Abstract - As the most wide-area industrial network, the power system can be modeled as a graph with edges and vertices, which represent the lines and buses of the power grid respectively. Further methodologies such as complex network theory may help in identifying the vulnerability of power grid, analyzing the contingency, preventing cascading blackouts and so on. When power system is integrated with distributed generation (DG), decentralized generation at distribution level replaces some of the centralized generation at transmission level. DG units are able to improve the reliability of the power system, shorten the electrical distance between the sources and loads, alleviate the long-distance large-capacity transmission, and increase the efficiency. This paper proposes several vulnerability indices, such as structural vulnerability index (SVI), contingency vulnerability index (CVI) and operational vulnerability index (OVI) to evaluate the impact of DG to power system vulnerability. The simulation in DigSILENT/PowerFactory is conducted to assess the vulnerability of a 93-bus test power system, identify the vulnerable lines and buses, evaluate the improvement of the vulnerability index when the network is integrated with DG units, and may further to optimize the planning DG units in the future.

Index Terms—Complex network theory, contingency vulnerability index, distribution generation, operational vulnerability index, structural vulnerability index.

I. INTRODUCTION

The networks of power system, often called power grids, have been regarded as one of the most important infrastructures whose security should be paid more and more concern. However, in recent years, several large blackouts occurred in USA and Europe, which have resulted in direct loss up to billions of dollars [1-3]. These blackouts expose the potential problems of current analysis methods for power systems. So far, most work of power system analysis has focused on only one aspect of such blackouts. Although, these approaches have made impressive advances in the understanding of each aspect, such as voltage stability analysis, transient stability analysis and frequency stability analysis, it does not provide a framework for understanding the overall phenomena. It is reasonable to go beyond these traditional deterministic bottom-up descriptions, instead to be in favour of statistical top-down approaches. The vulnerability is the ability of a network continuing to provide key services during random failures or intentional attacks. Further technology, i.e. complex network theory provides a feasible way to study the vulnerability of power grids, which has drawn the link between the topological structure and the vulnerability of networks [4].

The first systematic study about complex network theory appeared in late 1990s, having the goal of studying the properties of large networks that behave as complex systems [5-8]. Complex network theory has received considerable attention recently since the investigation of the small-world networks [5] and the scale-free networks [7], as their characteristics have been discovered in many real networks including power grids. With its recent considerable progress, complex network theory can be of interest in assessing the vulnerability of power grids [9-14]. The concept of global efficiency was widely used to assess the vulnerability or locate critical components for networked infrastructures [16, 17]. Furthermore, the cascading failure model was also directly applied to power grids analysis [18-20]. These above studies provided a new direction for analyzing the power grids.

Although the complex network theory has made so much progress, few researchers have used it to explore the impact of large-scale DG to transmission system [21]. In case of Danish power system, a significant proportion of today’s installed capacity is decentralized generation (about 40% of total capacity), such as wind turbines and combined heat and power (CHP) units, which are mostly connected to the distribution system, as shown in Fig. 1 [22]. Further, more onshore wind farms are expected to be connected to the distribution system below 100kV. Compared with conventional power grid which is only supplied by centralized power plant (CPP), DG units mainly supply part of the local load, while contributing much less to remote loads.

In Section II, some principles of complex network theory are introduced. In Section III, three vulnerability indices i.e. SVI, CVI and OVI are proposed to explore the DG impact on power grid. The simulation and result of a 93-bus test system
are shown in Section IV. Section V gives concluding comments.

Fig. 1. Production capacities at each voltage level in Western Danish power system [22].

II. PRINCIPLE OF COMPLEX NETWORK THEORY

The complex network theory has gained wide acceptance and has been successfully applied in analysis of power systems. In the complex network theory, each bus of the power system which may be a power source or a power sink, can be modelled as a vertex (or node), and each transmission line and transformer can be modelled as an edge (or line), in which power flow may be transmitted between its terminal buses in the forward or reverse direction.

So the power grid can be abstracted as a directed and weighted graph \( Y = \{B, L, W\} \) where \( B \) \((\text{dim}\{B\}=N_B)\) is the set of vertices (or nodes) and \( L \) \((\text{dim}\{L\}=N_L)\) is the set of edges (or lines) with an associated set of weight \( W \). Each vertex \( B_i \) can be identified by \( i \), and each edge \( L_{ij} \) represents a connection going from vertex \( i \) to vertex \( j \) with associated weight \( w_{ij} \) [23].

The following efficiency index (EI) has been widely applied to evaluate the transmission efficiency of a power grid [24].

\[
EI = \frac{1}{n_g n_l} \sum_{i=1}^{n_g} \sum_{j=1}^{n_l} \frac{1}{d_{ij}}
\]

(1)

where \( n_g \) and \( n_l \) represent the number of generator and the number of load respectively, \( V_g \) and \( V_l \) are the sets of generators and the sets of loads respectively, the geodesic distance \( d_{ij} = \min \{d_{i'j'}\} \) represents the least number of transmission lines in the shortest transmission path between a specific generator \( G_i \) and a specific load \( L_j \). The distribution of geodesic distance is usually used to measure the network connectivity. The lower number of \( d_{ij} \) means the lower distance or closer connectivity between sources and loads, which implies the higher efficiency of the power grid.

III. VULNERABILITY ASSESSMENT INDICES

A. Equivalent Impedance Between Generation and Load

In the basic circuit theory, the node current equation is used to compute the voltages of the nodes and the currents of branches in a network.

\[
ZI = V
\]

(2)

where \( Z \) is the node impedance matrix. It can be written in the extended form as given in (3).

\[
\begin{bmatrix}
z_{i1} & z_{i2} & \cdots & z_{in} \\
z_{21} & z_{22} & \cdots & z_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
z_{ni} & z_{n2} & \cdots & z_{nn}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix}
= 
\begin{bmatrix}
v_1 \\
v_2 \\
\vdots \\
v_n
\end{bmatrix}
\]

(3)

The equivalent impedance \( Z_{eqij} \) between generation bus \( i \) and load bus \( j \) can represent the difficulty in transmitting a unit current from bus \( i \) and bus \( j \), as shown in Fig. 2.

Assuming a unit current is injected at bus \( i \) and withdrawn at bus \( j \), while no other current is injected or withdrawn at other buses.

\[
I_i = 1 \quad \text{and} \quad I_j = -1
\]

(4)

\[
\begin{bmatrix}
z_{ij} & \cdots & z_{ij} & 1 & 0 & \cdots & 0 \\
\vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots \\
z_{ij} & \cdots & z_{ij} & -1 & 0 & \cdots & 0
\end{bmatrix}
\begin{bmatrix}
v_i \\
\vdots \\
v_j
\end{bmatrix}
\]

(5)

So the equivalent impedance \( Z_{eqij} \) can be calculated as

\[
Z_{eqij} = V_i /1 = V_i - V_j = (z_{ij} - z_0) - (z_{ij} - z_0) = z_{ij} - 2z_0 + z_0
\]

(6)

where \( z_{ij} \) is the \( i \)-th, \( j \)-th element of the impedance matrix.

B. Structural Vulnerability Index (SVI)

The efficiency index EI in (1) assumes that the electric power is only transmitted through the shortest path, in which \( d_{ij} \) does not represent the electrical characteristic. However, this assumption may be far away from the reality in power systems. The power flow from a specific generation at bus \( i \) to a specific load at bus \( j \) is distributed all over various transmission lines as determined by topology and electrical performance in the network.

Besides, the capacity of generator and the load in (1) are not considered, which should act as the weight of the transmission relationship from the generation \( i \) and a load \( j \).

From the structural point of view, the main factors of the power system vulnerability should be based on the inherent characteristics, such as the topological relationship, the impedance of the transmission lines and transformers as well
as the capacity of the generators. The varying power system operation conditions has little affect on the structural characteristics. Therefore, this paper proposes a novel SVI for power system with large amount of DG.

\[
SVI = \frac{1}{n_k \sum_{i \in A} \frac{P_{gl}}{P_{max}} \exp\left(Z_{eqij}\right)}
\]  

(7)

where \( P_{gl} \) is the capacity of generation at node \( i \), \( P_{max} \) is the maximum load at node \( j \), and \( Z_{eqij} \) is the electric distance (equivalent impedance) between node \( i \) and node \( j \).

Furthermore, compared with the conventional power system, the power system integrated with large amount of DG has relative more generation capacity in the low voltage level. The DG mainly supports the relatively local load demand, contributing much less for remote loads. This characteristic is taken in to account for SVI in (7), where \( P_{gl}/\exp(\text{Z}_{eqij}) \) approximately represents the contribution from generation bus \( i \) to load bus \( j \). The contribution from the generation to the load should exponentially decrease with the increase of the impedance between them. When the DG and load are connected to the same node, \( Z_{eqij} = 0 \), \( P_{gl}/\exp(\text{Z}_{eqij}) = 1 \), which means that DG has the highest priority to satisfy the local load demand.

So SVI is more effective and more accurate than EI as defined by (1) for evaluating the changes of power system transmission efficiency before and after integrated with DG units. The higher SVI means the higher the transmission efficiency, which was widely used to assess the vulnerability or locate critical components for networked infrastructures.

C. Contingency Vulnerability Index (CVI)

The contingency in the power system is most possibly a fault followed by a trip of transmission line or transformer by protection devices. When contingencies take place in the power grid, the tripping of transmission lines or transformers possibly result in the severe deterioration of power transmission performance. It is because that the deletion of the components increase the electrical distance between sources and loads. So the SVI will likely decrease after the removal of transmission lines or transformers.

CVI is used to evaluate the criticality of a contingency as defined by (8), which is the reduction percentage of SVI related to the network structure variation. Furthermore, the decrease in the percentage of SVI in N-1 contingency can also be used to identify the vulnerable point in the power grid.

\[
CVI = \frac{SVI_0 - SVI}{SVI_0} \times 100\%
\]  

(8)

The higher CVI represents the contingency is more critical or the power is more vulnerable after removal of this component. So the operator from TSO should pay more attention on the contingencies in this transmission line or load and CVI also provides an index for preventing cascading trips.

D. Operational Vulnerability Index (OVI)

OVI is the index based on the operational conditions. The most significant impact of the DG on power grid is that to reduce the long-distance large-capacity power transmission thus to increase the power transmission efficiency. OVI is proposed to evaluate the operational vulnerability, as defined in (9).

\[
OVI = \frac{\sum_{i \in G} z_i \times p_i}{\sum_{i \in G} z_i}
\]  

(9)

where \( p_i \) represents the active power in the transmission line \( i \), and the impedance \( z_i \) is the weight of line \( i \). The smaller the OVI implies the less amount of the long-distance large-capacity transmission for active power and high transmission efficiency is in the network.

IV. SIMULATION AND RESULT

A. Test Power System

A 93-bus test model that representing a power system integrated with large amount of DG units is used to evaluate the effect of the DG to power system vulnerability, as shown in Fig. 3. The model has 124 components (edges) composed of 112 lines and 12 transformers. Almost every node in the distribution system underneath the transmission level is integrated with DG units, such as PV, onshore wind farms and CHP plants, however, 12 CPPs are connected to the transmission system. The test system is simulated by DgSILENT/PowerFactory, in which the balanced positive sequence AC load flow is calculated by adjusting the power output of DG and CPP.

B. Structural Vulnerability Assessment

In order to evaluate the impact of DG penetration level for power transmission efficiency in the structural vulnerability
assessment, the share of DG output is gradually increased to replace the generation of CPP. The load is the maximum load of each load bus, remaining invariant before and after DG integration.

As shown in Fig. 4, with the increase in the penetration level of DG from 0% to 100%, the SVI linearly increases from 0.006 to 0.021, which means that the transmission efficiency improves with the increase of DG penetration level.

C. Contingency Vulnerability Assessment

In order to evaluate the impact of DG on the ability of power system to resist the N-1 contingency, SVI is calculated before and after the removal of one component (transmission line or transformer). As shown in Fig. 5, the x-axis shows the number of components corresponding to every transformers and lines in Fig. 3. It is obvious that the SVI integrated with 100% DG is much higher than that without DG before and after N-1 contingency. Besides, the decrease of SVI after N-1 contingency with DG is generally lower than that of the case without DG integration. This means the power grid is stronger with DG integration regarding N-1 contingency than that without DG. This result also testifies that DG helps to improve the vulnerability of the power grid after contingencies.

As mentioned in the Section III, CVI can also be used to evaluate the criticality of a contingency. From another point of view, CVI helps to identify the vulnerable points of a power grid. As shown in Fig. 6, without DG integration, Line 71-72, Line 42-50 and Line 16-11 are the 3 most vulnerable components in the power grid. They have been marked in the test power system in Fig. 3. The blue line in Fig.6 shows that the criticality of the contingencies in these 3 lines is evidently reduced because of DG integration, which means that the power system is more resistant to contingencies with DG integration.

Fig. 7 shows the descending ordering of CVI. It is important to note here that, with DG integration, most of components have much lower CVI, but some components have higher CVI, which means the power grid is more vulnerable after contingency take places at only a few points after DG integration. The maximum values and mean values of CVI with and without DG integration are shown in Table I.

![Fig. 4. The SVI with respect to the DG penetration level.](image)

![Fig. 5. The SVI before and after N-1 contingency.](image)

![Fig. 6. The CVI with and without DG integration.](image)

![Fig. 7. The descending sort of CVI.](image)

![Fig. 8. The 10 most vulnerable components.](image)
D. Operational Vulnerability Assessment

As mentioned earlier, the most significant impact of DG on power grid is to reduce the long-distance large-capacity power transmission. OVI is the index to evaluate the power transmission efficiency, meaning that the weighted average value of active power in all the lines and transformers, which is based on AC power flow. The lower OVI implies that lesser the transmission distance of active power, higher the efficiency of the network. In order to evaluate the impact of DG capacity to power flow pattern, the overall load should not be changed, and the DG penetration level is increased to replace the CPP output. Fig.9 shows the value of OVI with respect to the DG penetration level.

With the increase penetration level of DG from 0% to 100%, the OVI decreases from 126.66MW to 41.17MW. The lower active power transmission average value implies the lower power loss and thus the high transmission efficiency.

Fig. 9. The OVI with respect to the DG penetration level.

V. CONCLUSION AND DISCUSSION

This paper proposed three vulnerability indices, structural vulnerability index (SVI), contingency vulnerability index (CVI) and operational vulnerability index (OVI). SVI is the index to evaluate the topological vulnerability of the whole power grid structure. CVI is the index to identify the vulnerable component in a power grid. OVI is the index related the operation states of power system which is based on power flow.

In conclusion, the DG units are able to improve the reliability of the power system, shorten the electrical distance between the sources and loads, alleviate the long-distance large-capacity transmission, and increase the efficiency.

This vulnerability evaluation based on basic complex network theory is also able to provide a constraint to optimize the planning of future DG units. Besides, in emergency condition of power system, these vulnerability indices based on load flow can also helps to prevent cascading failures.

ACKNOWLEDGEMENTS

This work was supported in parts by Danske Strategiske Forskningscentres (DSF 09-067255) “Development of a Secure, Economic and Environmentally-friendly Modern Power System”.

REFERENCES