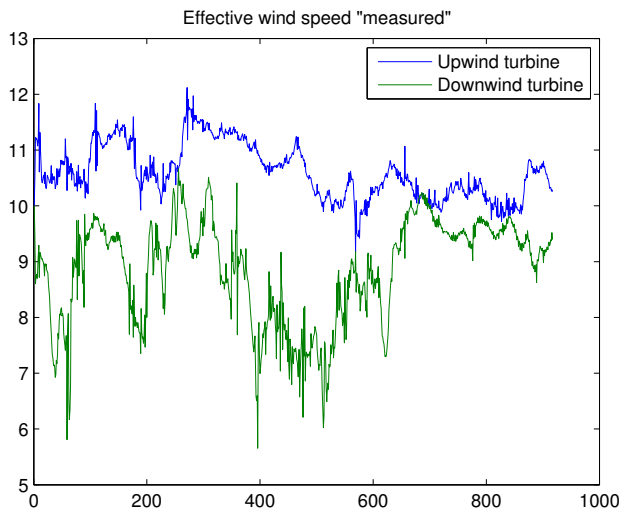

Figure 4. EWS calculated from measurements in figure 2–3.

Downwind turbine EWS is the model output and the first model input is the upwind turbine EWS. The second input should relate to turbine loading. Each turbine in a farm has a local controller that secures safe and optimal operation by actuation of blade pitch and/or generator torque depending on average wind speed. Control of the turbines by a farm level controller can be done by changing each turbines power set point. From a control engineering point of view choosing the upwind turbine power set point as the second input would hence be the immediate choice. The physics of the process, however, indicate that the trust coefficient, C_T, for the wake generating turbine is also a choice as outlined below.

There are a number of static models for wakes [Frandsen et al., 2006]. One of the simplest is the Jensen model [Jensen, 1983]:

$$v_d = (1 - kC_T)\, v_u \ , \tag{1}$$

$$k = \frac{1}{2\left(1 + \frac{d}{2D}\right)} \ , \tag{2}$$

where v_d is the wind speed at a distance d downwind from the wake generating turbine, v_u is the EWS, D the rotor diameter and C_T is the trust coefficient of the wake generating turbine. For a specific turbine the trust coefficient is a function of blade pitch and the ratio between blade tip speed and wind speed. This can all be calculated [Burton et al., 2008] from the standard signals shown in figure 2. The wake factor in (2) is only dependent on farm geometry, for the farm used in this experiment it is $k = 0.1364$. The resulting C_T for the two turbines is shown in figure 5. Figure 5 clearly shows the large variation in C_T for the upwind turbine due to the power reference excitation. C_T will be zero for zero power and increase to $\frac{8}{9}$ at maximum power and it can not exceed 1. Eq. (1) shows that the wake can be controlled by the upwind turbine power reference.

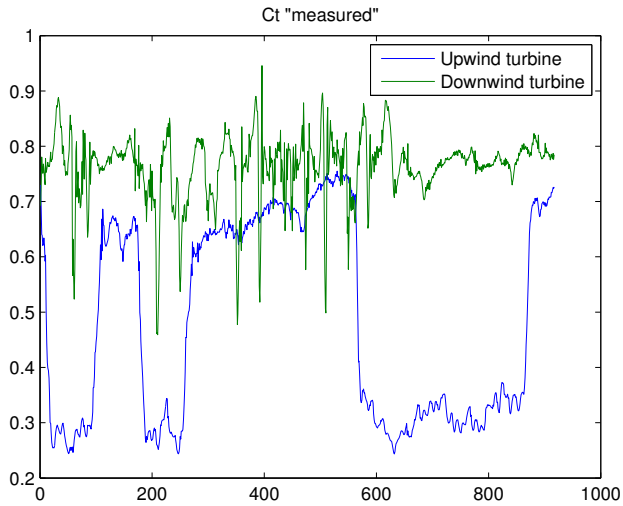

Figure 5. C_T calculated from measurements in figure 2–3.

Based on the above, both upwind power reference and C_T can be used as input together with the upwind EWS. Also, it might be worthwhile to include the upwind wind direction from the nacelle measurements. However, experience indicates that this measurement might not be calibrated to compass direction.

3.2 Dynamics

The dynamical part of the model has two components. When the loading and hence the C_T of the upwind turbine changes the wake is assumed to develop as a first order system with the time constant [Sørensen and Madsen, 2006]:

$$\tau = \frac{3D}{2v_m} , \tag{3}$$

where v_m is the 10 minutes average wind speed. The wake has to travel with the flow to the next turbine which according to Larsen et al. [2008] approximately results in the delay:

$$t_d = \frac{d}{v_m} \tag{4}$$

4. SYSTEM IDENTIFICATION

Based on the above different model structures can be suggested for a dynamic model.

- Extend the Jensen wake model (1) to a dynamic model in the simplest possible way.
- Include filtering of the upwind EWS input.
- Include filtering of the upwind C_T input.
- Include upwind power reference instead of C_T.
- Include the upwind wind direction to modify the wake.

Even more models can be generated from combining the choices above. Many of these has been tested. The more interesting and successful ones are presented below. Initially the model orders was limited to one to keep things simple but also because of the limited amount of samples.

4.1 The multiplicative model

In Madjidian and Rantzer [2011] a dynamic wake model covering all turbines in a row has been developed. This model is particularly suitable for distributed control design as it only needs communication between neighboring turbines. If only two turbines are considered and the upwind faces the free flow, the model in Madjidian and Rantzer [2011] reduces to (5) below. This model can also be interpreted as the simplest possible extension of the Jensen wake model (1) into a dynamic model.

$$v_d(t) = (1 - kC_T(t - t_d))v_u(t - t_d) + w(t - t_d) , \tag{5}$$

$$w(t - t_d) \triangleq \frac{1}{1 + dq^{-1}} e(t) , \; e(t) \in \mathrm{NID}(0, \sigma_e^2) \tag{6}$$

Above, w is a stochastic process that approximates the wind turbulence spectrum. Here a simple AR(1) process is used for w. In the model the up and downwind wind speeds are the EWS at the turbines. In the estimated models the delay parameter t_d is estimated as (4) i.e. the inter turbine distance divided by the average wind speed which is 49. Other methods such as DELAYEST from Matlab, which uses multiple ARX models, have been tested but gave worse results. For longer time series the delay has to be estimated in a adaptive manner.

The system identification toolbox for Matlab [Ljung, 2010] is used for the actual estimation. To use the standard procedures for estimating k in (5) it is convenient to define the auxiliary output y as follows:

$$y(t) = v_d(t) - v_u(t - t_d) , \; u(t) = C_T(t - t_d)v_u(t - t_d) \tag{7}$$

then

$$y(t) = -ku(t) + w(t - t_d) \tag{8}$$

The estimation results for the multiplicative model (5) are shown in table 2. The first row in the table marked ARX is where the noise w in (5) and (7) is assumed white such that (7) becomes a ARX model structure. The second row is where w is assumed to be a LP filtered white noise so the model becomes a Box Jenkins (BJ) model structure. The main results for these two model structures are quite similar. However, residual test show some differences. Both the ARX and the BJ model gives no correlation between residuals i.e. one step prediction errors and input. But as expected the BJ has close to white residuals whereas the ARX model shows a large LP type auto correlation in the residuals. Most important the multiplicative model gives at least some positive fit. The estimate of the k parameter at 0.353 is much higher than the so called Jensen model giving k at 0.136. The data hence suggest that the effect of derating being larger than prescribed by the Jensen model.

Model	$\hat{k}$	$\hat{d}$	Rms-S	Fit-S	Rms-P	Fit-P
Mult ARX	0.353		0.71	0.224	0.71	0.224
Mult BJ	0.356	-0.88	0.71	0.224	0.71	0.228

Table 2. Parameter estimates and performance for the multiplicative models. Rms-S is rms values for simulation errors and Rms-P is for 30 seconds prediction errors.

The simulation performance for the multiplicative model is seen in figure 6. Clearly the simulation follows the measurements to some extent but it is difficult to identify the effect of the power reference changes. The reason for

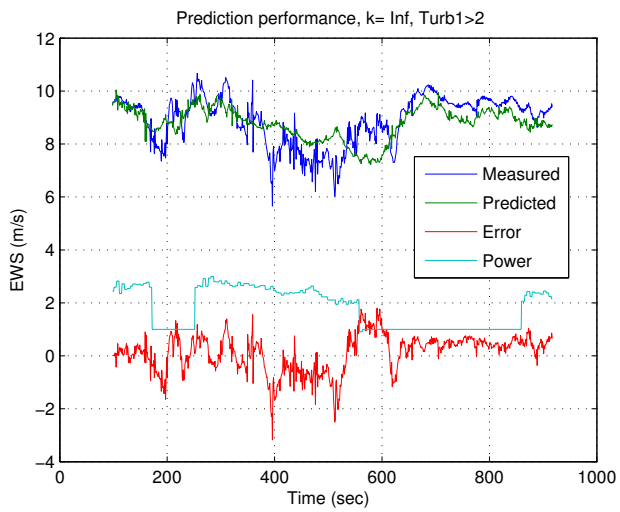

Figure 6. Simulation performance for the multiplicative BJ model. The power is on the upwind turbine. The other signals are for the downwind turbine.

this is mainly that the model assumes the downwind wind speed to be a pure scaling of the upwind wind speed plus some added turbulence. From a physical point of

view the faster variations in the upwind wind speed will be expected to change or disappear before reaching the downwind turbine. This therefore suggests LP filtering of the upwind wind speed which is the topic of the next section.

4.2 General transfer function models

When leaving the well motivated but rather inflexible model structure (5) the above discussion suggest to include LP filtering of input variables. From a physical point of view it can not apriori be assumed that the dynamics related to upwind EWS, wake and turbulence is the same. Consequently, individual transfer functions have to be tested.

The interesting models are listed below:

$$v_d(t) = \frac{1}{1+aq^{-1}}\left(bv_u(t-t_d)+e(t)\right) \tag{9}$$

$$v_d(t) = \frac{b}{1+fq^{-1}}v_u(t-t_d) + \frac{1+cq^{-1}}{1+dq^{-1}}e(t) \tag{10}$$

$$\begin{aligned} v_d(t) =& \frac{1}{1+aq^{-1}}\big(b_v v_u(t-t_d) \\ &+ b_c C_T(t-t_d)v_u(t-t_d) + e(t)\big) \end{aligned} \tag{11}$$

$$\begin{aligned} v_d(t) =& \frac{b_v}{1+f_v q^{-1}}v_u(t-t_d) \\ &+ \frac{b_c}{1+f_c q^{-1}}C_T(t-t_d)v_u(t-t_d) \\ &+ \frac{1+cq^{-1}}{1+dq^{-1}}e(t) \end{aligned} \tag{12}$$

$$e(t) \in \mathrm{NID}(0,\sigma_e^2) \tag{13}$$

The first two (9) and (10) are single input single output (SISO) models with only upwind EWS to predict downwind EWS. These models are only included to test the improvement from including loading. The first one (9) is a ARX model with common dynamics to test the improvement of the BJ model (10). The next two are multiple input (MISO) models. The models (11) and (12) includes the extra input vC_T to describe the wake effect due to loading. The thrust coefficient C_T could be included in a non linear model. By adding the extra input vC_T the model is kept linear and the wake is proportional to v as in the physical model (1). In (12) the loading is measured by the trust coefficient as suggested by physics [Jensen, 1983]. It has also been tested to replace C_T by the upwind generator power divided by nominal power as this also measures loading.

From a wind turbine engineering point of view time constants is a common way to present dynamics. Therefore they are included in the results according to the ZOH transformation:

$$\begin{aligned} G_d(q^{-1}) &= \frac{bq^{-1}}{1+fq^{-1}} \Rightarrow \\ G_c(s) &= \frac{b}{1+f}\frac{1}{1+\tau s}\ ,\ \tau = -\frac{1}{\ln(-f)} \end{aligned} \tag{14}$$

Also the size of the wake dependence on loading is of interest in wind engineering. Therefore a factor corresponding to k in (1) is calculated for the model (12). This is done by rewriting the stationary version of (12) in a form similar to (1) as follows.

$$\begin{aligned} v_d &= k_v v_u + k_c C_T v_u \\ &= (1+k_v-1)v_u + k_c C_T v_u \\ &= v_u + k_c C_T v_u + (k_v-1)v_u \quad \Rightarrow \\ &= v_u + \left(k_c + \frac{k_v-1}{C_T}\right)C_T v_u \end{aligned} \tag{15}$$

$$k \sim -\left(k_c + \frac{k_v-1}{C_T}\right)\ ,\ k_v = \frac{b_v}{1+f_v}\ ,\ k_c = \frac{b_c}{1+f_c} \tag{16}$$

An estimate of k from (16) is obtained by inserting all the parameter estimates and then the average value for C_T.

The results are given in table 3. The ARX SISO model has a bad fit. The BJ SISO model improves this a little due to different dynamics for input and disturbance/turbulence. Including the upwind turbine loading in the model via C_T significantly improves the model as the rms values are close to half of the previous models rms. The BJ model is again superior due to individual dynamics. The first steady state gain for the BJ MISO model is close to 1 as expected. The second steady state gain can be compared to the k parameter as discussed above. The wake factor $k \sim 0.38$ is similar to the one in table 2 for the multiplicative model and thus much higher than the Jensen parameter. There is a quite heavy LP filtering of the upwind EWS as $\tau_v \sim 131$. The dynamics related to the wake dependence on loading is faster with $\tau_c \sim 17$ which can be compared with the dynamic inflow time constant from (3) which here is $\frac{3\times 90}{2\times 10.6} = 12.7$. The disturbance/turbulence time constant seconds can be compared to the value $\frac{2L}{\pi v_m} = \frac{2\times 170.1}{\pi 10.6} = 10.2$ used in the Kaimal spectrum in Knudsen et al. [2011]. This value is for point wind speed which is the reason why it is smaller compared to the values here around 22 second which is for EWS i.e. rotor averaged wind speed. The MISO models are also clearly superior to the multiplicative models. The smallest rms values are obtained in the last rows of table 3 where the power is used for measuring the loading. This does not comply with physics and furthermore the very fast dynamics for the upwind EWS input is a bit surprising.

Finally the simulation performance of the best BJ MISO models is shown in figure 7. To see the effect of the power reference changes this signal scaled to MW is also included. Residual test for the BJ MISO model (12) has also been performed. Both residual auto correlation and cross correlation between residual and input variables are within significance limits.

4.3 Non linear models including wind direction

The MISO models (11)–(12) are non linear in the two inputs v_u and C_T but they are linear when the inputs are chosen as v_u and $v_u C_T$. This linearity is hard to retain when including the wind direction measured on the upwind turbine. The wake profile in the lateral direction is assumed to have a bell shape around the wake center. A good way to account for this is to include this profile as a Gaussian function multiplied with the wake i.e. to exchange the wake term in e.g. (12) with

$$C_T(t-t_d)v_u(t-t_d)e^{-\left(\frac{\theta(t-t_d)-\theta_b}{\theta_w}\right)^2}\ , \tag{17}$$

Model	k_v	k_c	k	τ_v	τ_c	τ_d	Rms-S	Fit-S	Rms-P	Fit-P
ARX SISO	0.836			21.3			1.13	-0.225	0.97	-0.051
BJ SISO	0.838			6.6		84.5	1.09	-0.198	0.84	0.073
ARX MISO-CT	1.065	-0.481	0.378	7.5			0.643	0.301	0.64	0.303
BJ MISO-CT	1.072	-0.502	0.387	130.8	17.2	22.7	0.585	0.364	0.585	0.364
ARX MISO-P	1.065	-0.428		6.4			0.584	0.365	0.583	0.366
BJ MISO-P	1.079	-0.443		2.1	7.6	20.9	0.564	0.386	0.560	0.390
BJ MISO-CT-Dir-NL	1.075	-0.592	0.473	47.4	20.4	19.4	0.542	0.410	0.559	0.392

Table 3. Parameter estimates and performance for the models.

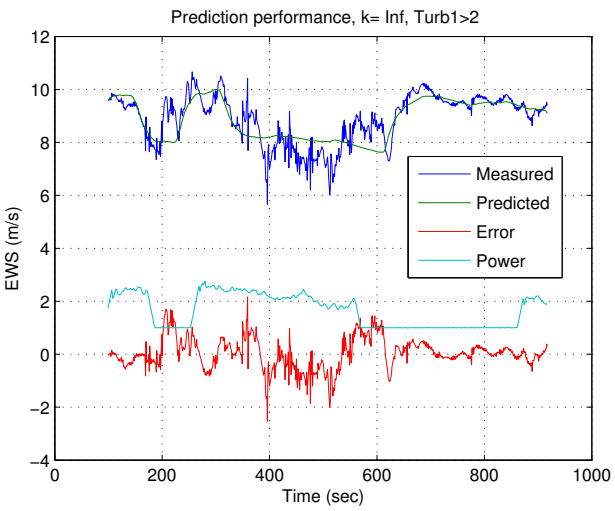

Figure 7. Simulation performance for the first order BJ model including upwind EWS and thrust coefficient.

where θ is the upwind wind direction, θ_b is a possible bias and θ_w is a measure of the wake width. A simple search shows that improvement from this is possible as $\theta_b = 1$ and $\theta_w = 5$ degrees gives even better results than without the wind direction included as seen in the last row in table 3.

5. CONCLUSION

Control of individual turbines in a farm is recognises as a important tool for optimisation of overall farm production and minimization of fatigue. To do this in practise it is crucial to have verified models for the effect upwind turbines have on downwind turbines. To the authors knowledge this is the first investigation using system identification methods based on data from real commercial wind farms.

Unique experimental data that include the upwind turbine load cycling has allowed the definition and analysis of a series of wake models. The wake effect is significant and clearly visible in the time series. This is also verified by the improvement from including upwind turbine loading into the models. The wake effect can be measured by the k factor as expressed in the static Jensen wake model. For the present measurement conditions this factor is $k = 0.136$. The corresponding dynamic wake factor estimated here is at least twice as large. The results confirms the wake dynamics to be a combination of average transport delay plus first order dynamic inflow. Because of the limited amount of data the models are limited to first order, but they include physical inspired non linear terms in the inputs upwind wind speed, thrust coefficient and wind direction. The superior model structure is a Box Jenkins type non linear model.

While these results indicate the effectiveness of system identification in establishing models in this challenging area, it is very important to stress that this is very preliminary results based on only one data series lasting only for 15 minutes and that no cross validation has been performed.

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