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# Diversity-Multiplexing Trade-off for Coordinated Direct and Relay Schemes

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**Abstract**

The recent years have brought a significant body of research on wireless Two-Way Relaying (TWR), where the use of network coding brings an evident advantage in terms of data rates. Yet, TWR scenarios represent only a special case and it is of interest to devise similar techniques in more general multi-flow scenarios. Such techniques can leverage on the two principles used in Wireless Network Coding to design throughput-efficient schemes: (1) aggregation of communication flows and (2) embracing and subsequently cancel/mitigate the interference. Using these principles, we investigate Coordinated Direct/Relay (CDR) schemes, which involve two flows, of a direct and a relayed user. In this paper we characterize a CDR scheme by deriving/bounding the Diversity-Multiplexing Trade-off (DMT) function. Two cases are considered. In the first case a transmitter knows the Channel State Information (CSI) of all the links in the network, while in the second case each node knows only CSI of the links towards its neighbors. The results show that the new CDR scheme outperforms the reference scheme in terms of DMT characterization. Several interesting features are identified with respect to the impact of the CSI knowledge to the improvement in diversity or multiplexing brought by the CDR scheme.

## I. Introduction

### A. *Motivation*

Relay–based transmission, has been a subject of extensive research efforts in the recent years, due to its potential to extend cellular coverage or increase diversity. Several relaying modes

have been established, such as Amplify-and-Forward (AF) [1], Decode-and-Forward (DF) [2] and Compress-and-Forward (CF) [3], etc. These modes have been used as building blocks to devise relaying techniques in relaying scenarios with one, two, or multiple communication flows [4].

In particular, Two-Way Relay (TWR) using Wireless Network Coding (WNC) has recently attracted a significant interest, due to the evident throughput benefits in wireless networks [5]–[7]. Schemes applying WNC in TWR scenarios have been extensively discussed, analyzed and evaluated in many different aspects. Yet, the TWR scenarios are essentially limited, as they represent a special case of the general scenario in which multiple communication flows need to be concurrently transmitted over a shared wireless medium. In order to generalize the transmission schemes applied in TWR scenarios, it is instructive to identify the two basic principles used in designing throughput–efficient schemes with WNC: (1) *aggregation of communication flows*: WNC operates by having a set of flows sent/processed jointly; (2) *intentional cancelable interference using side information*: flows are allowed to interfere over the wireless channel, knowing *a priori* that the interference can be cancelled by the destination via side information.

Using these principles one can design novel transmission schemes in other multi-flow scenarios that involve wireless relaying. A particularly promising scenario is a network with a base station (BS), a relay station (RS), a relayed user (U) and a direct user (V). One possible configuration of multiple traffic flows is depicted on Fig. 1, where user U receives downlink traffic from the BS, while V sends uplink traffic to the BS. In the first step the BS transmits to the relay RS. For the reference scheme $E$, in the second step RS transmits to user U and in the third step V transmits to the BS. However, using the principles behind WNC, we can do better in the following way. By recognizing that the BS knows *a priori* what RS will send in the second step because the relayed signal from RS is what was originally transmitted by the BS, the new scheme $S$ operates by combining both transmissions RS$\rightarrow$U and V$\rightarrow$BS in the same step. Then BS can cancel the signal sent by the RS and obtain a "clean" message from V. This scheme has been proposed in [8]. The same scenario can be used as a basis to design transmission schemes for other configurations of traffic flows. One example is when U has an uplink traffic towards BS

through the RS, while V has a downlink traffic from the BS. The latter and two more schemes have been discussed and analyzed in [9]. Detailed analysis of the achievable rate regions in these traffic scenarios is provided in [4] (AF) and [10] (DF).

So far, the devised CDR schemes have been analyzed only through the prism of *spectrum efficiency* [4], [8]–[10]. In order to get insight into the *diversity* aspect of the scheme, in this paper we carry out analysis of the Diversity-Multiplexing Trade-off (DMT) functions [11], [12]. In [12], a DMT analysis is provided leading to an adaptive optimization of the different transmission phase durations of TWR Network Coding. We focus on the scenario and the traffic configuration depicted on Fig. 1 (from now on referred to simply as "the scenario") and we derive the DMT functions for both the reference scheme and the scheme $S$ from [8]. We are using DF as a relaying method, which means that the relay is able to re–encode the information that is relayed with another codebook. The transmissions rates from the BS to the RS and from the RS to user U are not necessarily identical, leading to time intervals that are not necessarily equal. **The essential difference with [12] is that in the two-phase two-way relaying scheme in [12], the interference is a priori known and completely cancelled while in this paper part of the interference is cancelled and part is treated as noise. We thus introduce $\beta$ as a power exponent to distinguish the transmit powers of different sources. As a technical difference, we need to optimize over multiple variables that represent the time intervals, rather than one variable as in [12].**

The DMT function of $S$ depends on three different transmissions, two of which (RS→U and V→BS) are taking part simultaneously. In fact, due to the use of DF, the transmissions of RS and V in the second step of the scheme $S$ are not necessarily completely overlapping. The durations/overlaps of all transmissions are therefore different and subject to optimization, through which one minimizes the outage probability.

### *B. Main Contributions*

The main contributions of this paper can be described as follows. We consider two possible configurations of channel knowledge in the network. In the first case, termed *CSI-L* (Channel

State Information for the related Link channels), each node knows only the CSI of the links towards its neighbors. In the second model, termed *CSI-A* (Channel State Information for All channels), the information of all channels is available at all stations. The transmission strategies and the DMT functions are significantly different for the two cases. We analytically calculate the DMT function of the reference scheme, denoted by $E$, and the CDR scheme in CSI-L model. In CSI-A model the DMT functions are bounded.

- In the case of CSI-L, due to the fact that not all channels in the network are known, the durations of the different transmissions cannot be optimized for the instantaneous fading realizations, i.e. the actual channels that are valid during the time block in which the transmission takes place. Instead, for given rates, these durations are fixed and optimized according to the statistics of the individual links.
- In the case of CSI-A, for each fading realization one can optimize the durations of the different steps. Outage occurs if, for the fading realizations in the observed time block, there are no time durations that can satisfy the required rates.

The rest of the paper is organized as follows. Section II presents the system model used. We describe and calculate the maximal achievable rates of the reference and CDR schemes in Section III. Section IV calculates and bounds the DMT functions of the schemes. Section V presents and discusses the numerical results and Section VI concludes the paper.

## II. System Model

We consider a scenario with one base station (BS), one relay (RS), and two users (U and V), see Fig. 1. All stations have a single antenna and all transmissions have a normalized bandwidth of 1 Hz. Each of the complex channels $h_i, i \in \{1, 2, 3, 4\}$, is reciprocal and a realization of a block fading Rayleigh distribution. The variance of each channel is set to 1. We consider two channel models. In the first one, only the transmitter and the receiver of a transmission know the channel of that transmission (CSI-L). In the second one, information of all channels is available at all stations (CSI-A). The direct channel BS–U is assumed weak and U gets the information from BS only through the decoded/forwarded signal from RS.

BS has to send message $s_1$ to user U via relay RS and receive message $s_2$ from user V directly. Because we have a relayed downlink and a direct uplink, there are three relevant links: BS→RS, RS→U, V→BS, corresponding to the channels $h_1$, $h_2$ and $h_3$, respectively. We assume that the transmit powers of the corresponding transmitters are $P_B$, $P_R$, $P_V$, respectively. In addition, we assume $P_B = P_R = \rho$ and $P_V = \rho^\beta$ [14]. The noise at all stations has a complex circular Gaussian distribution with variance $\sigma_N^2$ set to 1. We denote the number of symbols in codeword $x$ as $|x|$, $\log$ denotes the base-2 logarithm and $\triangleq$ serves to define a notation via an equation.

In the reference scheme denoted as $E$, all transmissions are orthogonally multiplexed in time. In the CDR scheme denoted as $S$, transmissions of the two messages are partially overlapped in time. This overlap time is chosen in such a way that the information about the interference is exploited as much as possible. We use $R_U^i$ and $R_V^i$, $i \in \{E, S\}$ to denote the maximal achievable rates averaged over the whole duration of scheme $i$ for users U and V respectively.

Each transmission period may involve one or two transmissions with different rates. Different transmission durations are characterized by $\lambda$, $\theta$ and $\mu$, $0 < \lambda, \theta, \mu < 1$ which are defined as follows. In both schemes, there are $\lambda N$ symbols in the V→BS hop transmission where $N$ is the total number of symbols in the whole scheme. In scheme $E$, there are totally $\theta N$ symbols in the RS→U and V→BS hop transmissions. In scheme $S$, there are $\mu N$ symbols in the RS→U hop transmission.

Finally, we use $y_k[j]$ and $z_k[j]$ to denote the received and noise signals respectively at station $k$, $k \in \{B, R, U, V\}$ where B, R, U and V are corresponding subscripts for BS, RS, users U and V, in time interval $j$, $j \in \{1, 2, 3\}$ and $q_U^i[j]$ and $q_V^i[j]$ as instantaneous transmission rates in time interval $j$ for user U and V respectively in scheme $i$, $i \in \{E, S\}$. Denote $\gamma_i = \frac{|h_i|^2}{\sigma_N^2}$ with $i \in \{1, 2, 3, 4\}$. The notation summary is given in Table I.

## III. Scheme Description

We describe and calculate the achievable rates for two users in the reference scheme $E$ and the CDR scheme $S$.

TABLE I
NOTATION SUMMARY

| Notation | Description |
|---|---|
| $E$, $S$ | reference, CDR schemes |
| $h_i$ | channel coefficient |
| $\sigma_N^2 = 1$ | noise power |
| $P_B = P_R = \rho$ | transmit power of the BS and RS |
| $P_V = \rho^\beta$ | transmit power of user V |
| $R_U^i, R_V^i$ | maximal achievable rates for user U, V in scheme $i$ |
| $\lambda, \theta, \mu$ | tranmission durations |
| $N$ | symbol number in a scheme |
| $y_k[i], z_k[i]$ | received and noise signals at node $k$ in time interval $i$ |
| $q_U^i[j], q_V^i[j]$ | instantaneous rates for users U, V in time interval $j$ of scheme $i$ |
| $\gamma_i = \frac{\lvert h_i \rvert^2}{\sigma_N^2}$ | normalized channel gain |
| $r_U, r_V$ | multiplexing gains for users U, V |
| $R_U^t, R_V^t$ | target rates for users U, V |
| $\mathcal{O}_{U_k}^i, \mathcal{O}_{V_k}^i$ | outage events for the $k$-th transmission of users U, V in scheme $i$ |
| $d_{U_k}^i, d_{V_k}^i$ | diversity gain for the $k$-th transmission of users U, V in scheme $i$ |
| $d_{\mathrm{L}}^i$, | diversity gain for scheme $i$ in CSI-L |
| $d_{\mathrm{A}}^{i\mathrm{UB}}, d_{\mathrm{A}}^{i\mathrm{LB}}$ | upper and lower bound of DMT function of scheme $i$ in CSI-A |
| $r_{j,\mathrm{L}o}^i, r_{j,\mathrm{A}o}^i$ | maximum multiplexing gain for user $j$ in scheme $i$ in CSI-L, CSI-A |

### A. Reference Scheme

The reference scheme consists of three transmission phases. In the first transmission phase, BS encodes $s_1$ to $x_1$ at a rate $q_U^E[1]$ and transmits it to RS as seen in Fig. 1. RS receives $y_R[1] = h_1 x_1 + z_R[1]$. In the second transmission phase, RS decodes $x_1$ to $s_1$, re-encodes it to $x_1^R$ at a rate $q_U^E[2]$ and transmits it to user U. U receives $y_U[2] = h_2 x_1^R + z_U[2]$. In the third transmission phase, user V encodes $s_2$ to $x_2$ at a rate $q_V^E[3]$ and transmits it to BS. BS receives $y_B[3] = h_3 x_2 + z_B[3]$. Since $x_1$, $x_2$ and $x_3$ are transmitted with power $P_B$, $P_R$ and $P_V$, the rates $q_U^E[1]$, $q_U^E[2]$ and $q_V^E[3]$ are selected as the maximal rates over the corresponding channels $q_U^E[1] = \log(1 + \rho\gamma_1) \triangleq C_1$, $q_U^E[2] = \log(1 + \rho\gamma_2) \triangleq C_2$ and $q_V^E[3] = \log\left(1 + \rho^\beta \gamma_3\right) \triangleq C_3$.

Since all transmissions are performed separately, the duration of the BS→RS transmission is $(1-\theta)N$. The BS→RS, RS→U and V→BS transmissions therefore have durations of $(1-\theta)N$, $(\theta-\lambda)N$ and $\lambda N$ symbols respectively. On the other hand, the corresponding maximal rates are $C_1$, $C_2$ and $C_3$ for each of the respective transmissions. Thus the maximal rates for the transmissions on the links BS→RS, RS→U and V→BS are

$$R_{U_1}^E = \frac{(1-\theta)NC_1}{N} = (1-\theta)C_1, \quad R_{U_2}^E = (\theta-\lambda)C_2, \quad R_V^E = \lambda C_3. \tag{1}$$

*B. CDR Scheme*

The RS→U and V→BS transmissions can be conducted simultaneously because the interfering signal from RS to BS is known at BS and can be cancelled. Therefore compared to the reference scheme, the maximal rates will be increased.

In the first time interval, BS transmits $x_1$ to RS (see Fig. 1), RS receives $y_R[1] = h_1x_1 + z_R[1]$ and decodes $x_1$ to $s_1$. The maximal achievable rate of this transmission is therefore $q_U^S[1] = C_1$. In the second time interval, two cases are distinguished depending on the relative duration of transmissions RS→U and V→BS.

*1) $\mu \geq \lambda$:* In the second time interval, user V encodes $s_2$ to $x_2$ and RS divides $s_1$ into two sub-messages $s_{1,1}$ and $s_{1,2}$ so that after re-encoding to $x_{1,1}^R$ and $x_{1,2}^R$, we have $|x_{1,1}^R| = \lambda N$. RS transmits $x_{1,1}^R$ to user U and user V transmits $x_2$ to BS simultaneously. BS thus receives $y_B[2] = h_1x_{1,1}^R + h_3x_2 + z_B[2]$. Since BS knows $x_1$, it can cancel the contribution of $x_1$ and decodes its desired signal $x_2$ with maximal achievable rate $C_3$. In the meantime, user U receives $y_U[2] = h_2x_{1,1}^R + h_4x_2 + z_U[2]$. There are two decoding options for user U:

- No-Interference Cancellation (NIC): User U decodes $x_{1,1}^R$ treating $x_2$ as noise. The maximal achievable rate for transmitting $x_{1,1}^R$ is $q_U^S[2] = \log\left(1 + \frac{\rho\gamma_2}{\rho^\beta\gamma_4+1}\right) \triangleq C_{2-4}$. The transmitting rate of $x_2$ is selected as $q_V^S[2] = C_3$.
- Interference Cancellation (IC): User U first decodes $x_2$ treating $x_{1,1}^R$ as noise. It is possible only if the transmitting rate of $x_2$ is not larger than $\log\left(1 + \frac{\rho^\beta\gamma_4}{\rho\gamma_2+1}\right) \triangleq C_{4-2}$. Hence the transmitting rate of $x_2$ is selected as $q_V^S[2] = \min\{C_3, C_{4-2}\}$. Then user U cancels $x_2$'s

contribution in $y_U[2]$ and decodes $x_{1,1}$ interference-free. Hence the maximal achievable rate for transmitting $x_{1,1}$ is $q_U^S[2] = C_2$.

A trade-off between those two options can be achieved by multiplexing them by time. It is actually one point in the diagonal edge of the Multiple Access Channel (MAC) rate region when user U treats the interference from user V as another signal to decode [13]. In terms of sum-rate, this trade-off solution can be better than both NIC and IC decoding options above. However, when computing the DMT, the transmit powers tend to infinity. Depending on which of RS and user V's powers goes to infinity faster, the only relevant decoding options are either NIC or IC.

In the third time interval, RS transmits $x_{1,2}^R$ to user U interference-free with rate $q_U^S[3] = C_2$.

The durations for the three time slots are therefore $(1-\mu)N$, $\lambda N$ and $(\mu-\lambda)N$ symbols respectively. The maximal achievable rates averaged over the whole duration of the scheme are,

- For links BS→RS, RS→U and V→BS in the NIC option, $R_{U_1}^S = (1-\mu)C_1$, $R_{U_2}^S = \lambda C_{2-4} + (\mu-\lambda)C_2$ and $R_V^S = \lambda C_3$.
- For links BS→RS, RS→U, V→U and V→BS in the IC option, $R_{U_1}^S = (1-\mu)C_1$, $R_{U_2}^S = \mu C_2$ and $R_V^S = \lambda \min(C_3, C_{4-2}) + (\mu-\lambda)\min(C_3, C_4)$.

Selecting a decoding option will be discussed later on.

*2) $\mu < \lambda$:* In the second time interval, RS encodes $s_1$ to $x_1^R$ and user V divides $s_2$ into two sub-messages $s_{2,1}$ and $s_{2,2}$ so that after re-encoding to $x_{2,1}$ and $x_{2,2}$, we have $|x_{2,1}| = \mu N$. RS transmits $x_1^R$ to user U and user V transmits $x_{2,1}$ to BS simultaneously. The NIC and IC options are conducted similarly as in the case $\mu \geq \lambda$. In the third time interval, user V transmit $x_{2,2}$ to BS.

The durations for the three time slots are $(1-\lambda)N$, $\mu N$ and $(\lambda-\mu)N$ symbols respectively. The maximal achievable rates averaged over the whole duration of the scheme are,

- For links BS→RS, RS→U and V→BS in the NIC option, $R_{U_1}^S = (1-\lambda)C_1$, $R_{U_2}^S = \mu C_{2-4}$ and $R_V^S = \lambda C_3$.
- For links BS→RS, RS→U, V→U and V→BS in the IC option, $R_{U_1}^S = (1-\lambda)C_1$, $R_{U_2}^S = \mu C_2$ and $R_V^S = \mu \min\{C_3, C_{4-2}\} + (\lambda-\mu)\min(C_3, C_4)$ with $C_4 \triangleq \log\left(1+\rho^\beta \gamma_4\right)$.

## IV. Diversity-Multiplexing Trade-off (DMT) Analysis

We first introduce the DMT definition and notations in section IV-A and derive the DMT functions in both channel modes CSI-L and CSI-A for $E$ in section IV-B and $S$ in section IV-C.

### A. DMT Definition and Notations

A scheme is said to achieve spatial multiplexing gain $r$ and diversity gain $d$ if

$$\lim_{\zeta\to\infty} \frac{R(\zeta)}{\log\zeta} = r \quad \text{and} \quad \lim_{\zeta\to\infty} -\frac{\log P_e(\zeta)}{\log\zeta} = d \tag{2}$$

where $R(\zeta)$ is the maximal achievable rate, $\zeta$ is the corresponding average SNR and $P_e$ is the average outage probability.

Throughout the rest of the paper, we use the symbol $\doteq$ to denote exponential equality, i.e. we write $f(\zeta) \doteq \zeta^b$ then $\lim_{\zeta\to\infty} \frac{\log f(\zeta)}{\log\zeta} = b$. Notations and $\dot{\geq}$, $\dot{\leq}$ are similarly defined. Therefore, the second equation in (2) can be written as

$$P_e(\zeta) \doteq \zeta^{-d}. \tag{3}$$

We also denote $(x)^+ = \max(0, x)$. Because $\lim_{\zeta\to\infty} \frac{\zeta^a+\zeta^b}{\zeta^{\max(a,b)}} = 1$, we often use $\zeta^a+\zeta^b \doteq \zeta^{\max(a,b)}$ or $\zeta^a + 1 = \zeta^a + \zeta^0 \doteq \zeta^{\max(a,0)} = \zeta^{a^+}$. Moreover,

$$\Pr[\gamma_i < x] = \int_0^x e^{-t}\mathrm{d}t = 1 - e^{-x} \text{ and } \lim_{x\to 0} \frac{1-e^{-x}}{x} = 1. \tag{4}$$

Therefore,

$$\Pr\left[\gamma_i < \zeta^{-a}\right] \doteq \zeta^{-a^+}. \tag{5}$$

We investigate the outage probability and how fast it decays with respect to $\log P_i$, $i \in \{B, R, V\}$, when the system tries to achieve a certain target rate pair $(R_U^t, R_V^t)$.

Since the stations have different transmit powers, the target rates have different expressions. We calculate the DMT for the scheme when the target rate for user $i$ is given by $R_i^t = r_i \log P_j$ where $r_i$ is the corresponding multiplexing gain and $P_j$ is the transmit power of the transmitter i.e. for direct uplink $P_j = P_V = \rho^\beta$ and for relayed downlink $P_j = P_B = \rho$. We assume that $\beta$ is always known in both cases of CSI-L and CSI-A.

The scheme is in outage when the maximal achievable rate for user U or user V is lower than the respective target rate. The maximal achievable rate is computed over the whole duration of the scheme and depends on time-multiplexing factors $(\lambda, \theta)$ for scheme $E$ and $(\lambda, \mu)$ for scheme $S$. According to the CSI models that we have adopted, an outage is not due to the lack of knowledge of the instantaneous value of channel at the transmitter, but rather to the values of the multiplexing factors. For example, in scheme $S$, if the transmission from the BS is allocated a short duration compared to the 2 other simultaneous transmissions, the rate to user $U$ computed over the whole duration of the scheme will likely be small and below the target rate.

In the CSI-L model of scheme $E$, an outage is defined if the pair $(\lambda, \theta)$ cannot support a target rate pair $(R_U^t, R_V^t)$. The outage probability and then the DMT function are calculated as functions of $\lambda$ and $\theta$. The DMT function is maximized based on $\lambda$ and $\theta$. In the CSI-A model of scheme $E$, all channels are available thus for a certain channel realization, $\lambda$ and $\theta$ are calculated so that they can support a target rate pair for the given fading realization. An outage is defined as there is not any pair $(\lambda, \theta)$ which can support the target rate pair [12], [15], [16]. A similar procedure is carried out in both channel models of scheme $S$ with parameters $(\lambda, \mu)$.

### *B. Reference Scheme $E$*

#### *1) CSI-L:*

*Proposition 4.1:* The DMT function of scheme $E$ in CSI-L channel model is given by $d_{\mathrm{L}}^E = \min\left\{d_{U_1}^E, d_{U_2}^E, d_V^E\right\}$ in which

$$d_{U_1}^E = \left(1 - \tfrac{r_U}{1-\theta}\right)^+, \quad d_{U_2}^E = \left(1 - \tfrac{r_U}{\theta-\lambda}\right)^+, \quad d_V^E = \beta\left(1 - \tfrac{r_V}{\lambda}\right)^+ \tag{6}$$

are the DMT functions of three transmissions BS→RS, RS→U and V→BS in the scheme respectively.

*Proof:* The scheme is in outage, denoted as $\mathcal{O}^E$, when either $R_U^t$ is larger than the maximal achievable rate $R_U^E$ for user $U$ or $R_V^t$ is larger than the maximal achievable rate $R_V^E$ for user $V$. Recalling that $R_U^E = \min(R_{U_1}^E, R_{U_2}^E)$, the system is in outage when one of the following

conditions occur

$$\mathcal{O}_{U_1}^E : R_{U_1}^E < R_U^t, \quad \mathcal{O}_{U_2}^E : R_{U_2}^E < R_U^t, \quad \mathcal{O}_V^E : R_V^E < R_V^t. \tag{7}$$

Hence $\mathcal{O}_{\mathrm{L}}^E = \mathcal{O}_{U_1}^E \cup \mathcal{O}_{U_2}^E \cup \mathcal{O}_V^E$, where subscript L refers to CSI-L. The probability of the first event is

$$\Pr\left[\mathcal{O}_{U_1}^E\right] = \Pr[(1+\rho\gamma_1)^{1-\theta} < \rho^{r_U}] \doteq \Pr[(\rho\gamma_1)^{1-\theta} < \rho^{r_U}] = \Pr[\gamma_1 < \rho^{\frac{r_U}{1-\theta}-1}]. \tag{8}$$

The second equality in (8) is obtained when letting $\rho$ tend to infinity. This probability decays only when $\frac{r_U}{1-\theta} - 1 < 0$. It means that we have a positive diversity gain corresponding to this transmission only when $1 - \frac{r_U}{1-\theta} > 0$. Using (5), we can write $\Pr\left[\mathcal{O}_{U_1}^E\right] \doteq \rho^{-\left(1-\frac{r_U}{1-\theta}\right)^+}$. Similarly, the probability of the second event is $\Pr\left[\mathcal{O}_{U_2}^E\right] \doteq \rho^{-\left(1-\frac{r_U}{\theta-\lambda}\right)^+}$. We have $\Pr\left[\mathcal{O}_V^E\right] \doteq \rho^{-\beta\left(1-\frac{r_V}{\lambda}\right)^+}$. Therefore,

$$\Pr[\mathcal{O}_{\mathrm{L}}^E] \;\doteq\; \Pr[\mathcal{O}_{U_1}^E] + \Pr[\mathcal{O}_{U_2}^E] + \Pr[\mathcal{O}_V^E] \doteq \rho^{\max\left\{-d_{U_1}^E, -d_{U_2}^E, -d_V^E\right\}} = \rho^{-\min\left\{d_{U_1}^E, d_{U_2}^E, d_V^E\right\}}. \tag{9}$$

The first equality comes from the fact that the events $\mathcal{O}_{U_1}^E$, $\mathcal{O}_{U_2}^E$ and $\mathcal{O}_V^E$ are independent due to the independence of $\gamma_1$, $\gamma_2$ and $\gamma_3$ and that the product of any two or three of the probabilities $\Pr[\mathcal{O}_{U_1}^E]$, $\Pr[\mathcal{O}_{U_2}^E]$ and $\Pr[\mathcal{O}_V^E]$ decays faster than each of them. The second equality is due to that the smaller terms are negligible and the probability is determined by the largest term in the sum when $\rho \to \infty$. According to the definition in (3), the diversity gain is therefore $d_{\mathrm{L}}^E = \min\left(d_{U_1}^E, d_{U_2}^E, d_V^E\right) \triangleq f_{\mathrm{L}}^E(\theta, \lambda, r_U, r_V)$. ■

The information in CSI-L channel model is necessary and enough to achieve the result above. At a station, the information of all channels is not available thus determining the transmission durations according to the current channel realization is impossible. Therefore, as shown above, the results in (6) is obtained by averaging all channel realizations. The optimal transmission durations will be then optimized based on these averaged results. On the other hand, the information of the channels to the neighbors of a station is still necessary such that it can determine the instantaneous rates to transmit and receive e.g. in the second time interval of scheme $E$, the RS needs to know $h_2$ to determine $q_U^E[2] = C_2$. However this information does

not help to determine the optimal transmission durations as all channels are assumed to be independent.

*Proposition 4.2:* The DMT function of scheme $E$ in CSI-L channel model with optimized transmission durations is given by

$$d_{\mathrm{Lo}}^{E} = \left(1 - \frac{r_U}{1-\theta_o}\right)^+ = \left(1 - \frac{r_U}{\theta_o - \lambda_o}\right)^+ = \beta\left(1 - \frac{r_V}{\lambda_o}\right)^+. \tag{10}$$

in which $\theta_o = \frac{\lambda_o+1}{2}$, $\lambda_o = \frac{1-\beta-\beta r_V-2r_U+\sqrt{\Delta}}{2(1-\beta)}$ and $\Delta = (1-\beta-\beta r_V-2r_U)^2 + 4(1-\beta)\beta r_V$.

*Proof:* Each individual diversity component $\left(d_{U_1}^E, d_{U_2}^E, d_V^E\right)$ increases when the corresponding time assigned to each transmission, $(1-\theta)N$, $(\theta-\lambda)N$ and $\lambda N$ respectively, increases. The time durations assigned to the 3 transmission hops should be balanced such that the diversity gain of the whole scheme is maximized. Note that increasing the diversity gain at a certain multiplexing gain also makes the multiplexing gain at a certain other diversity gain increased. Therefore maximizing diversity gain is equivalent to maximizing the multiplexing gain. Hence, with finite $\beta$, the optimal values $\lambda_o$ and $\theta_o$ are roots of the equations $d_{U_1}^E = d_{U_2}^E = d_V^E$. The equations $d_{U_1}^E = d_{U_2}^E = d_V^E$ are equivalent to $(1-\beta)\lambda^2 - (1-\beta-\beta r_V-2r_U)\lambda - \beta r_V = 0$ and $\theta = \frac{1+\lambda}{2}$. Because $\Delta = (1-\beta-\beta r_V-2r_U)^2 + 4(1-\beta)\beta r_V \geq 0$, there are always real roots. We consider two cases

- $\beta \leq 1$, the two roots satisfies $\lambda_1\lambda_2 = -\beta(1-\beta)r_V \leq 0$, there is hence one positive root.
- $\beta > 1$, $\lambda_1\lambda_2 = -\beta(1-\beta)r_V > 0$ and $\lambda_1 + \lambda_2 = \frac{1-\beta-\beta r_V-2r_U}{1-\beta} > 0$, there are hence two positive roots.

We select the root as written in Proposition (4.2) which satisfies $0 < \lambda < \theta < 1$. ■

*2) CSI-A:* For the CSI-A case we have only derived the upper bound, similar to the approach in [11], [12].

*Proposition 4.3:* The upper bound of the DMT function of scheme $E$ in CSI-A channel model is given by

$$d_{\mathrm{A}}^{E_{\mathrm{UB}}} = \begin{cases} \left(\frac{\beta(1-2r_U-r_V)}{1-2r_U}\right)^+ & \text{if} \quad \beta \leq 1 \\ \left(\frac{1-2r_U-r_V}{1-r_U-r_V}\right)^+ & \text{if} \quad \beta > 1 \end{cases} \tag{11}$$

*Proof:* When all channels are available at all stations, the value of $\lambda$ and $\theta$ can be selected according to all CSIs. User U and BS can decode the messages $s_1$ and $s_2$ respectively when

$$R_{U_1}^E \geq R_U^t, \quad R_{U_2}^E \geq R_U^t, \quad R_V^E \geq R_V^t \tag{12}$$

Denoting $a_1 \triangleq \frac{r_U \log \rho}{\log(1+\rho\gamma_1)}$, $a_2 \triangleq \frac{r_U \log \rho}{\log(1+\rho\gamma_2)}$ and $a_3 \triangleq \frac{r_V \log \rho^\beta}{\log(1+\rho^\beta\gamma_3)}$, equation (12) is equivalent to

$$\theta \leq 1 - a_1, \quad \lambda \geq a_3 \quad \text{and} \quad \theta \geq \lambda + a_2 \tag{13}$$

We illustrate condition (13) in Fig. 2 with a coordinate system of $\lambda$ and $\theta$ in which the shaded area represents the values of $\lambda$ and $\theta$ which satisfy the conditions in (13). This non-outage area does not exist if and only if the point $(a_3, 1-a_1)$ lies below the line $\theta = \lambda + a_2$. This condition is equivalent to $1 - a_1 < a_3 + a_2$ or

$$a_1 + a_2 + a_3 > 1. \tag{14}$$

Equation (14) is the condition for an outage event, denoted as $\mathcal{O}_{\mathrm{A}}^E$, for the CSI-A model. Denote $\alpha_i = -\frac{\log \gamma_i}{\log \rho}$, $i \in \{1,2,3,4\}$ therefore $a_1 \doteq \frac{r_U \log \rho}{\log(\rho\gamma_1)} = \frac{r_U}{1-\alpha_1}$, $a_2 \doteq \frac{r_U \log \rho}{\log(\rho\gamma_2)} = \frac{r_U}{1-\alpha_2}$ and $a_3 \doteq \frac{r_V \log \rho^\beta}{\log(\rho^\beta\gamma_3)} \doteq \frac{r_V}{1-\frac{\alpha_3}{\beta}}$ hence

$$\Pr[\mathcal{O}_{\mathrm{A}}^E] = \Pr\left[\frac{r_U}{1-\alpha_1} + \frac{r_U}{1-\alpha_2} + \frac{r_V}{1-\frac{\alpha_3}{\beta}} > 1\right]. \tag{15}$$

When $\rho \to \infty$, $\alpha_i \to 0$, $\alpha_1\alpha_2$, $\alpha_2\alpha_3$, $\alpha_3\alpha_1$ and $\alpha_1\alpha_2\alpha_3$ decay faster than $\alpha_1$, $\alpha_2$ or $\alpha_3$ hence

$$\begin{aligned} \Pr[\mathcal{O}_{\mathrm{A}}^E] \;\; \doteq \;\; & \Pr\left[r_U(\beta - \beta\alpha_2 - \alpha_3) + r_U(\beta - \alpha_1 - \alpha_3)\right. \\ & \left. + r_V(\beta - \beta\alpha_1 - \beta\alpha_2) > \beta - \beta\alpha_1 - \beta\alpha_2 - \alpha_3\right]. \end{aligned} \tag{16}$$

Because the time is divided for two transmissions of user U and one tranmission of user V we have $r_U + r_V < 1$ and $2r_U < 1$. Using these conditions, we have two cases below respectively. We therefore obtain lower bounds on the outage probability and thus upper bounds on the DMT functions for both cases.

$$\Pr[\mathcal{O}_{\mathrm{A}}^E] \dot{\geq} \begin{cases} \Pr\left[\alpha_3 > \frac{\beta(1-2r_U-r_V)}{1-2r_U}\right] & = \Pr\left[\gamma_3 < \rho^{-\left(\frac{\beta(1-2r_U-r_V)}{1-2r_U}\right)^+}\right] \quad \text{if} \quad \beta \leq 1 \\ \Pr\left[\alpha_1 > \frac{(1-2r_U-r_V)}{1-r_U-r_V}\right] & = \Pr\left[\gamma_1 < \rho^{-\left(\frac{1-2r_U-r_V}{1-r_U-r_V}\right)^+}\right] \quad \text{if} \quad \beta > 1 \end{cases} \tag{17}$$

The upper bound of the DMT function is therefore given by (11). ■

It is obvious that $\Pr[\mathcal{O}_{\mathrm{A}}^{E}]$ monotonically decreases with $\rho$ whenever $\frac{\beta(1-3r)}{1-2r}$ is positive. Therefore $-\log\rho\Pr[\mathcal{O}_{\mathrm{A}}^{E}]$ monotonically increases with $\rho$. A lower bound of the DMT function can be hence obtained by calculating $\Pr[\mathcal{O}_{\mathrm{A}}^{E}]$ at a fixed value of $\rho=\rho_o$. The higher $\rho_o$ is, the tighter the lower bound is. From (15), the condition $\frac{r_U}{1-\alpha_1}+\frac{r_U}{1-\alpha_2}+\frac{r_V}{1-\frac{\alpha_3}{\beta}}>1$ is equivalent to $\alpha_3>m_E$ with $m_E=\beta\left(1-\frac{r_V}{1-\frac{r_U}{1-\alpha_1}-\frac{r_U}{1-\alpha_2}}\right)$. It means that an outage occur when $\gamma_3<\rho_o^{-m_E}$ therefore we have

$$d_{\mathrm{A}}^{E_{\mathrm{LB}}}=-\log_{\rho_o}\int_0^{\infty}e^{-\gamma_1}\mathrm{d}\gamma_1\int_0^{\infty}e^{-\gamma_2}\mathrm{d}\gamma_2\int_0^{\rho_o^{-m_E}}e^{-\gamma_3}\mathrm{d}\gamma_3. \tag{18}$$

This bound is numerically calculated and given in section V.

### C. Coordinated Direct and Relay Scheme $S$

In the CDR scheme $S$, there are two decoding options at user U. When $\beta\leq 1$, the NIC option treating the interference from user V (with transmit power $\rho^{\beta}$) as noise is better than the IC option decoding the interference first. On the other hand, in the NIC option, the three transmission hops BS→RS, RS→U and V→BS have corresponding normalized rates $R_{U_1}^{S}=(1-\max(\mu,\lambda))C_1$, $R_{U_2}^{S}=\min(\mu,\lambda)C_{2-4}+(\mu-\min(\mu,\lambda))C_2$ and $R_V^S=\lambda C_3$. We have

- If $\mu\geq\lambda$, $R_{U_1}^{S}=(1-\mu)C_1$, $R_{U_2}^{S}=\lambda C_{2-4}+(\mu-\lambda)C_2$, $R_V^S=\lambda C_3$.
- If $\mu<\lambda$, $R_{U_1}^{S}=(1-\lambda)C_1$, $R_{U_2}^{S}=\lambda C_{2-4}$, $R_V^S=\lambda C_3$.

Hence $R_{U_2(\mu\geq\lambda)}^{S}\geq R_{U_2(\mu<\lambda)}^{S}$, $R_{V(\mu\geq\lambda)}^{S}=R_{V(\mu<\lambda)}^{S}$. On the other hand, because $\lambda$ and $\mu$ take similar roles in $R_{U_1}^{S}$, the case $\mu\geq\lambda$ is better than the case $\mu<\lambda$ in the NIC option.

When $\beta>1$, the IC option is better than the NIC option and the case $\mu<\lambda$ is better than the case $\mu\geq\lambda$ in the IC option. To summarize, in each CSI model, we consider two cases: If $\beta\leq 1$, we use the NIC option with $\mu\geq\lambda$ and if $\beta>1$, we use the IC option with $\mu<\lambda$.

#### 1) CSI-L:

*Proposition 4.4:* The DMT function of scheme $S$ in CSI-L channel model is given by

- If $\beta \leq 1$, $d_L^S = \min\left\{d_{U_1}^S, d_{U_2}^S, d_V^S\right\}$ in which

$$d_{U_1}^S = \left(1 - \tfrac{r_U}{1-\mu}\right)^+, \quad d_V^S = \beta\left(1 - \tfrac{r_V}{\lambda}\right)^+,$$
$$d_{U_2}^S = \begin{cases} \left(1 - \frac{r_U}{\mu-\lambda}\right)^+ & \text{if} \quad r_U \leq \beta(\mu-\lambda) \\ \left(1 - \frac{\beta\lambda}{\mu} - \frac{r_U}{\mu}\right)^+ & \text{if} \quad r_U > \beta(\mu-\lambda). \end{cases} \tag{19}$$

- If $\beta > 1$, $d_L^S = \min\left\{d_{U_1}^S, d_{U_2}^S, d_{V_1}^S, d_{V_2}^S\right\}$ in which

$$d_{U_1}^S = \left(1 - \tfrac{r_U}{1-\lambda}\right)^+, \quad d_{U_2}^S = \left(1 - \tfrac{r_U}{\mu}\right)^+, \quad d_{V_2}^S = \beta\left(1 - \tfrac{r_V}{\lambda}\right)^+,$$
$$d_{V_1}^S = \begin{cases} \beta\left(1 - \frac{r_V}{\lambda-\mu}\right)^+ & \text{if} \quad r_V \leq \frac{\lambda-\mu}{\beta} \\ \left(\beta - \frac{\mu}{\lambda} - \frac{\beta r_V}{\lambda}\right)^+ & \text{if} \quad r_V > \frac{\lambda-\mu}{\beta}. \end{cases} \tag{20}$$

*Proof:* We consider two cases below

- $\beta \leq 1$: The scheme is in outage when one of the three link transmissions BS→RS, RS→U and V→BS is in outage; the corresponding events are denoted by $\mathcal{O}_{U_1}^S$, $\mathcal{O}_{U_2}^S$ and $\mathcal{O}_V^S$. According to section III-B1, $R_{U_1}^S = R_{U_1}^E$ and $R_V^S = R_V^E$ therefore $\Pr[\mathcal{O}_{U_1}^S] = \Pr[\mathcal{O}_{U_1}^E]$ and $\Pr[\mathcal{O}_V^S] = \Pr[\mathcal{O}_V^E]$ which are already calculated in the scheme $E$. Here $x_{1,1}^R$ and $x_{1,2}^R$ are jointly decoded therefore only one outage event for the RS–user U is defined.

$$\Pr[\mathcal{O}_{U_2}^S] \;=\; \Pr\left[R_{U_2}^S < R_U^t\right] \doteq \Pr\left[\left(1 + \tfrac{\gamma_2\rho}{\gamma_4\rho^\beta}\right)^\lambda (1+\gamma_2\rho)^{\mu-\lambda} < \rho^{r_U}\right] \tag{21}$$

We consider four cases below in which the outage events are denoted as $\mathcal{O}_L^{S_1}$, $\mathcal{O}_L^{S_2}$, $\mathcal{O}_L^{S_3}$ and $\mathcal{O}_L^{S_4}$ respectively.

  - $\frac{\gamma_2\rho}{\gamma_4\rho^\beta} \leq 1$ and $\gamma_2\rho \leq 1$: In this case, the outage always occurs. We have $\Pr\left[\mathcal{O}_L^{S_1}\right] = \rho^{-1}$.
  - $\frac{\gamma_2\rho}{\gamma_4\rho^\beta} > 1$ and $\gamma_2\rho \leq 1$: It is in outage when $\left(\frac{\gamma_2\rho}{\gamma_4\rho\beta}\right)^\lambda < \rho^{r_U}$. Combinning the conditions we have $\Pr\left[\mathcal{O}_L^{S_2}\right] \doteq \min\left\{\rho^{-1}, \rho^{\frac{r_U}{\lambda}+\beta-1}\right\} \doteq \rho^{-1}$
  - $\frac{\gamma_2\rho}{\gamma_4\rho^\beta} \leq 1$ and $\gamma_2\rho > 1$: It is in outage when $(\gamma_2\rho)^{\mu-\lambda} < \rho^{r_U}$. Combinning the conditions we have

$$\Pr\left[\mathcal{O}_L^{S_3}\right] \doteq \rho^{-\max\left\{1-\beta, 1-\frac{r_U}{\mu-\lambda}\right\}^+} \doteq \begin{cases} \rho^{-\left(1-\frac{r_U}{\mu-\lambda}\right)^+ 1} & \text{if} \quad r_U \leq \beta(\mu-\lambda) \\ \rho^{-(1-\beta)} & \text{if} \quad r_U > \beta(\mu-\lambda) \end{cases} \tag{22}$$

– $\frac{\gamma_2\rho}{\gamma_4\rho^\beta} > 1$ and $\gamma_2\rho > 1$: It is in outage when $\left(\frac{\gamma_2\rho}{\gamma_4\rho\beta}\right)^\lambda (\gamma_2\rho)^{\mu-\lambda} < \rho^{r_U}$. Combinning the conditions we have

$$\Pr\left[\mathcal{O}_L^{S_4}\right] \doteq \begin{cases} 0 & \text{if} \quad r_U \leq \beta(\mu-\lambda) \\ \rho^{-\left(1-\frac{\beta\lambda}{\mu}-\frac{r_U}{\mu}\right)^+} & \text{if} \quad r_U > \beta(\mu-\lambda) \end{cases} \tag{23}$$

$$\begin{aligned} \Pr\left[\mathcal{O}_L\right] &= \Pr\left[\mathcal{O}_L^{S_1} \cup \mathcal{O}_L^{S_2} \cup \mathcal{O}_L^{S_3} \cup \mathcal{O}_L^{S_4}\right] \doteq \Pr\left[\mathcal{O}_L^{S_1}\right] + \Pr\left[\mathcal{O}_L^{S_2}\right] + \Pr\left[\mathcal{O}_L^{S_3}\right] + \Pr\left[\mathcal{O}_L^{S_4}\right] \\ &\doteq \begin{cases} \rho^{-\left(1-\frac{r_U}{\mu-\lambda}\right)^+} & \text{if} \quad r_U \leq \beta(\mu-\lambda) \\ \rho^{-\left(1-\frac{\beta\lambda}{\mu}-\frac{r_U}{\mu}\right)^+} & \text{if} \quad r_U > \beta(\mu-\lambda) \end{cases} \end{aligned} \tag{24}$$

The DMT function for this case is therefore given in Proposition 4.4.

- $\beta > 1$: In this case, the scheme is in outage when one of the following event occurs

$$R_{U_1}^S \geq R_U^t, \quad R_{U_2}^S \geq R_U^t, \quad \min\left(R_{V_1}^S, R_{V_2}^S\right) \geq R_V^t \tag{25}$$

with $R_{U_1}^S$, $R_{U_2}^S$, $R_{V_1}^S$ and $R_{V_2}^S$ given in section III-B2. With similar steps as in the case $\beta \leq 1$, we have the DMT function as Proposition 4.4.

■

*Proposition 4.5:* The DMT function of scheme $S$ with optimized transmissions' durations in CSI-L channel model is given by

$$d_{Lo}^S = \begin{cases} \left(1-\frac{r_U}{1-\mu_a}\right)^+ & \text{if} \quad \beta \leq \beta_{a,b} \\ \left(1-\frac{r_U}{1-\mu_b}\right)^+ & \text{if} \quad \beta_{a,b} \leq \beta > \beta_{b,c} \\ \left(1-\frac{r_U}{1-\mu_c}\right)^+ & \text{if} \quad \beta_{b,c} \leq \beta > 1 \\ \left(1-\frac{r_U}{1-\lambda_d}\right)^+ & \text{if} \quad 1 \leq \beta \leq \beta_{d,e} \\ \left(1-\frac{r_U}{1-\lambda_e}\right)^+ & \text{if} \quad \beta_{d,e} \leq \beta \leq \beta_{e,f} \\ (1-2r_U)^+ & \text{if} \quad \beta > \beta_{e,f} \end{cases} \tag{26}$$

where $\mu_a = \lambda_a = \frac{-\beta-\beta r_V+1-r_U+\sqrt{(\beta+\beta r_V-1+r_U)^2+4\beta(1-\beta)r_V}}{2(1-\beta)}$,
$\mu_b = \frac{\beta\lambda_b+r_U}{\beta\lambda_b+2r_U}$, $\lambda_b = \frac{1-\beta-2r_U+\sqrt{(\beta-1+2r_U)^2+4\beta^2 r_V}}{2\beta}$,
$\mu_c = \frac{\lambda_c+1}{2}$, $\lambda_c = \frac{1-\beta-\beta r_V-2r_U+\sqrt{(1-\beta-\beta r_V-2r_U)^2+4(1-\beta)\beta r_V}}{2(1-\beta)}$,
$\mu_d = 1-\lambda_d$, $\lambda_d = \frac{3\beta-3+2r_U+\beta r_V-\sqrt{(3\beta-3+2r_U+\beta r_V)^2-8(\beta-1)(\beta+\beta r_V+r_U-1)}}{4(\beta-1)}$,

$\mu_e = 1 - \lambda_e$, $\lambda_e = \frac{r_U+\beta+1+\beta r_V - \sqrt{(r_U+\beta+1+\beta r_V)^2 - 4\beta(1+\beta r_V)}}{2\beta}$,

$\mu_f = \lambda_f = \frac{1}{2}$,

$\beta_{a,b}$, $\beta_{b,c}$, $\beta_{d,e}$ and $\beta_{e,f}$ are the roots of the equations $\mu_a(\beta) = \mu_b(\beta)$, $\mu_b(\beta) = \mu_c(\beta)$, $\lambda_d(\beta) = \lambda_e(\beta)$ and $\lambda_e(\beta) = \frac{1}{2}$ respectively.

*Proof:* Similarly to scheme E in CSI-L, the optimal values $\lambda_i$ and $\mu_i$, $i \in \{a, b, c, d\}$, are also roots of the equation $d^S_{U_1} = d^S_{U_2} = d^S_V$. ■

*2) CSI-A:*

*Proposition 4.6:* For scheme $S$ in the CSI-A model the upper bound of the DMT function of the CDR scheme $S$ in CSI-A channel model is given by

$$d_{\mathrm{A}}^{S_{\mathrm{UB}}} = \begin{cases} \beta\left(\frac{1-2r_U-\beta r_V}{1-2r_U}\right)^+ & \text{if} \quad \beta \le 1 \\ \left(\frac{1-\left(1+\frac{1}{\beta}\right)r_U - r_V}{1-\frac{r_U}{\beta}-r_V}\right)^+ & \text{if} \quad \beta > 1. \end{cases} \tag{27}$$

The proof is given in the Appendix. On the other hand, similarly to scheme $E$ and based on (32) and (36) a lower bound for scheme $S$ is given as follows.

$$d_{\mathrm{A}}^{S_{\mathrm{LB}}} = \begin{cases} -\log_{\rho_o} \int_0^\infty e^{-\gamma_1} \mathrm{d}\gamma_1 \int_0^\infty e^{-\gamma_2} \mathrm{d}\gamma_2 \int_0^{\rho_o^{-m_{S_1}}} e^{-\gamma_3} \mathrm{d}\gamma_3 & \text{if} \quad \beta \le 1 \\ -\log \rho_o \int_0^\infty e^{-\gamma_2} \mathrm{d}\gamma_2 \int_0^\infty e^{-\gamma_4} \mathrm{d}\gamma_4 \int_0^{\rho_o^{-m_{S_2}}} e^{-\gamma_1} \mathrm{d}\gamma_3 & \text{if} \quad \beta > 1 \end{cases} \tag{28}$$

where $m_{S_1} = \beta\left(1 - \frac{\beta r_V}{1-\frac{r_U}{1-\alpha_1}-\frac{r_U}{1-\alpha_2}}\right)$ and $m_{S_2} = 1 - \frac{r_U}{1-\frac{r_U}{\beta(1-\alpha_2)}-\frac{r_V}{1-\frac{\alpha_4}{\beta}}}$. These bounds are numerically calculated and given in section V.

### *D. Comparison between the schemes $E$ and $S$*

**We first consider the case of CSI-L and $\beta \le 1$. Substituting $\theta$ to $\mu$ in (19) and comparing (6) and (19), we see that $d^S_L = d^E_L$ when $r_U \le \beta(\mu - \lambda)$. The case $r_U \le \beta(\mu - \lambda)$ is divided into two cases. In the first case, $d^E_{U_2} \le d^E_{U_1}$, which is equivalent to $2\theta - 1 \le \lambda$, $d^E_{\mathrm{L}} = \min\{d^E_{U_2}, d^E_V\}$. Since $d^S_{U_2} \ge d^E_{U_2}$, by substitution and further manipulation, we get $d^S_{\mathrm{L}} > \min\{d^E_{U_2}, d^S_V\} = \min\{d^E_{U_2}, d^E_V\} = d^E_{\mathrm{L}}$ and therefore $d^S_{\mathrm{L}} \ge d^E_{\mathrm{L}}$. In the second case, $d^E_{\mathrm{L}} = \min\{d^E_{U_1}, d^E_V\}$ and we have $d^S_{\mathrm{L}} > \min\{d^E_{U_1}, d^S_V\} = d^E_{\mathrm{L}}$. The case with CSI-L and $\beta > 1$ is treated similarly and we also have $d^S_{\mathrm{L}} \ge d^E_{\mathrm{L}}$. With CSI-A, we can easily compare (11) and (27) see that $d^S_A \ge d^E_A$.**

**Regarding the maximum multiplexing gain (MMG), we see that a given scheme achieves the same MMG for both CSI models. Therefore, the knowledge of all the channels does not improve the MMG. To see this, denote $r^E_{i,\mathrm{Lo}}$, $r^S_{i,\mathrm{Lo}}$, $r^E_{i,\mathrm{Ao}}$ and $r^S_{i,\mathrm{Ao}}$, $i \in \{U, V\}$, as the corresponding MMGs of scheme $E$ and $S$ in CSI-L and CSI-A models, respectively, of the $i-$th user. The MMGs are the smallest roots $r_U$ and $r_V$ of the equations $d(r_U, r_V) = 0$ where $d(r_U, r_V)$ is the corresponding DMT function. The MMG pair including the MMGs of two users U and V is a curve represented by a certain equation $\mathrm{f}(r_U, r_V) = 0$. With the scheme $E$ in CSI-L, the MMGs satisfy $r^E_{U,\mathrm{Lo}} = 1 - \theta_o(r^E_{U,\mathrm{Lo}}, r^E_{V,\mathrm{Lo}})$ where $\theta_o$ is given in Proposition 4.2. The equation gives $r^E_{V,\mathrm{Lo}} = 1-2r^E_{U,\mathrm{Lo}}$ which is also simmilar to the equation for MMGs of scheme $E$ in CSI-A $r^E_{V,\mathrm{Ao}} = 1-2r^E_{U,\mathrm{Ao}}$. For the scheme $S$ in CSI-L, the MMGs satisfy $r^S_{U,\mathrm{Lo}} = 1 - \mu_j(r^S_{U,\mathrm{Lo}}, r^S_{V,\mathrm{Lo}})$, $j \in \{a, b, ..., f\}$ if $\beta \leq 1$ and $r^S_{U,\mathrm{Lo}} = 1 - \lambda_j(r^S_{U,\mathrm{Lo}}, r^S_{V,\mathrm{Lo}})$ and if $\beta > 1$, where $\mu_j$ and $\lambda_j$ are given in Proposition 4.5. The equations give**

$$r^S_{V,\mathrm{Ao}} = \begin{cases} \frac{1-2r_U}{\beta} & \text{if } \beta \leq 1 \\ 1 - \left(1 + \frac{1}{\beta}\right) r_U & \text{if } \beta > 1. \end{cases} \tag{29}$$

**which are also the equations for MMGs of scheme $S$ in CSI-A. Clearly, the MMGs of scheme $S$ are larger than MMGs of scheme $E$.**

## V. Numerical Results

The following numerical results show the DMT functions of scheme $S$ and $E$ in the two models CSI-L and CSI-A. In the CSI-L model, $d^E_L$ and $d^S_L$ are functions of $(\lambda, \theta)$ and $(\lambda, \mu)$ respectively, $d^E_{Lo}$ and $d^S_{Lo}$ are the optimal functions when optimized with respect to $(\lambda, \theta)$ and $(\lambda, \mu)$ respectively. In the CSI-A model, the lower bounds of the DMT functions of schemes $E$ and $S$ are numerically calculated with large enough $\rho = \rho_o$. In both cases of $\beta \leq 1$ and $\beta > 1$, the lower bounds are quite close to their corresponding upper bounds.

In Fig. 3, the DMT functions of scheme $S$ and $E$ in CSI-L and CSI-A are shown for the case with $r_V = \frac{3}{2}r_U$ and $\beta = 0.7$. In case of $\beta > 1$, by a similar variable substitution, we can see a similar comparison between $d^S_L$ and $d^E_L$. For CSI-A, the results with $\beta = 2.3$ and $r_V = \frac{3}{4}r_U$ are

shown in Fig. 4. From (6), (11), (19), (20) and (27), we observe that all schemes have the same maximum diversity gain of $\min(1, \beta)$, regardless of the CSI model.

The reference scheme gives the same MMG for both CSI-L and CSI-A channel models. The CDR scheme also has the same MMG for both CSI models but with a different value with the reference scheme. Therefore, at higher multiplexing gains, knowing all channels does not help significantly while applying the CDR scheme gives a significant improvement as shown in Fig. 3 and 4. At lower multiplexing gains, knowing all channels brings a large gain while applying the CDR scheme does not give a significant improvement.

We consider now the influence of parameter $\beta$ on the diversity gain. As we can see with CSI-A, in (11) and (27), when $\beta$ tends to 1, scheme $S$ loses its superiority to scheme $E$ because the interference from user V to user U is as strong as the desired signal thus both NIC or IC options consisting in treating interference as noise or decoding and cancelling it are not efficient. This also holds for CSI-L. The difference between CSI-L and CSI-A achieves its maximum at $\beta = 1$. Thus knowing all channels benefits the most when $\beta = 1$. These are reflected in Fig. 5 with a multiplexing gain $r$ fixed at 0.2.

The optimized DMT function of scheme $E$ in CSI-L is given by a single expression in Proposition 4.2 while the optimized DMT function of scheme $S$ in CSI-L is given by many expressions for cases of $\beta$ Proposition 4.5. Therefore as shown in Fig. 6 where the optimal $\mu$ and $\lambda$ of scheme $S$ and $E$ in CSI-L are shown also with fixed $r = 0.2$ and varied $\beta$, there are several corresponding discontinuities in case of scheme $S$.

Regarding the limit of the diversity gain when $\beta$ tends to infinity, as shown in Fig. 5, increasing $\beta$ over 1 does not give any improvement for scheme $E$ in CSI-A. The reason is that its diversity gain reaches and not affected by $\beta$ as seen in (11). On the other hand for scheme $S$ in CSI-A, the limit is higher at $\lim_{\beta\to\infty} d_A^{S_{UB}} = \left(\frac{1-r_U-r_V}{1-r_V}\right)^+ = 0.75$. For scheme $E$ in CSI-L, a very high $\beta$ means a very high $d_V^E = \beta(1 - \frac{r_V}{\lambda})^+$ in (6). The diversity gain of the scheme $d_L^E = \min\left\{d_{U_1}^E, d_{U_2}^E, d_V^E\right\}$ is hence limited by $d_{U_1}^E$ and $d_{U_2}^E$ which corresponds to transmission duration ratios $1-\theta$ and $\theta-\lambda$. Therefore $\lambda$ should be reduced as much as possible to make $1-\theta$ and $\theta-\lambda$ higher. However, $\lambda$ must be kept larger than or equal to $r_V = r$ to make $\left(1 - \frac{r_V}{\lambda}\right)$ not

negative. Hence the optimal value is $\lambda_o = r_V = 0.2$. The optimal $\theta$ can be calculated based on equation $d^E_{U_1} = d^E_{U_2}$ which is equivalent to $1-\mu = \mu - \lambda$ from which we obtain $\mu_o = \frac{1+r_V}{2} = 0.6$ therefore $\lim_{\beta\to\infty} d^E_L = 1 - \frac{0.2}{1-0.4} = 0.5$ as seen in the figure. The case with scheme $S$ in CSI-L is dealt similarly. The limit of diversity gain of scheme $S$ is higher than that of scheme $E$.

## VI. Conclusion

In this paper, we have considered a network with a base station, a relay station, a relayed user and a direct user in two models with different channel knowledge levels. The Diversity-Multiplexing Trade-off (DMT) functions of the Coordinated Direct and Relay (CDR) scheme and of the corresponding conventional scheme are either analytically calculated or bounded. The transmission durations in both schemes are optimized accordingly. The results reveal that at low diversity gains, knowledge of all channels at a station does not improve the DMT function. Nevertheless, upgrading the conventional scheme to the CDR scheme brings improvement in terms of the multiplexing gain. Conversely, at high diversity gains, upgrading does not help while knowing all channels can improve the DMT function.

## Appendix

## Proof of Proposition 4.6

We consider the two cases $\beta \leq 1$ and $\beta \geq 1$.

### A. $\beta \leq 1$

Similarly to the CSI-L case, we consider $\mu \geq \lambda$. From the expressions for $R^S_{U_1}$, $R^S_{U_2}$ and $R^S_V$, we derive the condition for no outage as follows

$$\begin{cases} \min\left\{(1-\mu)\log(1+\rho\gamma_1), (\mu-\lambda)\log(1+\rho\gamma_2) + \lambda\left(1+\frac{\rho\gamma_2}{\rho^\beta\gamma_4+1}\right)\right\} & \geq \quad r_U \log\rho \\ \lambda\log(1+\rho^\beta\gamma_3) & \geq \quad r_V\log\rho^\beta. \end{cases} \tag{30}$$

It is equivalent to

$$\mu \leq 1 - a_1, \quad \lambda \geq a_3 \quad \text{and} \quad \mu \geq \max(\lambda(1-a_4)+a_2, \lambda) \tag{31}$$

with $a_1$, $a_2$ and $a_3$ as denoted in section IV-B2 and $a_4 = \frac{\log\left(1+\frac{\rho\gamma_2}{\rho^\beta\gamma_4+1}\right)}{\log(1+\rho\gamma_2)}$. The no-outage area is demonstrated in Fig. 7a. Using the method in section IV-B2, we can derive that (31) never occurs if the point $(a_3, 1-a_1)$ lies below one of the lines $\mu = \lambda$ and $\mu = \lambda(1-a_4)+a_2$ or $1-a_1 < b_1 = \max(a_2 + a_3(1-a_4), a_3)$. In this case, we cannot find any $(\lambda, \mu)$ to accommodate the target rate pair. The condition is equivalent to $\mathcal{O}_{\mathrm{A}}^{S} = \mathcal{O}_{\mathrm{A}}^{S_1} \cup \mathcal{O}_{\mathrm{A}}^{S_2}$ where $\mathcal{O}_{\mathrm{A}}^{S_1} \triangleq \{1-a_1 < a_3\}$ and $\mathcal{O}_{\mathrm{A}}^{S_2} \triangleq \{1-a_1 < a_2 + a_3(1-a_4)\}$. Similarly to scheme $E$ in CSI-A, we have $\Pr\left[\mathcal{O}_{\mathrm{A}}^{S_1}\right] \dot{\geq} \rho^{-\beta\left(\frac{1-r_U-r_V}{1-r_U}\right)^+} \triangleq d_{\mathrm{A}}^{S_1}$. On the other hand, because $\lim_{\rho\to\infty} a_4 = 1-\beta$, we have

$$\begin{aligned}
&\Pr\left[\mathcal{O}_{\mathrm{A}}^{S_2}\right] = \Pr\left[\tfrac{r_U}{1-\alpha_1} + \tfrac{r_U}{1-\alpha_2} + \tfrac{\beta r_V}{1-\frac{\alpha_3}{\beta}} > 1\right] \\
&= \Pr\left[\beta\alpha_1 + \beta\alpha_2 + \alpha_3 - \beta\alpha_1(r_U+\beta r_V) - \beta\alpha_2(r_U+\beta r_V) - 2\alpha_3 r_U > \beta - 2\beta r_U - \beta^2 r_V\right] \\
&\dot{\geq} \Pr\left[\alpha_3 - 2\alpha_3 r_U > \beta - 2\beta r_U - \beta^2 r_V\right] \\
&= \Pr\left[\alpha_3 > \beta\tfrac{1-2r_U-\beta r_V}{1-2r_U}\right] \doteq \rho^{-\beta\left(\frac{(1-2r_U-\beta r_V)}{1-2r_U}\right)^+}.
\end{aligned} \tag{32}$$

$$\Pr\left[\mathcal{O}_{\mathrm{A}}^{S}\right] \doteq \Pr\left[\mathcal{O}_{\mathrm{A}}^{S_1}\right] + \Pr\left[\mathcal{O}_{\mathrm{A}}^{S_2}\right] \doteq \max\left\{\rho^{-\beta\left(\frac{1-r_U-r_V}{1-r_U}\right)^+}, \rho^{-\beta\left(\frac{(1-2r_U-\beta r_V)}{1-2r_U}\right)^+}\right\} = \rho^{-\beta\left(\frac{(1-2r_U-\beta r_V)}{1-2r_U}\right)^+}. \tag{33}$$

The DMT function is therefore given in (27).

### *B.* $\beta > 1$

We consider $\mu < \lambda$. The condition for no outage is

$$\begin{cases} \min\left\{(1-\lambda)\log(1+\rho\gamma_1), \mu\log(1+\rho^\beta\gamma_2)\right\} & \geq \quad r_U \log\rho \\ \min\left\{(\lambda-\mu)\log(1+\rho^\beta\gamma_4) + \mu\left(1+\frac{\rho^\beta\gamma_4}{\rho\gamma_2+1}\right), \lambda\log(1+\rho^\beta\gamma_3)\right\} & \geq \quad r_V \log\rho^\beta. \end{cases} \tag{34}$$

It is equivalent to

$$\lambda \leq 1-a_1, \quad \mu \geq a_2, \quad \lambda \geq \max(\mu(1-a_5)+a_6, \mu) \quad \text{and} \quad \lambda \geq a_3 \tag{35}$$

with $a_1$, $a_2$, $a_3$ as denoted in section IV-B2, $a_5 = \frac{\log\left(1+\frac{\rho^\beta\gamma_4}{\rho\gamma_2+1}\right)}{\log(1+\rho^\beta\gamma_4)}$ and $a_6 = \frac{\log\left(1+\rho^\beta\right)}{\log(1+\rho^\beta\gamma_4)}$. The no-outage area is demonstrated in Fig. 7b. Using the method in section IV-B2, we can see that (35) never occurs if (1) the point $(a_3, 1-a_1)$ lies above one of the lines $\lambda = \mu$ and $\lambda = \mu(1-a_5)+a_6$ or (2) $a_3 < 1-a_1$. The condition is equivalent to $1-a_1 < \max(a_2, a_6 + a_2(1-a_5), a_3)$.

The outage condition is denoted as $\mathcal{O}_{\mathrm{A}}^{S} = \mathcal{O}_{\mathrm{A}}^{S_1} \cup \mathcal{O}_{\mathrm{A}}^{S_2} \cup \mathcal{O}_{\mathrm{A}}^{S_3}$ where $\mathcal{O}_{\mathrm{A}}^{S_1} \triangleq [1 - a_1 < a_2]$, $\mathcal{O}_{\mathrm{A}}^{S_2} \triangleq [1 - a_1 < a_6 + a_2(1 - a_5)]$ and $\mathcal{O}_{\mathrm{A}}^{S_3} \triangleq [1 - a_1 < a_3]$ in which $\Pr\left[\mathcal{O}_{\mathrm{A}}^{S_1}\right] \doteq \rho^{-\left(\frac{1-2r_U}{1-r_U}\right)^+}$ and $\Pr\left[\mathcal{O}_{\mathrm{A}}^{S_3}\right] \doteq \rho^{-\beta\left(\frac{1-r_U-r_V}{1-r_U}\right)^+}$. On the other hand, because $\lim_{\rho\to\infty} a_5 = 1 - \frac{1}{\beta}$, we have

$$\begin{aligned}
&\Pr\left[\mathcal{O}_{\mathrm{A}}^{S_2}\right] = \Pr\left[\tfrac{r_U}{1-\alpha_1} + \tfrac{r_V}{1-\frac{\alpha_4}{\beta}} + \tfrac{r_U}{\beta(1-\alpha_2)} > 1\right] \\
&\doteq \Pr\left[\alpha_1(\beta - \beta r_V - r_U) + \alpha_2(\beta - \beta r_U - \beta r_V) + \alpha_4(1 - r_U - \tfrac{r_U}{\beta}) > \beta - \beta r_U - \beta r_V - r_U\right] \\
&\dot{\geq} \Pr\left[\alpha_1(\beta - \beta r_V - r_U) > \beta - \beta r_U - \beta r_V - r_U\right] \\
&= \Pr\left[\alpha_1 > \tfrac{\beta-(\beta+1)r_U-\beta r_V}{\beta-r_U-\beta r_V}\right] \dot{\geq} \rho^{-\left(\frac{1-\left(1+\frac{1}{\beta}\right)r_U-r_V}{1-\frac{r_U}{\beta}-r_V}\right)^+}.
\end{aligned} \tag{36}$$

Hence

$$\begin{aligned}
&\Pr\left[\mathcal{O}_{\mathrm{A}}^{S}\right] \doteq \Pr\left[\mathcal{O}_{\mathrm{A}}^{S_1}\right] + \Pr\left[\mathcal{O}_{\mathrm{A}}^{S_2}\right] + \Pr\left[\mathcal{O}_{\mathrm{A}}^{S_3}\right] \\
&\doteq \max\left\{\rho^{-\left(\frac{1-2r_U}{1-r_U}\right)^+}, \rho^{-\beta\left(\frac{1-r_U-r_V}{1-r_U}\right)^+}, \rho^{-\left(\frac{1-\left(1+\frac{1}{\beta}\right)r_U-r_V}{1-\frac{r_U}{\beta}-r_V}\right)^+}\right\} = \rho^{-\left(\frac{1-\left(1+\frac{1}{\beta}\right)r_U-r_V}{1-\frac{r_U}{\beta}-r_V}\right)^+}.
\end{aligned} \tag{37}$$

The upper bound of the DMT function in this case is therefore given in (27).

## ACKNOWLEDGMENT

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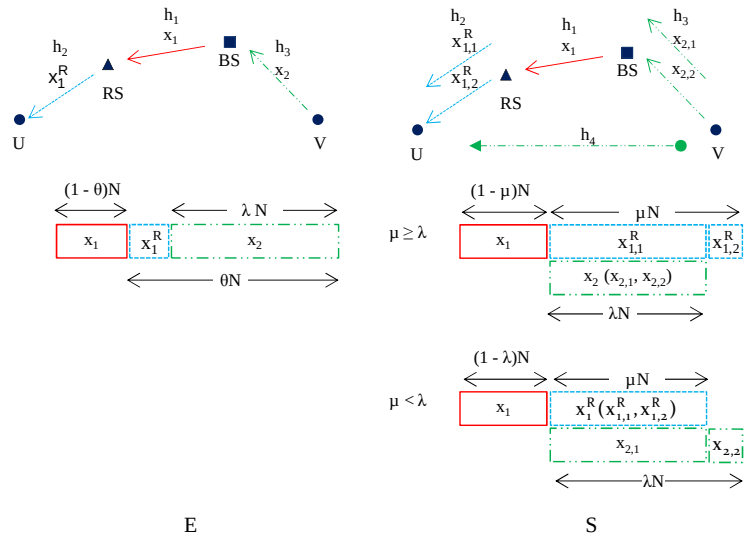


Fig. 1. Reference $E$ and $S$ CDR Schemes. In each scheme, the time interval of the transmission represented by an arrow above is represented by a rectangular below with the same color and the transmissions represented by the rectangles in the same column are conducted simultaneously.

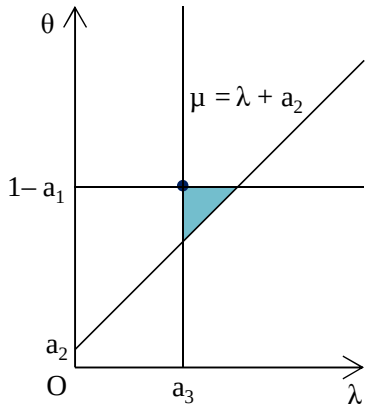


Fig. 2. Only in the shaded triangle in the middle, the scheme is not in an outage ($\lambda \geq a_3$, $\theta \leq 1 - a_1$ and $\theta \geq \lambda + a_2$).

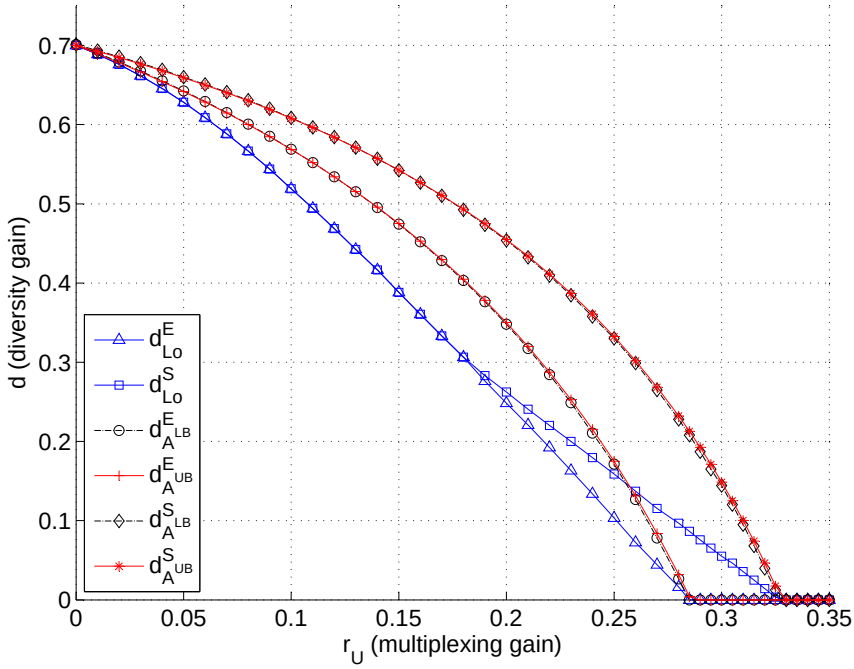


Fig. 3. The optimal DMT functions of $E$ and $S$ in CSI-L, $d_{\mathrm{Lo}}(r)$, and the lower and upper bounds of the DMT functions of $E$ and $S$ in CSI-A with $r_V = \frac{3}{2} r_U$ and $\beta = 0.7$.

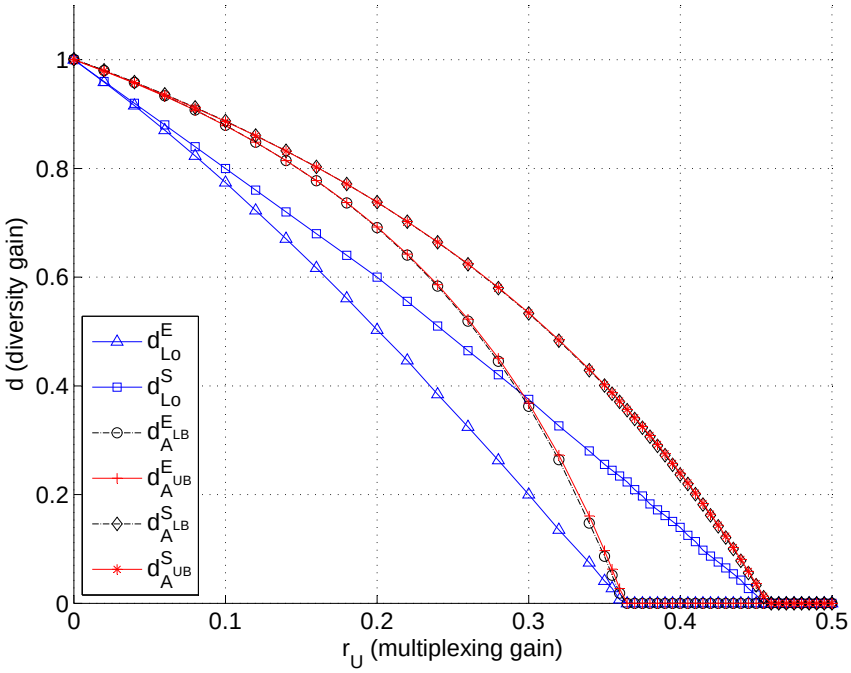


Fig. 4. The optimal DMT functions of $E$ and $S$ in CSI-L, $d_{\mathrm{Lo}}(r)$, and the lower and upper bounds of the DMT functions of $E$ and $S$ in CSI-A with $r_V = \frac{3}{4} r_U$ and $\beta = 2.3$.

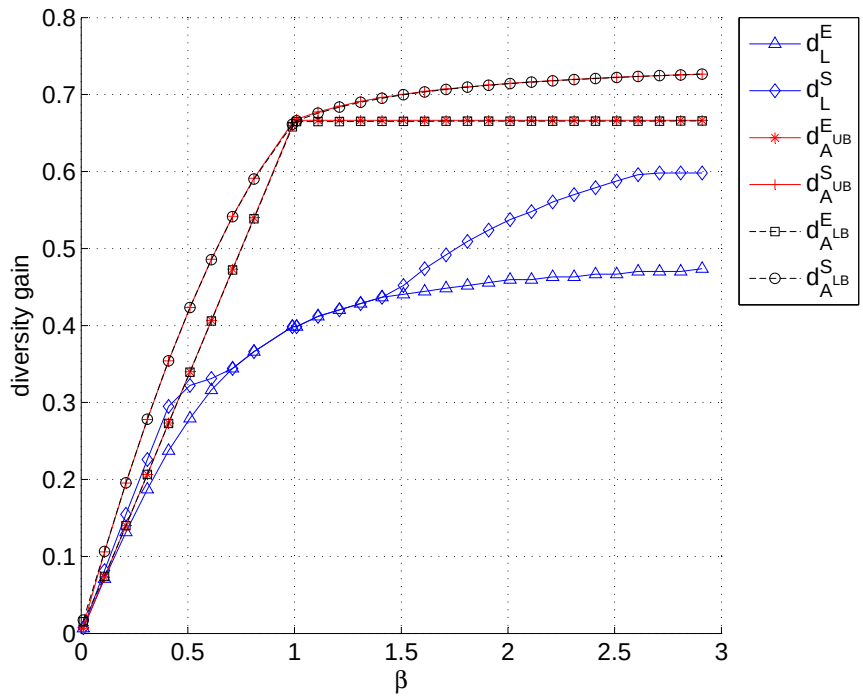


Fig. 5. The optimal DMT functions in CSI-L, $d_{\mathrm{Lo}}(r)$, of $E$ and $S$ and the lower and upper bounds of the DMT functions of $E$ and $S$ in CSI-A with fixed $r_U = r_V = 0.2$.

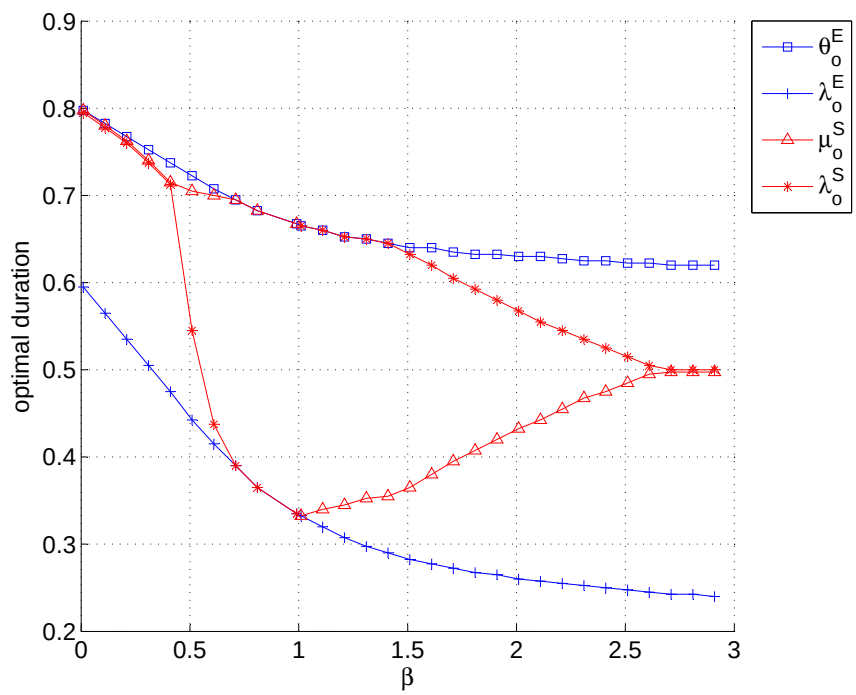


Fig. 6. The optimized tranmissions durations of scheme $E$ ($\theta$ and $\lambda$) and scheme $S$ ($\mu$ and $\lambda$) in CSI-L with fixed $r_U = r_V = 0.2$.

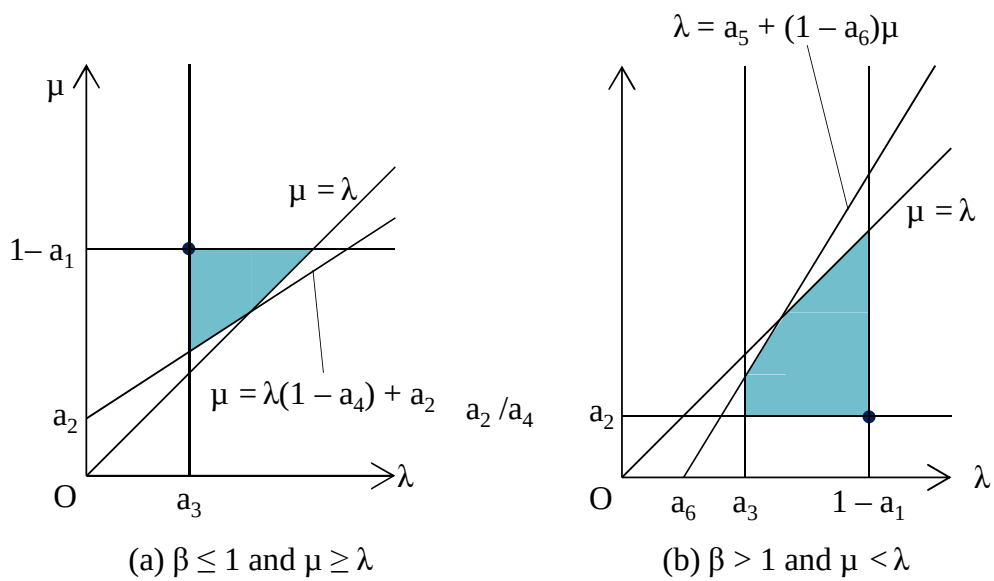


Fig. 7. Non-outage areas for $\mu \geq \lambda$ and $\mu < \lambda$ in CDR $S$.