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Political Equilibrium, Citizen-Candidates and Expressive Preferences

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Abstract: Recently a number of papers have considered the citizen-candidate model originally developed by Osborne and Slivinski (1996). We use a variant of the citizen-candidate model to show the implications of expressive preferences in an environment in which voters vote sincerely and policy is a combination of candidates’ announced policies. Using this set up we are able to show that for given costs of candidacy the number of candidates in equilibrium can differ quite sharply.

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1. Introduction

The standard spatial equilibrium model of voting is one of the finest examples of the dispersion of economic methods into essentially non-economic research issues. The standard Downsian model starts out by assuming that agents use fairly stereotyped economic reasoning to reach decisions in "the political market". The model simply postulates that agents' behaviour in the political arena closely resemble their behaviour in traditional markets. Although extremely popular in formal analysis of equilibrium in systems with democratic institutions this conceptualisation of individual behaviour in politics has been questioned ever since the model first appeared. The critique of behavioural postulates in the model has been concerned both with voter behaviour and with the instrumental aims of political candidates. While authors such as Hibbs (1977), Calvert (1985), Alesina (1988) and, more recently, Schultz (1996), Roemer (1996) and Besley and Coate (1997) have questioned the Downsian view that parties act as vote-maximisers, others have questioned the behaviour of voters in the traditional model. Voting is naive in the Downsian model. At least two alterations of this assumption have been introduced subsequently; one which bring standard economic reasoning to the limit and one which rely on limits of economic reasoning in the study of mass elections. The "mainstream" way of changing voter behaviour seems to have been to introduce strategic voting. Although this development in the literature has produced some very impressive voting equilibria, eg. see Austen-Smith and Banks (1988) or Besley and Coate (1997), it seems to be a rather problematic road to travel. First of all, it seems to be the case that n-person strategical games are so complex that simplifying assumptions which more or less off-set the strategic behaviour has been undertaken in order to obtain tractability in the models. Second, and far more important, strategic voting seems to fit very badly with the incentive structure that is naturally imposed on the voting decision. Downs (1957) first mention the very small probability that any single voter will have any effect on the result of a mass election by voting. In the early literature there seems to be very little effort to incorporate the insight explicitly into the analysis by allowing agents to take their incentives to abstain from voting into consideration as an explicit part of the voting process. Instead, most scholars in the political economy tradition mention the problem in their introductions; Dennis Mueller (1989) even devote an entire chapter to the subject in his text-book on Public Choice Theory. Recently Brennan and Lomasky (1983) and Brennan and Hamlin (1998) have developed an alternative framework for voter behaviour which they term expressive voting. In their work,
voting is seen as expressive in the sense that citizens will need to be able to identify themselves with a political party in order to participate in the election as voters.

In the present study we establish yet another way of conceptualising individual behaviour in the political market. Although very different from the conceptualisation of Brennan et al, but in lack of a better name, we shall term the behavioural postulates in the present study "expressive" as well. We do not restrict ourselves to the behaviour of voters here; instead we impose "expressive" behaviour on all agents. This is done by using a variant of the citizen-candidate model developed independently by Osborne and Slivinski (1996) and Besley and Coate (1997). This set up, in which the number and location of political candidates is determined endogenously, seems to go well with the expressive behavioural postulate presented here. In addition, endogenous candidate determination seems far more plausible as a basic assumption than the standard two-party assumption. According to the set up in the citizen-candidate model, a society is defined by a continuum of citizens each identified by their preferences over the political issue space. Political equilibrium is the outcome of a three stage political process, where citizens choose whether to become candidates in the first stage, an election is held in the second stage and a policy is implemented in the third and final stage.

The particular model we use in the present study is a variant of the model presented in Hamlin and Hjortlund (1998) which itself is inspired by the work of Osborne and Slivinski (1996) and Besley and Coate (1997). The model presented here differs from all three models in several aspects. First of all, individual preferences have an expressive term. Individuals care both about policy and about the explicit, candidate based expression of policies. Second, while both Osborne and Slivinski and Besley and Coate use the familiar plurality rule as the basis for policy implementation, the policy implementation mechanism used in the present study is a variant of proportional representation, in which the implemented policy is a vote-weighed average of candidates' announced policy.

The proportional representation mechanism used here has been proposed earlier by Ortuno-Ortin (1997) in a traditional two-party model. In that study two ideological motivated parties act utility maximising in a single issue environment where the policy implementation mechanism and the
distribution of voters’ ideal points are known with certainty by both parties. Ortuno-Ortin shows that under these assumptions the convergence result of the original downian model disappears. Although the ex post implemented policy from the model resembles the same features as the convergence model, candidates’ platforms will not converge in equilibrium if candidates have different ideal policies. Unfortunately, proportional representation as modelled by Ortuno-Ortin (1997) leads to a situation where equilibrium positions of candidates changes more dramatically with changes in the distribution of voters ideal points than it is the case with traditional, plurality rule driven models. This feature of proportional representation remains the case in the present study. Thus, we have restricted ourselves to look at a uniform distribution of voters ideal points. Hamlin and Hjortlund (1998) offer a discussion of the changes induced by altering the distribution of voters ideal points in a model with proportional representation and Hjortlund (1998) contains an explicit analysis of non-uniform voter distributions under proportional representation. Before turning to the actual model, we briefly present the key-features separating this model from the traditional spatial equilibrium model.

1.1. Preferences
First, we postulate that agents have an expressive element in their utility functions. Brennan and Lomasky (1983) argue why this should be so in the case of voting. The basis of their argument stems from “the prediction of rational abstention.”1 Traditional economic reasoning cannot explain why people bother to vote in large electorate elections if there are positive costs associated with voting. The traditional model simply assume this insight away by postulating that all voters vote. Brennan and Hamlin (1998) on the other hand insist that the decision on whether to vote or not simply cannot be treated separately from the decision on whom to vote for. By holding this view, Brennan and Hamlin, building on Ledyard (1984), show that equilibrium with traditional instrumental voting must have a zero turnout. Consequently, there must be something else to voting. In their study, expressive voting is seen as one possible way of constructing this “something else”. In the expressive account voting is conceptualised as a way to express support for a given electoral option. Thus if for any expressively voting agent, no electoral option exist with which the voter identify herself she does not wish to express her support for any electoral

1Brennan and Lomasky (1983) and Mueller (1989) discuss the content of the prediction of rational abstention in detail.
option and abstains from voting. In the present study we change the expressive account somewhat. Voters vote sincerely for their preferred candidate, but their benefit from doing so depend on the distance between their ideal point and the announced policy of their preferred candidate in the set of candidates. Thus given any arbitrary set of candidates, the citizen who is located at the greatest distance from any candidate in the candidate set will have the strongest expressive incentive to become a political candidate herself. It can be argued that the expressive behaviour story, because of its relation to the paradox of rational abstention, does not generalise to candidates, but we hold the contrary here; i.e. we believe that expressive concerns exist even when instrumental incentives are important. In that way this model constitutes a far more clean break with traditional economic reasoning than the original expressive voting story because expressive incentives exist regardless of the presence of instrumental incentives. Citizens simply have a taste for expressing their views publicly as candidates. The expressive candidate hypothesis also seems to fit well with the observation that in most democracies there exist political candidates who continue to run for office even though they cannot have any realistic hope of affecting the outcome of the election. The traditional economic explanation of these observations has to go through strategical behaviour but, at least in multiparty systems, there seems to be little reason for strategic candidacy. Finally, if voters are inclined to vote not only instrumentally but expressively and if expressive concerns are different from instrumental concerns, there might be instrumental gains from simply expressing a policy. If for example expressive motivations include the individuals conception of candidates’ moral attitude, shear management competence etc, continuing candidacy through several elections might be rational for some candidates. Although our model does not take this argumentation explicitly into account, the fact that the present model uses a proportional representation mechanism to implement policies indeed do strengthen this argument.

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2Some papers, including Osborne and Slivinski (1996), find that some citizens might want to enter as a candidate even when this citizen does not have a positive probability of winning. Candidacy from such a citizen requires that candidacy from the citizen helps another, and from the strategic citizens perspective, preferred candidate into winning.

3The model presented in this paper is static. To capture the instrumental argument for expressive candidate behaviour one will have to employ a dynamic model in which voters can alter their decisions in the next period on the basis of the information gained in the present period. In short, the distribution of voters location in the political issues spaces will have to be changeable over time. In macro economic models such as Persson and Tabellini (1991) or Alesina and Rosenthal (1995) such a set up seems to be introductable simply by “renaming” some of the features of the models.
1.2. Voting rules

Plurality voting is by far the most popular voting rule in economic models of political equilibrium. There seems to be several reasons for this. First of all the plurality rule is very simple. Policy is equal to the announced policy of the candidate who obtains the highest vote share. It hardly gets any simpler than that. Second, one might suspect that the two-party or “near two party” systems in dominating western democracies such as the United States and United Kingdom have introduced some bias towards accepting the plurality rule as the proper voting mechanism in theoretical literature on political equilibrium. However attracting as an analytical foundation for equilibrium in theoretical models, the relation between plurality rule as a voting rule and plurality rule as a policy implementation mechanism seems to be rather unrealistic. Is it indeed the case that the plurality winner of real world elections have complete discretionary power to implement policy? Even in mainstream economic analysis, there seems to growing concern about whether this is the case or not. Although explicitly stating that his model is intended to describe the outcome in parliamentary systems, Ortuno-Ortins (1997) proportional representation rule seems to indicate otherwise. Likewise, recent work by Alesina and Rosenthal (1995,1996), Austen-Smith and Banks (1988), Persson, Roland and Tabellini (1997), Kollman, Miller and Page (1997), Hamlin and Hjortlund (1998) and Hjortlund and Zambelli (1998) all use policy implementation mechanisms which are different from the crude winner take all mechanism. In many ways it seems to be a realistic assumption that all elected candidates do have some influence over policy, at least by restraining the plurality winners possibility to do what ever she wants. Clearly there can be only one American president at any given time, but that does not mean that the power of the president is dictatorial in nature. Before leaving the discussion, we should mention that the alternative used in the present paper can be seen as a natural benchmark case, which in some sense can be seen as the opposite policy implementation.

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*It should be mentioned here that the downsian way of using plurality rule seems far more attractive because candidates only care about the number of votes they receive. The winner is simply the party who gets more votes than any other party. Thus “winning” can be interpreted to be just that and there is no need to mention policy at all. Hjortlund (1998b) offers a discussion on the distinction between voting rules and policy implementation mechanisms.*

*Dunleavy (1997) split parliaments up into three different categories according to their possibility to influence policy. According to his conceptualisation parliaments can either have power to build legislation, to influence legislation or no legislative power at all. According to Dunleavy most parliamentary systems belong to the second category, thus implying that parliaments should have, at least, a restraining effect on government behaviour.*
mechanism of plurality voting in which "the winner take all", cfr. Ortuno-Ortin (1997). We do not postulate that the pure proportional representation rule where policy is a vote-weighed average of candidates announcements on a single dimensional issue space is more realistic than the plurality rule. Rather it should be seen one possible implementation mechanism which of course can be improved by further research into the relations between candidates' platforms, voting outcomes and implemented policies.

1.3. Citizen-candidates

The present model also differs from most work by building on the notion of citizen-candidates. The citizen-candidate model has been developed independently by Osborne and Slivinski (1996) and Besley and Coate (1997). Hamlin and Hjortlund (1998) offers an extension of the citizen-candidate model building on proportional representation and the model in the present paper is a variant of that particular model. The fundamental difference between citizen-candidate models and traditional spatial equilibrium models is that while candidacy is described ad hoc to the system in the latter, it is determined inside the model in the former. While this feature makes the citizen-candidate model more universal in some respects, it introduces difficulties too. First of all, there seems to be few barriers on the entry decision when the model is compared to real world politics. This makes the model partial in the questions it is able to address. For example we are not able to describe any dynamics in this type of model and we cannot begin to say anything about how or why political parties, rather than political candidates are dominating in real-world politics. One might indeed suggest that the citizen-candidate model is far more capable of describing democratic systems where individuals, not organisations, run against each other. Such set ups can be found in local clubs, in companies' choice of a board, or in political parties' choice of policy platforms.

1.4. Environment and main results

The model we use can be described like this. Politics is unidimensional and there is a continuum of citizens characterised by their preferences over political outcomes who democratically have to choose a unique policy from the set of attainable policies. To do this citizens engage in a three stage game in which they decide whether to become candidates in the first stage, they vote in the second stage and a policy is implemented in the third stage. Candidacy is based on strategic
considerations but voting is naive. The policy implementation mechanism and the distribution of voters ideal points are common knowledge, ie. there is no uncertainty in the model. Preferences are such that agents in addition to the instrumental benefit depending on the outcome of the political game receive positive benefits from expressing their views as candidates.

The main findings that we are able to present following the described model are:

- One-candidate equilibria exist under fairly the same conditions regardless of whether there is an expressive element in voters utility functions or not.
- Two candidate equilibria exists inside a range of costs of candidacy, ie there exist a unique, positive cost of candidacy, \( c^* \), below which two-candidate equilibria cannot exist. This result stands in contrast to the main results presented by Hamlin and Hjortlund (1998).
- For given and appropriate low costs of candidacy the number of candidates in equilibrium is given by an interval, ie. for given costs we cannot predict an exact equilibrium number of candidates.
- The chosen utility functions has real implications for equilibrium when the policy implementation mechanism is of the winner take all type as well as when it is based on proportional representation. This makes it somewhat more difficult to discern the effects of the chosen voting rule relative to agents' utility functions. We are able to present some points, though, that underlines the inherent differences between the two opposite policy implementation rules.

The paper is structured as follows. In section two we set up the model and in section three the basic results are presented. In Section four we briefly present equilibrium conditions when the policy implementation mechanism is of the winner take all type and discuss the differences between the two mechanisms. Section five concludes. All proofs and examples can be found in appendix.

2. The model

We use a variant of the model presented in Hamlin and Hjortlund (1998). The political issue space is given by the segment of the real line \([0,1]\). Citizens are characterised by their ideal policy
and citizens' ideal policies are distributed uniformly across the political issue space.

Agents' utility depend on the policy implemented after each election. In addition, the distance between any agents' ideal point and the ideal point of the closest candidate is negatively related to agents' well-being. Thus if the i'th citizen chooses not to become candidate she receives a benefit, \( B_i \), as stated in (1).

\[
B_i = -|P - x_i| - \delta \{ \min \{ y_j - x_i \} \},
\]

where \( P \) is the policy resulting from the election and there are \( m \) candidates indexed by \( j=1,2,...,m \). Furthermore, \( \delta \) is a positive parameter indicating the valuation of expressive motivations relative to policy motivations. Note that we take \( \delta \) to be equal across citizens, ie. all citizens place the same weight on expressive benefits relative to instrumental benefits. With \( \delta=0 \) the model reduces to the model proposed by Hamlin and Hjortlund (1998).

Agents can choose to become candidates at a positive cost, \( c \). The pay-off to a candidate is equal to the negative of the distance between policy and the ideal point of the candidate plus the cost of candidacy, \( c \). The net-benefit from candidacy, then, is given by:

\[
|P_j - x_j| - |P - x_j| + \delta \{ \min \{ y_j - x_j \} \} - c
\]

where \( P_j \) and \( P \) denotes the policy without and with the j'th citizen as a candidate respectively and \( y_j \) is the candidate set less the j'th citizen-candidate, ie: \( \{ -j=1,2,(j-1),(j+1),m \} \). If no candidates run for office all citizens obtain a negative pay-off of size \(-z\).

Policy is determined as a vote-weighed average of candidates' announced policies. In our notation that is:

\[
P = \sum_{j} V_j x_j \quad j \in J
\]

where \( V_j \) is the voteshare obtained by the j'th candidate. This conceptualisation has been proposed by, among others, Ortuno-Ortit (1997) as a voting rule, but here we simply state it as
a outcome of political competition, regardless of the voting rule actually employed in the constitution. There are two connected assumptions at stake here. First we assume that the policy implementation mechanism, which mimic a legislative process by transforming an election result into policy, is perfectly proportional in nature. Second we assume that candidates are tied to defend their announced policies.  

The timing of events is as follows. First citizens decide whether to become candidates or not. We denote the decision to become a candidate E (enter) and the decision not to become a candidate N (Not enter). When a candidate set has formed an election is held in which all voters vote. Given the distribution of votes policy is determined according to (3) above. We are interested in Nash equilibrium in pure strategies. Thus, we define an equilibrium as a set of candidate decisions such that no citizen wishes to change her decision, given all other agents' decisions on candidacy and voting, and the policy following from these decisions:

**Definition one: Equilibrium**

Equilibrium is given by a strategy profile \( S, s_i \in \{E, N\} \) such that

(i) for all citizens playing \( E \), \( P_j - x_j / \P_j - x_j / + \delta \min \{ y_j - x_j \} - c \geq 0 \), and

(ii) for all citizens playing \( N \), \( P_i - x_i / \P_i - x_i / + \delta \min \{ y_j - x_i \} - c < 0 \).

3. Results

First of all, we reduce the notational complexity of the set-up by assuming that \( \delta = 1 \). Evaluated against common sense one might suggest a value of \( \delta \) which is somewhat lower than 1 in order to capture the salience of expressive gains relative to instrumental gains. The point here is that although changes in \( \delta \) does have an impact of the results obtained, the qualitative findings remain unaltered as long as \( \delta > 0 \).

Since the model is a variant of the model presented in Hamlin and Hjortlund (1998) we should expect the main findings to have some resemblance to the findings of that work. In many ways

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\( ^{6} \)If we do not have such an assumption time-inconsistency arises because once elected the ideal point of the j'th voter might not be the optimal policy to promote in the legislative process. Thus this model, unlike the citizen-candidate model with plurality rule inherits the time-inconsistency problem of the traditional two party model, see for example Alesina and Rosenthal (1995).
this is indeed the case. The conditions for one-candidate equilibria, for example, are quite trivial compared to those presented in Hamlin and Hjortlund. Consequently they can be found in appendix 1. Due to the uniform distribution of voters' ideal points we are able here to reproduce also the preliminary finding from Hamlin and Hjortlund (1998) that any middle of the road candidate, \( x_j \) will have no impact on the policy if more extreme candidates exist at both sides of \( x_j \):

**Lemma one:**

Let the distribution of citizens ideal points be uniform. Then, for any set of candidates in which the number of candidates is greater or equal to two, the implemented policy is equal to the policy that would have prevailed under a two-candidate game with the two extreme candidates in the candidate set running for office.

Lemma one can be found in Hamlin and Hjortlund (1998) and we have reported it here only because it helps us establish the conditions for equilibria with two or more candidates. Thus we are now able to establish the conditions for two-candidate equilibria.

**Proposition two**

Let a and b be the two candidates in a two-candidate equilibrium and assume that \( x_a < x_b \). Then (a,b) forms a two candidate equilibrium iff:

(i) \( c < \min \{ \left( x_b - x_a \right) + \frac{1}{2}(x_b^2 - x_a^2), \left[2(x_b - x_a) - \frac{1}{2}(x_b^2 - x_a^2)\right] \} \), AND

(ii) \( c > \max \{ \frac{1}{2}(x_b - x_a), [x_a + 1/2 x_b^2], [3/2 - 2x_a + 1/2 x_b^2] \} \)

Although in some ways similar to the two-candidate equilibria obtained by Hamlin and Hjortlund (1998) there seems to be crucial differences too. First of all, entry from a middle of the road candidate is possible, given that a and b are far enough apart from each-other. This is a simple consequence of the expressive term in the definition of citizens' loss functions. Second, for appropriate low c, no two-candidate equilibria exist. This results have some features in common with the results in Osborne and Slivinski (1996) where candidates are motivated by office rents and ideological concerns. Consider Figure 2 which depict possible pairs of candidate cost c and positions of \( x_b \) when candidate a is positioned at \( x_\ast \). For \( a = x_\ast \) to be part of a two-
candidate equilibrium we require that $x_c$ and $c$ obeys the conditions shown as a shaded area. The two sharpest rising curves originating at $x_a^*$ are the conditions securing that $a$ and $b$ will be willing to run against each other (condition (i) in proposition 2). Consequently costs of candidacy are bounded by these curves. The lower line originating from $x_a^*$ secures that $a$ and $b$ will deter entry from any citizen located between them. So for any pair of candidate locations $(x_a^*, x_b)$, $c$ must be higher than the indifference point given by the line. In the same way we can determine the minimum equilibrium values of $c$ in order to deter entry from any citizen located to the left of candidate $a$ or to the right of candidate $b$. Since the position of candidate $a$ is given, the cost necessary to avoid entry from an extremist citizen to the left of candidate $a$ is equal to $c^*$. Finally the downward-sloping curve originating at 1.5 is the condition which secures that no citizen to the right of $x_a$ want to enter, given existence of a candidate $b$, located at $x_b$. Since all conditions need to be fulfilled simultaneously, we end up with the shaded area. Note that for $x_a^* + x_b = 1$, the conditions on candidates incentives to enter is equal for both candidates. At the same point in the issue space, the conditions securing entry proofness from both extremes are also equivalent. This underlines the inherent symmetry in the model and we shall use this fact, when we study the model with plurality rule in the next section.

Figure two - Two-candidate equilibria under proportional representation
From figure 2 we also notice that the minimum cost of candidacy for which two-candidate equilibrium exists, differ for the positions of \( x_a \) and \( x_b \). The combination of \( x_a \) and \( x_b \), for which \( c \) is minimised in a two-candidate equilibrium is reported in Proposition 3.

**Proposition 3:** Minimum costs for which a two-candidate equilibrium exists.

Let \( a \) and \( b \) with ideal positions \( x_a < x_b \), be the two candidates running against each other. Then for a two-candidate equilibrium to exist we require that \( c > \frac{1}{2}(5^{1/2} - 2) \). The locations of \( a \) and \( b \) that minimise the necessary cost of candidacy are \( x_a = \frac{5^{1/2} - 2}{m-1} \) and \( x_b = 1 - \frac{5^{1/2} - 2}{m-1} \).

With the range of cost for which two-candidate equilibria exist we can concentrate on multi candidate equilibria. Setting \( c = 0 \) we immediately obtain the following result:

**Proposition 4:**

Let the costs of candidacy be equal to zero. Then only situations in which a continuum of candidates run for office will be an equilibrium.

Given proposition two and four it is interesting to find out whether the number of candidates in equilibrium depends on the size of \( c \) in a "continuous" way. It turns out that the range of candidacy costs for which equilibria with \( m \) candidates exist indeed do generalise. We can establish the following result:

**Proposition 5:**

Let \( m \) be the number of candidates in equilibrium and assume that \( m > 2 \). Then a \( m \)-numbered set of candidates can be an equilibrium only if \( c \) is in the interval given by:

\[
\sqrt{\left( \frac{m}{m-1} \right)^2 + \frac{1}{m-1}} - \frac{m}{m-1} + \frac{1}{2} \left\{ \sqrt{\left( \frac{m}{m-1} \right)^2 + \frac{1}{m-1}} - \frac{m}{m-1} \right\}^2 \leq c \leq \frac{1}{m-1}
\]

Even though we are not enabled to say anything about candidates locations from proposition 5, it is interesting because knowing the costs of candidacy does not give us an accurate prediction of the number of candidates in equilibrium. For example, let the costs of candidacy be given by 0.1. Then we are told that the system is equilibrium. How many candidates will the equilibrium
contain? Well, using proposition 5 as an algorithm straightforward calculation gives us the answer that we should expect between 6 and 11 candidates in equilibrium. It all depends on the location of the candidates. And this result seems to give us some comfort when we look at the variation of the number of candidates in real world democracies where we should expect the costs of candidacy to vary relatively little through otherwise comparable societies. Second, as \( c \) goes towards zero the possible number of candidates in equilibrium becomes increasingly large. If we let \( c \) be equal to 1/100 for example, we should expect up to 101 candidates in equilibrium. Thus by observing real world democracies we might even come close to being able to suggest a cost of candidacy in modern democracies. Conversely the model might be able to explain the varying number of parties one can observe in real-world democracies which seemingly introduce quite homogenous entry barriers in the form of constitutional set ups, opportunity costs of time used in political fora etc.

Figure 3 - The number of candidates in equilibrium for varying costs of candidacy

![Graph showing the number of candidates in equilibrium for varying costs of candidacy. The graph is labeled as Figure 3.](image)

Figure three shows the cost intervals for which an \( m \)-numbered set of candidates can be an equilibrium for \( m=(2,3,...10) \). The figure can be read both horizontally and vertically. Assume
for example that costs of candidacy is equal to $c^*$. Then from figure three we can see that we should expect between 3 and 5 candidates in equilibrium. On the other hand we can see from the figure that any equilibrium with say 5 candidates must imply that $0.10093 < c < 0.25$. We can also see from the figure that the lower is $c$, the greater is the possible variation of the number of candidates in equilibrium.

4. Are the results due to the voting rule or the formulation of agents utility?

As we explained in section one agents utility functions in this model differ quite sharply with the standardised formulation of utility in economic models of political equilibrium. Thus, the results we have presented in section 3 might be heavily depending on this conception of individual utility. In order to lay out how much the standard model changes when we add expressive benefits and, we should mention, in order to be able to compare the implications of proportional representation with the implications of plurality voting, we briefly present the model and its natural implications when the voting rule is of the plurality type.

The model with plurality voting is identical to the model presented in section two except from the changes in the policy-implementation rule. Thus, (3) now becomes:

(3a) $P = \{ x_j^* \in J \mid V_j^* = \max V_j \}$

This change of the model has the obvious implication that lemma one does not apply, since a middle of the road candidate who obtains a winning voteshare will have her preferred policy implemented. Consequently we will have to look for another way to establish the equilibrium conditions. It turns out that the conditions for one-candidate equilibrium are given by:

Proposition 6: One candidate equilibria:

Let a be a single candidate running for office. Let a candidate b with position $x_b = (1 - x_a - e)$, where $e$ is an arbitrary small and positive number, be the candidate located at the highest distance from candidate a who can win the election by running as a candidate herself. Then a is a one candidate equilibrium if:

(i) $c < c^*$ AND


(ii) if $x_a = 1/2$ and $c > 1/2$ OR

(iii) if $x_a < 1/2$, then $c > \max \{2(1-2x_a), 1/x_a \}$ OR

(iv) if $x_a > 1/2$, then $c > \max \{2(2x_a + c - 1), 1/x_a \}$

In much the same way we can establish the conditions for existence of two-candidate equilibria. There are essentially two types of two candidate equilibria; one in which the candidates tie and one in which one of the candidates win for sure. If we address the tying case first we have that:

Proposition 7: Two candidate equilibrium with plurality voting and tying candidates.

Let $a$ and $b$ be two candidates, such that $x_a < x_b$ and $x_a = 1-x_b$. Then $a$ and $b$ can form a two-candidate equilibria under plurality voting if:

(i) $3/2 - 3x_a > c$, AND either;

(iiia) $x_a \in (1/6; 1/2)$ and $c > \max \{2x_a - 1/2, 1/2 - x_a \}$, OR;

(iiib) $x_a \in [0; 1/6)$ and $c > 1 - 2x_a$, OR;

(iiic) $x_a = 1/6$ and $c > 4/9$

The combination of $x_a$ and $c$ for which two candidate equilibria of the type $x_a = 1-x_a$ exist is shown in Figure 4. We consider the issue space in the interval $(0, 1/2)$ here because $x_a$ is assumed to be less than $1/2$ and because the figure is symmetric around $1/2$. The upper bound on the shaded intervals is given by condition (i) of proposition 7, which secures that $x_a = 1-x_a$ is willing to run against $b$. The lower bound for $x_a \in (0, 1/6)$ is given by condition (iiib) and the thick part of the line $x_a = 1/6$ is the values of $c$ for which $a$ can be part of a two-candidate equilibrium with $x_a = 1-x_b$ when $x_a = 1/6$. The lower bound of cost $(4/9)$ is given by condition (iiic). Finally condition (iiia) gives us the lower bound on $c$ for $x_a \in (1/6, 1/2)$. The two lines correspond to the two conditions in condition (iiia) and the area above the thick part of the two lines is the set that satisfies the condition. The figure shows several interesting points. First, the range of candidate cost for which $x_a = 1-x_b$ can be an equilibrium is discontinuous around $x_a = 1/6$, $x_b = 5/6$. Second, there exist no symmetric two-candidate equilibria with $x_a \geq 0.4$ ($x_b \leq 0.6$). Thus, we need the position of the two candidates to be at least somewhat differentiated in equilibrium. In the traditional spatial voting model with plurality voting we should expect that candidates are not too far apart, because a middle of the road candidate then can enter and win the election outright. But this results points
towards the opposite. We require that the two candidates are located at some distance, not only because of their incentives to stay in the race, but also in order to avoid entry from extreme candidates who have no positive chance of winning the election.

Figure 4 - Two-candidate equilibria with winner take all and tying candidates

Two candidate equilibria with \( x_a \neq 1 - x_b \) are harder to describe in the set up we have used throughout this paper. It is so, because the conditions on entry-proofness split in to several cases. Thus, we will restrain ourselves to show that such equilibria exist and mention the different cases we need to consider in order to prove equilibrium.

Claim 1: Existence of two-candidate equilibria with \( x_a \neq 1 - x_b \) under plurality voting.
For appropriate costs of candidacy two-candidate equilibria under plurality voting exist in which \( x_a \neq 1 - x_b \).

While equilibria of the type \( x_a = 1 - x_b \) obeys the usual result underlying analysis of plurality rule that all candidates must have a positive chance of winning the election in equilibrium; ie. see for example Downs (1957) or, in the context of the citizen-candidate model, lemma 1 in Osborne and
Slivinski (1996), the second type of equilibria in which $x_0 \neq 1-x_0$ is different. The existence of non-symmetric equilibria is of course due to the fact that we have added expressive benefits to citizens utility functions. Thus, this works in much the same way as the introduction of proportional representation in Hamlin and Hjortlund (1998), albeit for other reasons. In the proportional representation set up there are no extreme meaning of "winners" and "losers", but here we have that in equilibrium one of the candidates will lose for sure and this is precisely what we mean by counter intuitive. Since some citizens will gain by becoming candidates out of expressive reasons in itself, there is nothing exceptional in the result. Thus concluding this section we are able to state that changes in either the voting rule or the specification of individual preferences is sufficient to overturn the standard plurality model conclusion that candidates obtain equal electoral support in equilibrium.

5. Concluding remarks

We have used a citizen-candidate model to study how the introduction of expressive preferences in citizens' utility functions change the set of possible electoral equilibria. We have done so by letting citizens have preferences for policy outcomes and policy expressions. The analysis leads us to draw the following conclusions. First, the existence of office rents are not necessary for the existence of multi-candidate equilibria in the citizen-candidate model with a uniform distribution of citizens ideal point. If citizens in addition to the standard conception of utility, gain something from expressing their preferences in itself, this might be enough to establish multi-candidate equilibria in democracies. Second, for given costs of candidacy the number of candidates in equilibrium might differ quite sharply across institutions (democracies) which are restrained by seemingly equal constraints. Finally we have shown that the introduction of expressive benefits in agents' utility functions destroy the basic result underlying traditional plurality voting models that candidates obtain equal support in equilibrium. The statement that agents' in their decision problems might trade off expressive benefits against instrumental benefits might seem quite unrealistic to some readers, but many political observers probably would agree that real world politics does provide examples of candidates who at the expense of ever winning an election, continue to run for office and thereby express their ideal policy. The model we have used is completely static, but it would, of course, be particularly interesting to see what happens to these trade-offs in a dynamic set up, in which the distribution of citizens ideal points changes over
time. Thus future research should be directed at combining a citizen-candidate type of model with the possibility of dynamic analysis.

6. References


Appendix one- proof of propositions and examples

*Proposition one - One candidate equilibrium*

A one-candidate equilibrium with candidate a running unopposed exists iff:

(i) $5/8 < c < z$ and

(ii) $a \epsilon \{2-(1+2c)^{1/2},(1+2c)^{1/2} - 1\}$.

**Proof of proposition 1**: First we prove proposition 1(i). It is clear that c must be less than z for a to be willing to run. Second, we postulate that no one-candidate equilibria exists for $c < 5/8$. This is simply due to the fact that for costs lower than this threshold value, there will be at least one other citizen who benefits from entering as a candidate. With a uniform distribution of voters ideal points, the most likely citizen is one located as far away from a as possible. This distance is minimised for $x_a = 1/2$. But consider a candidate located at 1. To keep such a candidate from running, (ii) of the equilibrium condition must hold. Now a candidate b, located at $x_b = 1$ will gain exactly $5/8$ by entering ($4/8$ from the expressive part of the equilibrium condition and an addition $1/8$ from the instrumental part of the equilibrium condition) given that a is a candidate. The second part of the proof is concerned with entry proofness and follows the same line of argument by considering the interval of $x_a$ for given costs c.

The conditions for existence of one-candidate equilibria are conceptually equal to the one-candidate equilibria in Hamlin and Hjortlund (1998), only here the lower bound of c for which a one-candidate equilibrium exists has been raised relative to the results presented in Hamlin and Hjortlund (1998). This difference is simply due to the fact that we have added an expressive part to citizens loss-function here and it has no particular interpretation besides that. Figure one
presents the possible one-candidate equilibria as a function of c.

**Figure one - one candidate equilibria with proportional representation**

**Lemma one:** Let the distribution of citizens' ideal points be uniform. Then, for any set of candidates in which the number of candidates is greater or equal to two, the implemented policy is equal to the policy that would have prevailed under a two-candidate game with the two extreme candidates in the candidate set running for office.

Proof of Lemma one: See Hamlin and Hjortlund (1998), lemma 2. The effect of adding expressive benefits have no policy impact, thus reducing the problem to the one posed by Hamlin and Hjortlund.

**Proposition two:** Let a and b be the two candidates in a two-candidate equilibrium and assume that \( x_a < x_b \). Then \((a,b)\) forms a two candidate equilibrium iff:

(i) \( c < \min \left\{ \left( x_b - x_a \right) + \frac{1}{2} \left( x_b^2 - x_a^2 \right) \right\} , (2(x_b - x_a) - \frac{1}{2} (x_b^2 - x_a^2) \} \), AND

(ii) \( c > \max \left\{ \frac{1}{2}(x_b - x_a), \left[ x_a + \frac{1}{2} x_b^2 \right] , \left[ \frac{3}{2} - 2x_b + \frac{1}{2} x_b^2 \right] \} \)
Proof of proposition 2: Condition (i) assures that a and b are willing to run against each other and thus corresponds to (i) in the equilibrium condition in section two. We simply insert \( x_a \) and \( x_b \) in condition (i) of definition 1. The first part of the right hand side of the inequality secures that a is willing to run against b and the second part secures that b is willing to run against a. Condition (ii) is the entry proofness condition. The first part of the inequality is needed in order to avoid entry from a candidate located between candidates \( x_a \) and \( x_b \). Such a candidate will join if (ii) in the equilibrium condition does not hold. Now, by lemma one we have that middle of the road candidates only obtain expressive benefits from entering as candidates. Notice that citizens located at \((x_a + x_b)/2\) will gain more by entering than any other middle of the road candidate. Thus, we need that \(|(x_a + x_b)/2 - x_a| < 0\) in order to avoid entry from such a candidate. Rearranging we obtain that \(c > 1/2(x_b - x_a)\). The next part of (ii) secures that no extreme candidate located to the left of a wants to enter, given candidacy of a and b. If \(x_a > (1 - x_b)\) citizens located at 0 will gain more than any other citizen by entering. Thus we can use (ii) in the equilibrium condition with insertion of a potential candidate located at 0 as the entry-threat. By doing so, we obtain that \(x_a + 1/2 x_a^2 - c < 0\) By symmetry we can do the same by solving for \(x_b\) if \(x_a < (1 - x_b)\). This is what we have done in the last part of the expression. See also Hamlin and Hjortlund (1998) proof of proposition two.

Proposition 3: Minimum costs for which a two-candidate equilibrium exists.

Let a and b with ideal positions \(x_a < x_b\), be the two candidates running against each other. Then for a two-candidate equilibrium to exist we require that \(c > 5^{1/2} - 2 + 1/2(5^{1/2} - 2)\). The locations of a and b that minimise the necessary cost of candidacy are \(x_a = 5^{1/2} - 2\) and \(x_b = 1 - (5^{1/2} - 2)\).

Proof of proposition 3: This is a straight forward minimisation-problem following condition (ii) in proposition two. It is clear that cost of entry from an extreme candidate is minimised at \(x_a = (1 - x_b)\). From proposition two we have that \(1/2 x_b - 1/2 x_a < c\) (entry from middle of the road candidate) and that \(x_a + 1/2 x_a^2 < c\), minimising the maximum cost of these two conditions and using the fact that \(x_a = (1 - x_b)\) we obtain that the minimum cost for which a two-candidate equilibria exist is equal to \(c_{min} = 5^{1/2} - 2 + 1/2(5^{1/2} - 2)^2\). This corresponds to the following location of candidates: \(x_a = 5^{1/2} - 2, x_b = 1 - x_a = 3 - 5^{1/2}\).
**Proposition 4:** Let the costs of candidacy be equal to zero. Then only situations in which a continuum of candidates run for office will be an equilibrium.

**Proof of proposition 4:** With c=0 part (ii) of the equilibrium condition can only be satisfied if there exist no citizen \( x \) such that \( \min |y_j - x| > 0 \). Consequently the only possible equilibrium is the one in which all citizens become candidates. Can this be an equilibrium? According to (i) in the equilibrium condition it can, because in this situation we have for all citizens that (i)=0.

**Proposition 5:** Let \( m \) be the number of candidates in equilibrium and assume that \( m>2 \). Then a \( m \)-numbered set of candidates can be an equilibrium only if \( c \) is in the interval given by:

\[
\sqrt{\left(\frac{m}{m-1}\right)^2 + \frac{1}{m-1} - \frac{m}{m-1} + \frac{1}{2} \left(\left(\frac{m}{m-1}\right)^2 + \frac{1}{m-1} - \frac{m}{m-1}\right)} \leq c \leq \frac{1}{m-1}
\]

**Proof of proposition 5:** The proof follows the same intuition as the proof of proposition two. We start out by proving the first part of the inequality and then we go on to prove the second part. Now let \( m \) be the maximum number of candidates in equilibrium. Using the method of proposition 2, we know that the position of candidates in an \( m \)-numbered equilibrium with the lowest possible costs of candidacy must obey the following conditions:

1. \( x_*, x_{*,+1/2} < c \)
2. \( \frac{1}{2}(x_b* - x_*)/(m-1) < c \)
3. \( x_a* = (1-x_a*) \)

where \( x_a* \) and \( x_b* \) denote the most extremist candidates, \( x_a* < x_b* \).

Substituting 3. in 2. and setting \( 2. = 1 \). yields that: \( 1/2 x_a*^2 + x_a*(1+(1/(m-1))-1/2(m-1)=0 \) or:

\( x_a* = ((m/(m-1)^2+1/(m-1))^{1/2} - m/(m-1)) \).

Define \( c* \) as the highest value of \( c \) for which \( x_a* \) is not part of an equilibrium. This implies that \( c* = x_a* + 1/2 x_a*^2 \) in turn yielding that \( c* = ((m/(m-1)^2+1/(m-1))^{1/2} - m/(m-1)+1/2(((m/(m-1)^2+1/(m-1))^{1/2} - m/(m-1))^2-m/(m-1)) \). Thus for an \( m \)-numbered equilibrium to exist we require that \( c > c* \). The position of candidates for which an \( m \)-numbered equilibrium exists is given by \( (x_a*, x_a* + 1/2(x_b* - x_a*)/(m-1)) \).
Now we show the upper bound of \( c \) for which an m-numbered equilibrium exists. This problem can be seen as the opposite problem, i.e., how high can \( c \) be for an m-numbered equilibrium to exist. Here we focus on the willingness to run condition rather than the entry proofness condition. It is quite simple to see that we should expect the extremist candidates to be located at the endpoints of the distributions. Similarly, we should expect the candidate immediately to the "right" of a to be located at \( x_a + c \), which gives such a citizen a net payoff from entering of exactly 0. We should expect the next candidate to be located at \( x_a + 2c \) etc. and the \( m \)'th candidate, \( x_m \), to be located at \( x_m = x_a + (m-1)c = 1 \). Thus by rearranging we obtain that for any m-numbered equilibrium \( c \leq 1/m - 1 \).

**Proposition 6:** One candidate equilibria: Let \( a \) be a single candidate running for office. Let a candidate \( b \) with position \( x_b = (1 - x_a - \epsilon) \), where \( \epsilon \) is an arbitrary small and positive number, be the candidate located at the highest distance from candidate \( a \) who can win the election by running as a candidate herself. Then \( a \) is a one candidate equilibrium if:

1. \( c < z \) AND
2. if \( x_a = 1/2 \) and \( c > 1/2 \) OR
3. if \( x_a < 1/2 \), then \( c > \max \{ 2(1 - \epsilon - 2x_a), /1 - x_a / \} \) OR
4. if \( x_a > 1/2 \), then \( c > \max \{ 2(2x_a + \epsilon - 1), /x_a / \} \)

**Proof of proposition 6:** (i) is the by now familiar condition to secure that \( a \) wants to run as a candidate. (ii) and (iii) are the conditions securing entry proofness for different values of \( a \). The second and third term in parentheses cover the situation where a candidate might want to enter out of expressive concerns alone. Such a candidate will always be located at either 0 or 1. Thus for \( a < 1/2 \) the maximum expressive benefit is equal to the distance \( 1 - x_a \). For \( x_a > 1/2 \) the maximum distance is simply \( x_a \). The first term in the parentheses covers entry proofness from citizens who would win the election if they ran as candidates, given candidacy of \( a \). Now for \( x_a < \frac{1}{2} \) \( (> \frac{1}{2}) \) a citizen located immediately to the left (right) of \( \{1 - x_a\} \) would gain more from candidacy than any other citizen. If we denote such a citizen \( x_a = (1 - x_a - \epsilon) \) \( ((1 - x_a + \epsilon)) \) and use definition one we get that for \( a < 1/2 \) \( (> 1/2) \), \( |a - b| - a + |a - b| - c < 0 \), or: \( c > \{ 2(1 - \epsilon - 2a), 1 - a \} \( (2(2a + \epsilon - 1), a \) ).
Proposition 7: Two candidate equilibrium with plurality voting and tying candidates: Let a and b be two candidates, such that \( x_a < x_b \) and \( x_a = 1 - x_b \). Then a and b can form a two-candidate equilibria under plurality voting if:

(i) \( \frac{3}{2} - 3x_a > c \), AND either;

(iiia) \( x_a \in (1/6; 1/2) \) and \( c > \max \{2x_a - 1/2, 1/2 - x_a \} \), OR;

(iiib) \( x_a \in (0; 1/6) \) and \( c > 1 - 2x_a \), OR;

(iiic) \( x_a = 1/6 \) and \( c > 4/9 \)

Proof of proposition 7: Condition (i) is the condition that secures that a and b are willing to run against each other. Conditions (iia), (iib) and (iic) guarantees entry proofness for different values of \( x_a \) (and \( x_b \) because \( x_b \) is given as a function of \( x_a \) \( x_b = 1 - x_a \)). First consider condition (i). Using (i) of definition 1 and remembering the change of policy implementation mechanism to allow for plurality voting we get that for b to be willing to run against a we require that:

\[
(x_b - x_0) + (x_b - x_0) - \frac{1}{2}(x_b - x_a) - c > 0.
\]

Now note that \( x_a = 1 - x_b \). We immediately obtain that \( c > \frac{3}{2} - 3x_a \). Turning to entry proofness if follows from the proposition that we have divided the condition into three sections defined by the size of \( x_a \). The dividing line is the value of \( x_a \) (and \( x_b \)) for which an entrant would win the election if she entered. For \( 1/6 < x_a < 1/2 \) there exist no citizen who can change the policy by becoming candidate (no one can affect the result positively, see Palfrey (1984) for a model which rely on this with strategic candidates). For \( x_a = 1/6 \) a citizen located at the median position (1/2) will have 1/3 probability of winning the election and for \( x_a < 1/6 \) there exist at least one citizen who would win the election if she entered. Let us examine the first case first. In order to avoid entrance we require that for all citizens not in the candidate set, \( c > \min \{ y_j - x_i / + P_n - x_i / - P_i - x_i / \} \). Note that since \( x_b = 1 - x_a \), the citizen who would gain most by entering is located either at the endpoints of the distribution, or at the median position. We analyse the situation for an entrant located at 0 first (because of the symmetry of the system we need only to consult one of the endpoints in order to establish our argument). Note that entry of a candidate located at 0 (1) change the expected policy outcome of the game; Without her candidacy a will win with probability 1/2 and b will win with probability 1/2, but with her candidacy the candidate which is located furthest away from her would win the election for sure. Thus for an entrant located at 0 we require that:

\[
(x_a - 0) + (1/2 - 0) - (x_b - 0) < c, \text{ or (using the fact that } x_b = 1 - x_a, c > 2x_a - 1/2.
\]

Now
consider entrance by a citizen located at the median position \( \frac{1}{2} \). Entry from such a citizen will leave the a priori expected outcome of the election unaltered. Thus to avoid entry by such a citizen we require that \( c > \frac{1}{2} - x_a \). This completes the proof of condition (iia) in the proposition.

Now consider what happens when \( x_a \in (0, 1/6) \). In this situation a citizen located at the median position \( \frac{1}{2} \) will always gain at least as much by becoming a candidate as any other citizen. It is so because of the symmetry of the system \( (x_a + x_b) / 2 = \frac{1}{2} \) whenever \( x_a = 1 - x_a \). Now it is no longer enough for us to use the expected policy defined in definition one as the proper measure of disutility. Instead we use the expected (ex post) disutility that an entrant located at \( \frac{1}{2} \) obtains by refusing to become a candidate. This is equal to \(- \frac{1}{2} / x_a - \frac{1}{2} / - \frac{1}{2} / x_b - \frac{1}{2} /, \text{ or simply } - \frac{x_a - \frac{1}{2}}{.} \)

Thus, in order to avoid entry by a citizen located at the median position, we require that: \( \frac{1}{2} - x_a / - x_a / 0 < c, \text{ or } c > 1 - 2 x_a \). This completes the proof of condition (iic). Finally we need to prove condition (iid). When \( x_a = 1/6 \) an entrant located at the median position will win the election with probability \( 1/3 \). Thus, using the same logic as above we need that \( \frac{1}{2} - x_a / + / 2/3 / 2/3 / x_a / - c / c, \text{ or using the fact that } x_a = 1/6, c > 4/9. \)

Example and outline of proof of claim 1: Consider the case in which \( x_a = 0.25 \) and \( x_b = 0.8 \). This is an equilibrium for \( c = 0.4 \). First, we immediately see that \( a \) and \( b \) is willing to run against each other (b for expressive reasons only). Now consider entry to the left of \( a \). The maximum gain is for a candidate located at \( 0 \). But this candidate will gain 0.25 in expressive pay-off plus a negative instrumental pay-off due to the fact that the election result alters from \( P = x_a \) to \( P = x_b \). Thus no candidate to the left of \( a \) wants to enter. Consider entry from a candidate to the right of \( x_b \). The citizen who gains most from entry is one located at \( 1 \). But such a citizen will gain a payoff of 0.2 because of the expressive term in the utility function. Since policy is unaltered (a continues to win) her net-payoff by entering is equal to \(-0.2 \). Finally, consider entry from a citizen located between \( x_a \) and \( x_b \). No citizen exists in this interval who can win the election by entering. Consequently only expressive benefits matters. But there exist no citizen \( d \) for whom \( \min \{ x_a, x_b \} / x_d / 0.4 \). We conclude that \( x_a = 0.25 \) and \( x_b = 0.8 \) is a two-candidate equilibrium for \( c = 0.4 \).

Now consider the conditions for equilibrium. For simplicity assume that \( x_a < x_b \) and \( x_a > 1 - x_b \), such that \( a \) will win the election for sure. Now in order for \( a \) and \( b \) to be willing to run against

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\(^3\text{It is so because the ex ante expected policy is equal to } \frac{1}{2} \text{ but this policy would never be reached, since either } a \text{ or } b \text{ is winning the election.} \)
each other we require, that;

(ia) for a, \( c < 2(x_a - x_b) \) and (ib) for b, \( c < (x_b - x_a) \), or simply;

(i) \( c < (x_b - x_a) \).

This is straight forward. a gains both instrumental and expressive benefits by becoming a candidate given candidacy of b, but b only gains expressive benefits given candidacy of a (because \( x_a > 1 - x_b \)). Next, consider the entry-proofness conditions. First look at entry from a candidate located at the endpoints of the distribution. We have four cases:

(a) entry from \( x_a > x_b \), (b) entry from \( x_d < x_a \) where d gains a majority by entering (since such a candidate always exist if such a candidate exists to the right of \( x_a \), we need not consider the latter), (c) entry from \( x_d < x_a \) where d does not win a majority but changes the election result and (d) entry from \( x_d < x_a \) where d does not change the election result by entering. We can state the following conditions for these cases:

(iiia): \( c > (1 - x_b) \), because a citizen located at 1 will gain more than any other candidate in the range.

(iiib): \( 2(x_d - x_a) < c \), where \( x_d^* = \min x \mid V_d > \max(V_a, V_b) \) because such a citizen gains both expressive and instrumental benefits.

(iiic): \( 2(x_a - x_b) < c \) (because a citizen located at 0 will gain more than anybody else)

(iiid): \( c > x_a \) (because a citizen located at 0 will gain more than anybody else)

We also need to consider entry from a candidate located between \( x_a \) and \( x_b \). This condition divides into 2 cases:

(e) entry from \( d \), where \( x_d < x_d < x_b \) and where d obtains a majority by entering, and (f) entry from \( d \), \( x_d < x_d < x_b \) where d does not obtains a majority by entering. Then we obtain the conditions:

(iiie): \( c > |x_d - x_a| + \min |(x_a, x_b) - x_d| \). The position of d depends on the position of candidates a and b. It is clear that \( x_d = (x_a + x_b)/2 \) will gain more in expressive terms than any other candidate, but this candidate might be unable to obtain a majority by entering, even though another such candidate exist. Being a bit more precise we require of d in this case (where \( x_d > 1 - x_b \)) that it is the rightmost candidate to secure a majority against a, ie: \( x_d = \max x \mid V_d > V_a \). (iiff): \( c > (x_d - x_a) \), where \( x_d = (x_a + x_b)/2 \), because a citizen located in this position will gain more than any other candidate entering.