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Published in:
I E E E Workshop on Signal Processing Advances in Wireless Communications. Proceedings

DOI (link to publication from Publisher):
10.1109/SPAWC.2011.5990468

Publication date:
2011

Document Version
Early version, also known as pre-print

Degrees of Freedom of Asymmetrical Multi-Way Relay Networks

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***Abstract*—In this paper, we introduce an asymmetrical multi-way relay channel where a base station conveys independent symbols to K different users while receiving independent symbols from the users. In this network, user i has M_i antennas ($i \in [1, \ldots, K]$), the base station has $\sum_{i=1}^{K} M_i$ antennas and the relay is equipped with N_R antennas. To study the capacity of this network, the degree of freedom (DOF) is characterized to be $2\sum_{i=1}^{K} M_i$ if $N_R \geq \sum_{i=1}^{K} M_i$. The DOF implies that the number of symbols any transmitter can deliver and receive is equivalent to its number of antennas. The DOF is achievable with a proposed joint multiple access and broadcast precoding design using the signal space alignment concept.**

I. Introduction

Two-way relay-aided communications have been studied to overcome the loss of spectral efficiency of their one-way counterpart [1]. The two-way relay channel has drawn great attention because of its attractive application to cellular networks. It has been recently generalized to include multiple links. The authors in [2] studied the relay-aided multi-pair communication network, where a relay node facilitates the communication between multiple pairs of users. Multiuser and multi-way relay communications were considered in [3], [4], where multiple users intend to exchange information with a shared relay terminal. Particularly in [4], a multiple-input multiple-output (MIMO) Gaussian Y channel with one relay and three users was introduced. An efficient signal space alignment for network coding scheme exploiting the concepts of interference alignment [5]–[7] and network coding [8], [9] is proposed to manage interference signals in the multiuser bi-directional system. The extension of the MIMO Y channel to the case where a general number of users simultaneously exchange messages with each other was investigated in [10]. An alternative scheme was applied to multi-link two-way relay channels in [11], where multiple two-way relay links are operating simultaneously. Moreover, the application to the multiuser multi-way relay channel was studied in [12], where each user transmits a multicast message to all the other users while receiving independent messages from the other users.

In this paper, we consider a multiuser MIMO relaying network [13] with K user equipments (UEs), one base station (BS) and one full-duplex relay, as in Fig. 1. In this network, each UE has arbitrary number of antennas, i.e. UE i is with M_i antennas, and the BS is equipped with $\sum_{i=1}^{K} M_i$ antennas. The relay is equipped with N_R antennas. We assume the distance between the BS and any UE is large and hence the received signal from the direct link is relatively weak and negligible. In this network, the BS conveys independent messages to the different UEs while receiving independent messages from the UEs. We call this network information flow as an *asymmetrical multi-way relay channel*. The motivation originates from a practical scenario in cellular networks where multiple cell-edge UEs establish interference-free bi-directional transmissions with the corresponding BS. Each UE has no intension to communicate with the other UEs but only communicates with a central node (the BS), which differs from the MIMO Y channel [10] and the multiuser multi-way relay channel [12] where only UEs are communicating with each other. In the symmetrical setup in [10], [12], each UE transmits (and receives) the same amount of information in one transmission. Hence, another distinction in our model

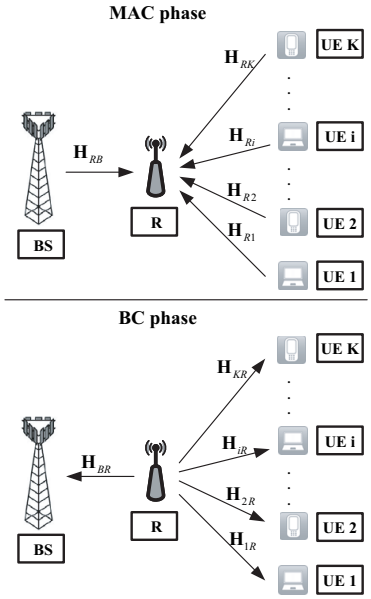

Fig. 1. Asymmetrical multi-way relay channels.

is that different UEs are allowed to deliver and receive different number of messages to and from the central node during one transmission time, which can naturally support various types of services for the UEs in practice. In the multiple access (MAC) phase, all the UEs and the BS concurrently transmit signals to the relay. Then in the broadcast (BC) phase, the relay sends information to the other nodes. These nodes will suffer from severe interference problems, i.e. each UE will receive signals from the BS intended to the other UEs, and also signals sent from the other UEs to the BS (the signals are forwarded by the relay in the BC phase). The requirement on interference management becomes crucial. The fundamental problems lie in how many messages the asymmetrical multi-way relay channel can support during one transmission time and how all the nodes cooperate to accomplish the transmission.

The main contribution of this paper is to characterize the degrees of freedom (DOF) for the asymmetrical multi-way relay channel from the information theoretical perspective. The DOF is found to be $2\sum_{i=1}^{K} M_i$ with a minimum number of antennas $N_R = \sum_{i=1}^{K} M_i$ at the relay. This further implies that the number of messages any transmission node can deliver and receive is equivalent to the number of antennas. With simple amplify-and-forward (AF) relay operation, we propose a joint MAC-BC precoding design using the signal space alignment concept to demonstrate the achievablity. To the best of our knowledge, our paper is the first one to give the DOF for this channel.

II. System Model

We describe the system model for asymmetrical multi-way relay channels (see Fig. 1), where we have one BS and K cell-edge UEs. The communication between the BS and the cell-edge UEs is supported by an intermediate AF relay. UE i is equipped with M_i antennas and the BS is equipped with $\sum_{i=1}^{K} M_i$ antennas. In the downlink, the BS intends to send out $\sum_{i=1}^{K} N_i$ independent symbols, N_i symbols for UE i ($N_i \leq M_i, i \in [1,\ldots,K]$), using the relay with N_R antennas. Meanwhile, there are also $\sum_{i=1}^{K} N_i$ symbols in the uplink direction, where UE i ($i \in [1,\ldots,K]$) wants to transmit N_i symbols to the BS. In this work, all the nodes are able to operate in the full-duplex mode, i.e. the BS, the UEs and the relay are able to transmit and receive signals simultaneously.

The whole transmission can be divided into two phases, the MAC phase and the BC phase, as in Fig. 1. We denote the jth symbol transmitted from the BS to UE i as s_{ij}^B and the jth symbol from UE i to the BS as s_j^i ($i \in [1,\ldots,K]$, $j \in [1,\ldots,N_i]$). Each symbol is with unit variance. We let $\mathbf{s}^B = [s_{11}^B \cdots s_{1(N_1)}^B \cdots s_{ij}^B \cdots s_{K(N_K)}^B]^{\mathrm{T}}$ and $\mathbf{s}^i = [s_1^i \cdots s_{N_i}^i]^{\mathrm{T}}$ to be the symbol vectors.

During the MAC phase, the BS and each UE simultaneously transmit the signals to the relay node. The BS transmits the $(\sum_{i=1}^{K} N_i) \times 1$ symbol vector $\mathbf{s}^B$ using the $(\sum_{i=1}^{K} M_i) \times (\sum_{i=1}^{K} N_i)$ precoding matrix $\mathbf{W}^B = [\mathbf{w}_{11}^B \cdots \mathbf{w}_{1(N_1)}^B \cdots \mathbf{w}_{ij}^B \cdots \mathbf{w}_{K(N_K)}^B]$. UE i ($i \in [1,\ldots,K]$) transmits the $N_i \times 1$ symbol vector $\mathbf{s}^i$ using the $M_i \times N_i$ precoding matrix $\mathbf{W}^i = [\mathbf{w}_1^i \cdots \mathbf{w}_{N_i}^i]$. We use $\mathbf{x}^B$ to represent the $(\sum_{i=1}^{K} M_i) \times 1$ transmit vector at the BS and $\mathbf{x}^i$ ($i \in [1,\ldots,K]$) to represent the $M_i \times 1$ transmit vector at UE i,

$$\mathbf{x}^B = \mathbf{W}^B \mathbf{s}^B,\ \mathbf{x}^i = \mathbf{W}^i \mathbf{s}^i, \quad i = 1,\ldots,K. \tag{1}$$

We have $\mathbb{E}[\mathbf{s}^B(\mathbf{s}^B)^{\mathrm{H}}] = \mathbf{I}_{\sum_{i=1}^{K} N_i}$ and $\mathbb{E}[\mathbf{s}^i(\mathbf{s}^i)^{\mathrm{H}}] = \mathbf{I}_{N_i}$ ($i \in [1,\ldots,K]$). Per-transmitter average power constraint is used,

$$\begin{aligned} &\mathbb{E}\{\mathrm{Tr}[\mathbf{x}^B(\mathbf{x}^B)^{\mathrm{H}}]\} = \mathrm{Tr}[\mathbf{W}^B(\mathbf{W}^B)^{\mathrm{H}}] \leq P^B, \\ &\mathbb{E}\{\mathrm{Tr}[\mathbf{x}^i(\mathbf{x}^i)^{\mathrm{H}}]\} = \mathrm{Tr}[\mathbf{W}^i(\mathbf{W}^i)^{\mathrm{H}}] \leq P^i \end{aligned} \tag{2}$$

We use $\mathbf{H}_{RB}$ and $\mathbf{H}_{Ri}$ to denote the $N_R \times (\sum_{i=1}^{K} M_i)$ channel matrix from the BS to the relay and the $N_R \times M_i$ channel matrix from UE i to the relay, respectively. It is assumed that the channel elements are generated from independent identically distributed (i.i.d) complex Gaussian random variables with zero mean and unit variance. The received signal vector at the relay is expressed as

$$\mathbf{y}^R = \mathbf{H}_{RB}\mathbf{x}^B + \sum_{i=1}^{K} \mathbf{H}_{Ri}\mathbf{x}^i + \mathbf{n}^R, \tag{3}$$

where $\mathbf{n}^R$ stands for the $N_R \times 1$ additive white Gaussian noise (AWGN) vector with unit variance ($\sigma^2 = 1$).

After the MAC phase, the relay produces the symbol vector $\mathbf{x}^R$ using the precoding matrix $\mathbf{W}^R$ with size $N_R \times (\sum_{i=1}^{K} M_i)$,

$$\mathbf{x}^R = \alpha \mathbf{W}^R \mathbf{y}^R, \tag{4}$$

where α stands for the power normalization coefficient satisfying the relay power constraint $\mathbb{E}\{\mathrm{Tr}[\mathbf{x}^R(\mathbf{x}^R)^{\mathrm{H}}]\} \leq P^R$, which leads to

$$\alpha = \sqrt{\frac{P^R}{\mathbb{E}\{\mathrm{Tr}[\mathbf{W}^R\mathbf{y}^R(\mathbf{y}^R)^{\mathrm{H}}(\mathbf{W}^R)^{\mathrm{H}}]\}}}. \tag{5}$$

In the BC phase, the relay broadcasts $\mathbf{x}_R$ to all the other nodes. Then, the received signal vectors at the BS and the UEs are written as ($i \in [1,\ldots,K]$)

$$\mathbf{y}^B = \mathbf{H}_{BR}\mathbf{x}^R + \mathbf{n}^B, \mathbf{y}^i = \mathbf{H}_{iR}\mathbf{x}^R + \mathbf{n}^i, \tag{6}$$

where $\mathbf{H}_{BR}$ and $\mathbf{H}_{iR}$ stand for the $(\sum_{i=1}^{K} M_i) \times N_R$ channel matrix from the relay to the BS and the $M_i \times N_R$

channel matrix from the relay to UE i, respectively. In addition, $\mathbf{n}^B$ denotes the AWGN vector at the BS with unit variance and $\mathbf{n}^i$ denotes the AWGN vector at UE i with unit variance. We consider the general case where each channel matrix is different. In this paper, channel state information (CSI) is available perfectly at all the nodes.

In the following, we will study the DOF of this network. The DOF is the expression of the pre-log factor on the capacity. Thus, it is the crucial metric for characterizing capacity behavior at high SNR. We define the DOF as the sum of the achievable DOFs,

$$\eta = \lim_{\mathrm{SNR}\to\infty} \frac{C(\mathrm{SNR})}{\log(\mathrm{SNR})} = d_B + \sum_{i=1}^{K} d_i, \tag{7}$$

where $C(\mathrm{SNR})$ denotes the total sum capacity given SNR, d_B and d_i represent the DOFs for information transfer from all the UEs to the BS and from the BS to the ith UE, respectively.

III. Degrees of Freedom of Asymmetrical Two-Way Relay Networks

In this section, we investigate the achievable DOF in the asymmetrical multi-way relay channel without direct links between the UEs and the BS. We first summarize the main result in the following theorem.

Theorem 1: The asymmetrical multi-way relay network with K UEs and one BS, where UE i has M_i antennas, both the BS and the relay have $\sum_{i=1}^{K} M_i$ antennas, is able to achieve the DOF equals to $2\sum_{i=1}^{K} M_i$, i.e.,

$$\begin{aligned} &d_B = \sum_{i=1}^{K} M_i, \quad d_i = M_i, \quad \sum_{i=1}^{K} d_i = \sum_{i=1}^{K} M_i, \\ &\eta = d_B + \sum_{i=1}^{K} d_i = 2\sum_{i=1}^{K} M_i. \end{aligned} \tag{8}$$

A. Converse

The proof of the converse result is trivial. Obviously, the DOF of the system is upper bounded by a MIMO two-way relay channel where two $(\sum_{i=1}^{K} M_i)$-antenna nodes want to communicate with each other through a full-duplex relay with $N_R = (\sum_{i=1}^{K} M_i)$ antennas. Therefore, it is straightforward to see that the DOF of the asymmetrical multi-way relay channel $\eta \le \sum_{i=1}^{K} M_i + \sum_{i=1}^{K} M_i = 2\sum_{i=1}^{K} M_i$.

Next, we show that this DOF upper bound is actually achievable based on the proposed joint MAC-BC precoding design using the signal space alignment concept.

B. Achievability

The whole transmission involves the MAC and the BC phases. We first assume that the BS sends out $\sum_{i=1}^{K} M_i$ symbols, M_i symbols for UE i ($i \in [1,\ldots,K]$) respectively; UE i transmits M_i symbols to the BS. Naturally, the proof of the achievability consists of the MAC phase, the relay precoding design and the BC phase.

1) MAC phase: In this phase, the BS conveys $\sum_{i=1}^{K} M_i$ independent symbols to the relay. At the same time, the relay also receives M_i symbols from the ith UE ($i \in [1,\ldots,K]$). In order to exploit the signal dimension at the relay efficiently, the precoding matrices are designed to align signal vectors from UEs and signal vectors from the BS. To be more specific, by designing precoding vectors $\mathbf{w}_{ij}^B$ of size $(\sum_{i=1}^{K} M_i) \times 1$ and $\mathbf{w}_j^i$ of size $M_i \times 1$, the signal vector of the j symbol sent from the BS to the ith UE s_{ij}^B are aligned with the signal vector of the j symbol transmitted from the ith UE to the BS s_j^i, ($i \in [1,\ldots,K]$, $j \in [1,\ldots,M_i]$), at the relay. The signal space alignment is expressed as

$$\mathrm{span}(\mathbf{H}_{RB}\mathbf{w}_{ij}^B) = \mathrm{span}(\mathbf{H}_{Ri}\mathbf{w}_j^i), \tag{9}$$

The relay is thereby able to detect the signals and encode them via the analog network coding (ANC) method [14] or the physical-layer network coding (PNC) method [15]. We denote $\overline{\mathbf{w}}_{ij}^B$ and $\overline{\mathbf{w}}_j^i$ to be the vectors representing the directions of $\mathbf{w}_{ij}^B$ and $\mathbf{w}_j^i$, i.e. $\mathbf{w}_{ij}^B = \beta\overline{\mathbf{w}}_{ij}^B$ and $\mathbf{w}_j^i = \gamma\overline{\mathbf{w}}_j^i$ where β and γ are two positive scalars. We further denote $\mathbf{b}_{ij}^R$ as a normalized vector in the intersection subspace constituted by the column space of $\mathbf{H}_{RB}$ and $\mathbf{H}_{Ri}$. Then, $\mathrm{span}(\mathbf{b}_{ij}^R) = \mathrm{span}(\mathbf{H}_{RB}\overline{\mathbf{w}}_{ij}^B) = \mathrm{span}(\mathbf{H}_{Ri}\overline{\mathbf{w}}_j^i)$. By using the Lemma 1 in [4], $\overline{\mathbf{w}}_{ij}^B$ and $\overline{\mathbf{w}}_j^i$ can be derived via solving

$$\begin{bmatrix} \mathbf{I}_{\sum_{i=1}^{K} M_i} & -\mathbf{H}_{Ri} & \mathbf{0} \\ \mathbf{I}_{\sum_{i=1}^{K} M_i} & \mathbf{0} & -\mathbf{H}_{RB} \end{bmatrix} \begin{bmatrix} \mathbf{b}_{ij}^R \\ \overline{\mathbf{w}}_j^i \\ \overline{\mathbf{w}}_{ij}^B \end{bmatrix} = \mathbf{A}\overline{\mathbf{w}} = \mathbf{0}. \tag{10}$$

Then $\mathbf{w}_{ij}^B$ and $\mathbf{w}_j^i$ satisfying the per-transmitter average power constraint in Eq. (2) are derived based on $\overline{\mathbf{w}}_{ij}^B$ and $\overline{\mathbf{w}}_j^i$. It is clear the matrix $\mathbf{A}$ of size $(2\sum_{i=1}^{K} M_i) \times (2\sum_{i=1}^{K} M_i + M_i)$ has M_i dimensional null space. Therefore, for UE i, the M_i precoding vector pairs at the UE $\mathbf{w}_j^i$ and at the BS $\mathbf{w}_{ij}^B$ ($j \in [1,\ldots,M_i]$) can be obtained explicitly with probability one.

We define $\mathbf{B} = [\mathbf{b}_{11}^R \cdots \mathbf{b}_{1(M_1)}^R \cdots \mathbf{b}_{ij}^R \cdots \mathbf{b}_{K(M_K)}^R]$ of size $(\sum_{i=1}^{K} M_i) \times (\sum_{i=1}^{K} M_i)$. It can be shown that the columns of $\mathbf{B}$ are linearly independent of each other. Then Eq. (3) can be rewritten as

$$\mathbf{y}^R = \mathbf{H}_{RB}\mathbf{x}^B + \sum_{i=1}^{K} \mathbf{H}_{Ri}\mathbf{x}^i + \mathbf{n}^R = \mathbf{B}\mathbf{s}^R + \mathbf{n}^R, \tag{11}$$

where $\mathbf{s}^R = [s_{11}^R \cdots s_{1(M_1)}^R \cdots s_{ij}^R \cdots s_{K(M_K)}^R]^{\mathrm{T}}$ of size $(\sum_{i=1}^K M_i) \times 1$ is defined as the network coded symbol vector at the relay. s_{ij}^R stands for the sum of the two aligned symbols $s_{ij}^R = a_{ij}^B s_{ij}^B + c_j^i s_j^i$, where a_{ij}^B and c_j^i are the normalized channel gains determined from $a_{ij}^B \mathbf{b}_{ij}^R = \mathbf{H}_{RB}\mathbf{w}_{ij}^B$ and $c_j^i \mathbf{b}_{ij}^R = \mathbf{H}_{Ri}\mathbf{w}_j^i$, respectively.

2) Relay precoding design and BC phase: In the BC phase, there are potential inter-user interference (IUI) signals received at each UE, created by the signals transmitted from the BS for the other UEs and the signals transmitted from the other UEs for the BS (the signals are also forwarded by the relay antennas). The design of the precoding at the relay is aiming at mitigating the IUI to all the UEs and guarantee correct decoding of the desired symbols from the BS in the BC phase. With the relay design, at UE i, only the signals delivered to and transmitted from the ith UE shall be obtained; the signals intended for the other UEs and the signals transmitted from the other UEs shall be nulled out. Then the M_i-antenna UE i is feasible to receive and detect M_i aligned signals from the relay. With the capability of self-interference cancelation, UE i is able to decode M_i independent symbols from the BS. We characterize the relay precoding matrix as

$$\mathbf{W}^R = \mathbf{G}\mathbf{F}^{\mathrm{H}}, \tag{12}$$

where $\mathbf{G} = [\mathbf{g}_1^1 \cdots \mathbf{g}_{M_1}^1 \cdots \mathbf{g}_j^i \cdots \mathbf{g}_{M_K}^K]$ and $\mathbf{F} = [\mathbf{f}_1^1 \cdots \mathbf{f}_{M_1}^1 \cdots \mathbf{f}_j^i \cdots \mathbf{f}_{M_K}^K]$ are both of size $(\sum_{i=1}^K M_i) \times (\sum_{i=1}^K M_i)$. We denote the BC channel matrix to all the UEs as $\mathbf{H}^{BC} = [\mathbf{H}_{1R}^{\mathrm{H}} \ \cdots \ \mathbf{H}_{iR}^{\mathrm{H}} \ \cdots \ \mathbf{H}_{KR}^{\mathrm{H}}]^{\mathrm{H}}$. Let us further denote $\mathbb{H}_i^{BC}$ as the matrix which excludes the channel matrix to the ith UE $\mathbf{H}_{iR}$ from the matrix $\mathbf{H}^{BC}$; $\mathbb{B}_{ij}$ as the matrix which excludes the aligned signal space vector $\mathbf{b}_{ij}^R$ from the matrix $\mathbf{B}$. For instance, $\mathbb{H}_2^{BC} = [\mathbf{H}_{1R}^{\mathrm{H}} \ \mathbf{H}_{3R}^{\mathrm{H}} \ \cdots \ \mathbf{H}_{KR}^{\mathrm{H}}]^{\mathrm{H}}$ $(K \geq 3)$ and $\mathbb{B}_{12} = [\mathbf{b}_{11}^R \ \mathbf{b}_{13}^R \cdots \mathbf{b}_{K(M_K)}^R]$ $(M_1 \geq 3)$. $\mathbf{G}$ and $\mathbf{F}$ are obtained from $(i \in [1, \ldots, K], j \in [1, \ldots, M_i])$

$$\mathbf{g}_j^i \in \mathrm{Null}(\mathbb{H}_i^{BC}), \ \mathbf{f}_j^i \in \mathrm{Null}((\mathbb{B}_{ij})^{\mathrm{H}}) \tag{13}$$

The existence of the solutions to Eq. (13) is given as follows. The matrix $\mathbb{H}_i^{BC}$ of size $[(\sum_{i=1}^K M_i) - M_i] \times (\sum_{i=1}^K M_i)$ has M_i dimensional null space. Therefore, $\mathbf{g}_j^i$ $(j \in [1, \ldots, M_i])$ can be derived with probability one. The matrix $(\mathbb{B}_{ij})^{\mathrm{H}}$ of size $[(\sum_{i=1}^K M_i) - 1] \times (\sum_{i=1}^K M_i)$ has one dimensional null space. Thus, the existence of f_j^i is also proved. It can be verified that the columns in $\mathbf{G}$ are linearly independent of each other; the columns in $\mathbf{F}$ are also linearly independent of each other.

The relay precoding design nulls the IUI and ensures UE i to be able to decode the M_i symbols intended from the BS. To show the relay precoding design can achieve IUI nulling at each UE, we first consider the transmitting symbols $\mathbf{x}^R$ at the relay. From Eq. (4), (11) and (12),

$$\begin{aligned}
\mathbf{x}^R &= \alpha \mathbf{W}^R \mathbf{y}^R \\
&= \alpha \mathbf{G}\mathbf{F}^{\mathrm{H}}\mathbf{B}\mathbf{s}^R + \alpha \mathbf{W}^R \mathbf{n}^R \\
&= \mathbf{G}\mathrm{diag}\left[(\mathbf{f}_1^1)^{\mathrm{H}}\mathbf{b}_{11}^R \cdots (\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R \cdots (\mathbf{f}_{M_K}^K)^{\mathrm{H}}\mathbf{b}_{K(M_K)}^R\right] \\
&\quad \alpha[s_{11}^R \cdots s_{ij}^R \cdots s_{K(M_K)}^R]^{\mathrm{T}} + \alpha \mathbf{W}^R \mathbf{n}^R \\
&= \alpha \mathbf{G} \begin{bmatrix} (\mathbf{f}_1^1)^{\mathrm{H}}\mathbf{b}_{11}^R s_{11}^R \\ \vdots \\ (\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R s_{ij}^R \\ \vdots \\ (\mathbf{f}_{M_K}^K)^{\mathrm{H}}\mathbf{b}_{K(M_K)}^R s_{K(M_K)}^R \end{bmatrix} + \alpha \mathbf{W}^R \mathbf{n}^R \\
&= \alpha \left(\sum_{i=1}^K \sum_{j=1}^{M_i} \mathbf{g}_j^i (\mathbf{f}_j^i)^{\mathrm{H}} \mathbf{b}_{ij}^R s_{ij}^R \right) + \alpha \mathbf{W}^R \mathbf{n}^R.
\end{aligned} \tag{14}$$

In the derivation of Eq. (14), we use $(\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{mn}^R = 0$ $(\forall m \neq i, \forall n \neq j)$ since $(\mathbf{f}_j^i)^{\mathrm{H}}\mathbb{B}_{ij} = \mathbf{0}$. $\mathbf{B}\mathbf{s}^R$ represents the mixed aligned signals from the BS and from all the UEs. $\mathbf{F}^{\mathrm{H}}$ acts as a decoupling matrix to decouple the mixed signals in $\mathbf{B}\mathbf{s}^R$ into parallel signals, where $(\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R s_{ij}^R$ stands for the decoupled aligned signal from the jth symbol from the BS to UE i and the jth symbol from UE i to the BS.

In the BC phase, the relay sends out $\mathbf{x}^R$ to all the other nodes. The received signals at UE i is

$$\begin{aligned}
\mathbf{y}^i &= \mathbf{H}_{iR}\mathbf{x}^R + \mathbf{n}^i \\
&= \alpha \mathbf{H}_{iR} \left(\sum_{i=1}^K \sum_{j=1}^{M_i} \mathbf{g}_j^i (\mathbf{f}_j^i)^{\mathrm{H}} \mathbf{b}_{ij}^R s_{ij}^R \right) + \alpha \mathbf{H}_{iR}\mathbf{W}^R\mathbf{n}^R + \mathbf{n}^i \\
&= \mathbf{H}_{iR}[\mathbf{g}_1^i (\mathbf{f}_1^i)^{\mathrm{H}}\mathbf{b}_{i1}^R \cdots \mathbf{g}_j^i (\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R \cdots \mathbf{g}_{M_i}^i (\mathbf{f}_{M_i}^i)^{\mathrm{H}}\mathbf{b}_{i(M_i)}^R] \\
&\quad \alpha[s_{i1}^R \cdots s_{ij}^R \cdots s_{i(M_i)}^R]^{\mathrm{T}} + \alpha \mathbf{H}_{iR}\mathbf{W}^R\mathbf{n}^R + \mathbf{n}^i \\
&= \mathbf{H}_{ef}^i [s_{i1}^R \cdots s_{ij}^R \cdots s_{i(M_i)}^R]^{\mathrm{T}} + \alpha \mathbf{H}_{iR}\mathbf{W}^R\mathbf{n}^R + \mathbf{n}^i,
\end{aligned} \tag{15}$$

where $\mathbf{H}_{ef}^i$ is the effective channel matrix at UE i. We use $\mathbf{H}_{iR}\mathbf{g}_j^m = \mathbf{0}$ $(\forall m \neq i)$ from $\mathbb{H}_i^{BC}\mathbf{g}_j^i = \mathbf{0}$ in the derivation of Eq. (15).

The design of $\mathbf{G}\mathbf{F}^{\mathrm{H}}$ guarantees the M_i-antenna UE i to receive only M_i aligned signals which include the symbols from the BS to UE i $([s_{i1}^R \cdots s_{ij}^R \cdots s_{i(M_i)}^R]^{\mathrm{T}})$. Therefore, the interference created by the other UEs is completely canceled out.

The vectors $\mathbf{g}_j^i (\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R$ $(j \in [1, \ldots, M_i])$ can be proved to be linearly independent of each other with high probability. It is thereby clear that the M_i columns in $\mathbf{H}_{ef}^i$ are linear independent of each other with high probability. The signal streams $[s_{i1}^R \cdots s_{ij}^R \cdots s_{i(M_i)}^R]^{\mathrm{T}}$ can be detected at UE i with a receive filter. Using self-interference cancelation, it is feasible for UE i to

TABLE I
DOFS FOR ASYMMETRICAL MULTI-WAY RELAY CHANNELS

K UEs	$(M_1, \cdots, M_K)$	DOF (proposed)
2	(2,2)	8
2	(2,3)	10
3	(2,2,2)	12
3	(2,3,4)	18

successfully decode $s_{i1}^B \cdots s_{ij}^B \cdots s_{i(M_i)}^B$ from the BS. As a result, the DOF $d_i = M_i$ is achieved at UE i. Thus, the DOF $\sum_{i=1}^K M_i$ is achieved from the K UEs.

Next, we show that the BS can detect $(\sum_{i=1}^K M_i)$ independent symbols from all the UEs. In the BC phase, the received signal vector at the BS is rewritten as

$$\begin{aligned}\mathbf{y}^B &= \mathbf{H}_{BR}\mathbf{x}^R + \mathbf{n}^B \\ &= \alpha \mathbf{H}_{BR} \begin{bmatrix} \left(\mathbf{g}_1^1 (\mathbf{f}_1^1)^{\mathrm{H}} \mathbf{b}_{11}^R\right)^{\mathrm{H}} \\ \vdots \\ \left(\mathbf{g}_j^i (\mathbf{f}_j^i)^{\mathrm{H}} \mathbf{b}_{ij}^R\right)^{\mathrm{H}} \\ \vdots \\ \left(\mathbf{g}_{M_K}^K (\mathbf{f}_{M_K}^K)^{\mathrm{H}} \mathbf{b}_{K(M_K)}^R\right)^{\mathrm{H}} \end{bmatrix}^{\mathrm{H}} \mathbf{s}^R \\ &\quad + \alpha \mathbf{H}_{BR}\mathbf{W}^R\mathbf{n}^R + \mathbf{n}^B \\ &= \mathbf{H}_{ef}^B \mathbf{s}^R + \alpha \mathbf{H}_{BR}\mathbf{W}^R\mathbf{n}^R + \mathbf{n}^B,\end{aligned} \tag{16}$$

where $\mathbf{H}_{ef}^B$ is the effective channel matrix at the BS. The $(\sum_{M=i}^K M_i)$ vectors $\mathbf{g}_1^1(\mathbf{f}_1^1)^{\mathrm{H}}\mathbf{b}_{11}^R \cdots \mathbf{g}_j^i(\mathbf{f}_j^i)^{\mathrm{H}}\mathbf{b}_{ij}^R \cdots \mathbf{g}_{M_K}^K(\mathbf{f}_{M_K}^K)^{\mathrm{H}}\mathbf{b}_{K(M_K)}^R$ are also found to be linearly independent of each other with high probability. Thus, the $(\sum_{M=i}^K M_i)$ columns in $\mathbf{H}_{ef}^B$ are linear independent of each other with high probability as well. $\mathbf{s}^R$ could be detected at the BS via a certain receive filter. With self-interference cancelation, s_j^i transmitted from UE i can be successfully decoded. Hence, the DOF $d_B = \sum_{i=1}^K M_i$ is achieved at the BS.

Finally, $(2\sum_{i=1}^K M_i)$ DOF is achieved in the asymmetrical multi-way relay channel. ■

Remark 1: Theorem 1 indicates that the number of messages any transmitter can deliver and receive is equal to its number of antennas. The BS is capable of decoding $N_i = M_i$ symbols from the ith UE, thereby in total $\sum_{i=1}^K M_i$ symbols from all the UEs; UE i is able to correctly decode $N_i = M_i$ symbols from the BS. Each M_i can be chosen as any positive integer and they shall not necessarily be the same. Table I shows the DOFs for different antenna configurations when two UEs or three UEs are supported in the system.

IV. CONCLUSION

In the multi-directional relay-aided communication network, the BS transmits independent symbols to K different users while receiving independent symbols from the users, via the relay node. We call this channel "the asymmetrical multi-way relay channel", where UE i has M_i antennas ($i \in [1, \ldots, K]$), the BS has $\sum_{i=1}^K M_i$ antennas and the relay is equipped with N_R antennas. We study the capacity of this channel by characterizing the DOF performance, which implies how many symbols the channel can support during one transmission time. If $N_R \geq \sum_{i=1}^K M_i$, the $2\sum_{i=1}^K M_i$ DOF indicates that the number of symbols any transmitter can deliver and receive is equivalent to its number of antennas. In this scenario, we propose an efficient joint MAC-BC precoding scheme to show how all the nodes cooperate to accomplish the transmission and achieve the DOF. In the MAC phase, the precoding matrices at all the UEs and the BS are designed to align signals to create network coded symbol vectors at the relay. The relay precoding design nulls the IUI and ensures all the UEs and the BS to be able to decode the corresponding desired symbols.

REFERENCES

[1] B. Rankov and A. Wittenben, "Spectral efficient protocols for half-duplex fading relay channels," *IEEE J. Sel. Areas. Commun.*, vol. 25, no. 2, pp. 379 – 389, February 2007.

[2] A. S. Avestimehr, M. A. Khajehnejad, A. Sezgin, and B. Hassibi, "Capacity region of the deterministic multi-pair bi-directional relay network," in *Proc. IEEE ITW*, June 2009.

[3] D. Gündüz, A. Yener, A. Goldsmith, and H. V. Poor, "The Multi-way Relay Channel," in *Proc. IEEE ISIT*, June 2009.

[4] N. Lee, J.-B. Lim, and H. Chun, "Degrees of Freedom of the MIMO Y Channel: Signal Space Alignment for Network Coding," *IEEE Trans. Inform. Theory*, vol. 56, no. 7, pp. 3332–3342, July 2010.

[5] M. A. Maddah-Ali, A. S. Motahari, and A. K. Khandani, "Communication over MIMO X channels: Interference alignment, decomposition, and performance analysis," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3457–3470, August 2008.

[6] S. A. Jafar and S. Shamai, "Degrees of Freddom for the MIMO X Channel," *IEEE Trans. Inform. Theory*, vol. 54, no. 1, pp. 151–170, January 2008.

[7] V. R. Cadambe and S. A. Jafar, "Interference alignment and degrees of freedom of the K-user interferernce channel," *IEEE Trans. Inform. Theory*, vol. 54, no. 8, pp. 3425–3441, August 2008.

[8] R. Ahlswede, N. Cai, S.-Y. R. Li, and R. W. Yeung, "Network information flow," *IEEE Trans. Inform. Theory*, vol. 46, no. 4, pp. 1204–1216, July 2000.

[9] T. Ho, M. Médard, R.Koetter, D. Karger, M. Effros, J. Shi, and B. Leong, "A Random Linear Network Coding Approach to Multicast," *IEEE Trans. Inform. Theory*, vol. 52, no. 10, pp. 4413–4430, October 2006.

[10] K. Lee, N. Lee, and I. Lee, "Feasibility Conditions of Signal Space Alignment for Network Coding on K-user MIMO Y channels," *to be presented at IEEE ICC*, June 2011.

[11] K. Lee, S.-H. Park, J.-S. Kim, and I. Lee, "Degrees of Freedom on MIMO Multi-Link Two-Way Relay Channels," in *Proc. IEEE Globecom*, December 2010.

[12] N. Lee and J. Chun, "Signal Space Alignment for an Encryption Message and Successive Network Code Decoding on the MIMO K-way Relay Channel," *to be presented at IEEE ICC*, June 2011.

[13] C.-B. Chae, T. Tang, J. R. W. Heath, and S. Cho, "MIMO relaying with linear processing for multiuser transmission in fixed relay networks," *IEEE Trans. Signal Proccess.*, vol. 56, no. 2, pp. 727–738, February 2008.

[14] S. Katti, S. Gollakota, and D. Katabi, "Embracing Wireless Interference: Analog Network Coding," in *Proc. SIGCOMM*, August 2007.

[15] S. Zhang, S.-C. Liew, and P. P. Lam, "Physical-layer network coding," in *Proc. MobiCom*, September 2006.