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The 40% Neoclassical Aggregate Theory of Production

Results of a Simulational Investigation

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Abstract. The assumption that production in an economic system may be described by an aggregate neoclassical production function is at the foundation of most modern equilibrium neoclassical business cycles and growth models. Its validity requires stringent assumptions on individual production functions and on market structure. In this article the likelihood that an aggregate neoclassical production function could emerge from a simple heterogeneous production system is measured through computer simulations. The conclusion is that there exists a not-too-small world for which the aggregate neoclassical theory of production does not hold.

Key words: Neoclassical production functions; Growth theory; Capital controversy; Wage-profit frontier; Computational economics.
JEL classification: 020; 030; 111; 620;

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1. **On the relation between the measurement of aggregated capital and its marginal productivity. Introductory remarks.**

During the last two decades a great deal of research has focussed on the improvement of neoclassical models intended to explain the mechanics of economic development and long-run growth. Extensive literature, following the now seminal works of Lucas (1988) and Romer (1986, 1990), has been published in specialized journals. Much of this material is now presented to students and scholars through several monographs on growth (For example Barro and Sala-i-Martin, 1995, Romer, D., 1996, Jones, 1998, Aghion and Howitt, 1998).

This literature on long run properties of economic systems stands on the work on growth initiated by Ramsey (1928), Solow (1956), Cass (1965) and Koopmans (1965), Samuelson (1958), Diamond (1965). It almost completely disregards the work on growth by von Neumann (1945-6), Harrod (1939, 1948), Robinson (1956), Kaldor (1960), Pasinetti (1962) and characterized by the assumption that the aggregate production may be represented by an aggregated constant returns to scale production function where the factors of production are aggregated physical and human capital, aggregated labour, aggregated knowledge and technology. An important requirement for the aggregated neoclassical production function is that as the utilization of the factors of production increases (decreases) its rate of return decreases (increases). With the underlying assumption of perfect competition, the physical rate of return on capital is then set to be equal to the interest rate and the physical rate of return on labour to be equal to the wage rate. In the cases where the perfect competition assumption is substituted with some oligopolistic or monopolistic assumptions for market structure, it is still the rate of return on capital and the rate of return on labour that sets the benchmark on which to compute 'actual' values.

The above approach to aggregate production first implemented for long-run models extends also to a wide class of short-run models (Kim, 1988, Lucas, 1987). These models, for example the real business cycles, originated by the work of Kydland and Prescott (1982), assumed again that there is a single thing called capital that can be put into a single production function that, together with labour, will produce total homogeneous output.

Although the level of aggregation of both the short-run and long-run models is obviously extremely high there is the general claim that they are based on solid micro foundations, i.e., on disaggregated structures. The underpinning assumption is that a Walrasian equilibrium of the

Debreu (1959) or Arrow and Hahn (1971) type holds at each point of time (or at each interval of time).

The tenability of the neoclassical theory of production relies on two fundamental postulates (Sato, 1974, p.355). The first, **(Sato)Postulate A**, or 'Wicksell effect', is the proposition that *a more capital intensive method of production tends to be adopted at a lower rate of interest (or higher wage rate).* The second, **(Sato)Postulate B**, or 'No-Reswitching', asserts that *no method of production recurs as the rate of interest falls*, i.e. once a method of production has become uneconomical at a certain rate of interest it will not become economical at a lower one.

Section 1.1, presents a brief historical discussion of the capital controversy and a discussion of the implications for current economic theory follows. In section 2, a formalism concerning the case of heterogeneous production with heterogeneous capital goods is provided with focus on the case in which different methods of production are available, on the idea of aggregate capital, on the capital-labor ratio and net national product. Subsequently an operational definition of the (Sato)neoclassical postulates is given. In section 3, artificial economies are generated through computer simulations and the representative variables are computed. The aim of the experiments is that of providing a measure of the likelihood that these artificially generated economies might exhibit neoclassical properties. The working of the algorithms, described in section 3.1, is demonstrated through a simple example, section 3.2, and more general results are provided in section 3.3. Finally conclusions are discussed in section §4. In the Appendix the results of the simulations are presented with some detail.

1.1 The '*surrogate*' production function and the capital controversy.

In the history of economic thought the theoretical question of whether different forms of physical capital goods can be grouped and measured by a scalar magnitude called capital whose marginal productivity is negatively related with respect to the profit rate has stimulated, at different points of time and with different intensities, interesting and heated debates[1].

At the beginning of the sixties, Samuelson devised a powerful tool, the 'surrogate' production function, to support the tenability of the above postulates. He wanted to show *"how certain quite complicated heterogeneous capital models will behave by treating them 'as if' they had come from a simple generating production function (even when we know they did not 'really' come from such a function)* (Samuelson, 1962, p.194)". Levhari (1965) used the 'surrogate' production function to establish the impossibility that a system producing n commodities which ceased to be profitable at a given interest rate (profit rate) should again become profitable at a lower interest rate.

These and other attempts were challenged and proven to be wrong by Bhaduri (1966), Pasinetti (1966) and Garegnani (1966) and others. In essence what they showed was that reswitching of techniques was in fact a logical possibility, i.e., they demonstrated that the above (Sato)Postulate-B would not always hold. Samuelson (1966) admitted the problem. He recognized that *"The phenomenon of switching back at a very low interest rate to a set of techniques that had seemed viable only at a very high interest rate* ***involves more than esoteric technicalities****. It shows that the simple tale told by Jevons, Böhm-Bawerk, Wicksell, and other neoclassical writers - alleging that, as the interest rate falls in consequence of abstention of present consumption in favour of future, technology must become in some sense more "roundabout," more "mechanized," and more "productive" -* ***cannot be universally valid***

[1] During this century we have witnessed (Ahmad, 1991, Ch.21) three debates on this and related issues. The first debate involved Böhm-Bawerk, J.B.Clark, Cassel, Landry and Fisher. The second involved among several Knight and Kaldor. The third, which is most pertinent to the arguments put forward in this paper - known as the debate between the two Cambridges (Cambridge-England attacking the neoclassical postulates and Cambridge-USA defending it) - involved Robinson, Sraffa. Pasinetti, Solow, Samuelson, Garegnani, Kahn, Hahn. It was first Robinson (1953-4) who challenged the neoclassical approach by claiming that the determination of the rate of return on capital (marginal productivity) could be obtained only in parallel with the determination of the value of capital, but the value of capital could be determined only when the rate of return was simultaneously determined. This would lead to a circularity and hence to the impossibility of determining an 'absolute' measurement for capital. Sraffa (1960) provided new foundations to this critique. See also Harcourt (1972).

(emphasis added, Samuelson, 1966, p.568)"[2].

Also Sato (1974), who wrote in defence of the neoclassical system had to admit that the neoclassical postulates did hold only under certain conditions. He argued "*that there is a not-too-small world in which the neoclassical postulate is perfectly valid. So long as we live in that world, we need not to give up the neoclassical postulate. ... Nonetheless, it is important to realize that there is another world in which the neoclassical postulate may not fare well or is contradicted. An empirical question is which of the two worlds is more probable.* (emphasis added, Sato, 1974, pp.383)". Analytical or empirical research on how big this world would be did not follow.

Much of the debate between the two Cambridges is centred on the issue of a proper micro foundation and on the long-run tendency for profit rates to converge towards a uniform value. Hahn (1975, 1982) and Bliss (1975) defended the neoclassical view by saying that in the general equilibrium theory of the Arrow-Debreu type the case in which a uniform rate of profit exists is a special one. But one could claim, following for example Garegnani (1976), Kurz (1985) and Kurz and Salvadori (1995, ch.14), that the multiplicity of profit rates is relative only to the short-run or unsettled case. In the long-run case, due to competitive forces, capital would be shifted towards production of the most profitable commodities and hence a uniform rate of profit would emerge. This is unlikely to occur in the Arrow-Debreu general equilibrium case[3].

Given the current tendency to see macroeconomic production as being well described by an aggregated neoclassical production function one could infer that in recent years, thanks to new

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[2] Samuelson was even more clear about the generality of the reswitching theorem, and hence about his own mistakes, when commenting the work of one of his students (Levhari, 1965): "*We wish to make clear for the record that the nonswitching theorem associated with us is definitely false* (Levhari and Samuelson, 1966, p.518)". It should be therefore convincing to anyone that the last capital debate, between the two Cambridges showed that the above postulates, in order to remain valid, required serious restrictions about the descriptive functional forms of the underlining productive economic system (Harcout, 1972).

[3]The question of the proper micro foundation of macro models has been addressed several times in the history of economic theory. The micro foundations of the Arrow-Hahn-Debreu type imply a unidirectional concept of time and unidirectional causality going from the factors of production to the production of the final consumption goods. But micro foundation is also present in other alternative classical approaches where the emphasis is posed on the circular process of production (for example in Quesnay's Tableau Economique). On this issue see also Geanakoplos (1987, pp.777-78).

evidence, *the spark of an ultimate truth has been struck*[4]. But this is not the case; on the contrary, Lucas himself writes in his celebrated paper on growth, after having used standard neoclassical assumptions all through the article: "*We can, after all, no more directly measure a society's holdings of physical capital than we can its human capital. The fiction of 'counting machines' is helpful in certain abstract contexts but not at all operational or useful in actual economies - even primitive ones. If this was the issue in the famous 'two Cambridges' controversy, then it has long since been resolved in favour of the [English side]*"(Lucas,1988, p.35-6). If this is the case there is good ground to wonder why one should follow Lucas' method and use an aggregate neoclassical production function. How can we measure the marginal productivity of capital without knowing, at least in theory, what capital is?

The choice of the proper model of development and growth has far too obvious and important implications for policy recommendations. The idea that the production level of an economy may be universally described by a neoclassical function of the Solow-Romer-Lucas type allows researchers to compare quite different economic systems, such as Bangladesh, South Korea, Japan, Russia and United States, in a simple manner, so as to allow them to give policy prescriptions that are, from the qualitative point of view, the same. In view of the above discussion, there is a serious doubt the general validity of the neoclassical postulates. It follows that giving the same policy advice to all the countries not only would be wrong but extremely dangerous. In fact, for some economic structures, the capital controversy showed that higher interest rates could possibly be associated with higher future consumption levels. In this case the policy prescriptions should be different from the standard ones. This is not at all to conclude that higher physical capital would reduce the production capacity of the economic system, but that the increase of the future production capacity of a system might be associated with a concomitant increase of the value of capital and of the interest rate.

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[4]Three quarters of a century ago Piero Sraffa wrote: "*A striking feature of the present position of economic science is the almost universal agreement at which economists have arrived regarding the theory of competitive value, which is inspired by the fundamental symmetry between the forces of demand and those of supply, and is based upon the assumption that the essential causes determining the prices of particular commodities might be simplified and grouped together by a pair of intersecting curves of collective demand and supply. This state of things is in such a marked contrast with the controversies on the theory of value by which political economy was characterized during the past century, that it might almost be thought that from these clashes of thought the spark of an ultimate truth had been struck (Sraffa, 1926, p.535, emphasis added)*". This seems to be still pertinent to the current state of the economics of production functions.

2. The characterization of an economy, the efficient factor price frontier and aggregate magnitudes.

Let us consider an economic system that is producing n basic goods and that is in a self-producing replacing state where the rate of profit and the wage rate are uniform (See Sraffa, 1960).

In matrix notation the system may be written as follows[5]:

$$\mathbf{Ap}(1+r) + \mathbf{L}w = \mathbf{Bp} \tag{1}$$

where: **A** is the matrix of the means of production (semi-positive and indecomposable), whose components a_{ij} are the amount of good j used as mean of production for the production of good i;

L is the labour vector whose components ℓ_i are the amount of labour used by industry i;

B is the production matrix, diagonal and semipositive definite, single production case, whose elements b_{ii} are the amount of good i produced by industry i;

p is the price vector whose elements p_i are the price of good i;

r is the uniform profit rate;

w is the uniform wage rate.

The above describes an adjusted situation, which we could call settled or long-run, in which the rate of profits, wage rates and prices are assumed to be uniform throughout the whole economy. The values, i.e., prices, are measured in terms of a *numerairé*, i.e., a bundle of goods:

$$\boldsymbol{\eta}'\mathbf{p} = 1 \tag{2}$$

where: $\boldsymbol{\eta}$ is a vector of positive weights of the goods belonging to the bundle.

If the surplus vector is semipositive definite, i.e., if $\mathbf{e}'(\mathbf{B}-\mathbf{A}) \geq 0$ (**e** is the unit vector or summation operator), and the vector of prices is also semipositive the system is a productive one. Any productive system like the above may in principle be re-proportioned so that the different industries would produce different quantities and as a whole would constitute another productive system. If the semipositive diagonal matrix **X** is the re-proportioning matrix, where each diagonal element i is the amount of re-proportioning of industry i, this new productive system can be expressed by the quadruple (**XB**, **XA**, **XL**, $\boldsymbol{\eta}$). The set of all the possible quadruples derived from

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[5] Vectors and matrices are written with bold characters and a prime denotes transposition.

the original quadruple $(\mathbf{B},\mathbf{A},\mathbf{L},\eta)$ through a productive transformation $\mathbf{X}$ is given by $\mathscr{E} = \{\mathbf{B},\mathbf{A},\mathbf{L},\eta\}$[6].

For a given profit rate, the vector of natural or production prices formed by (1) and (2) is determinate. Rearranging (1) we have that $\mathbf{p} = [(\mathbf{B}-\mathbf{A}(1+r))]^{-1}\,\mathbf{L}w$ and substituting into (2) we obtain the traditional wage-profit function:

$$(3) \qquad w(r,\mathscr{E}) = [\eta'[(\mathbf{B}-\mathbf{A}(1+r))]^{-1}\,\mathbf{L}]^{-1}$$

and prices:

$$(4) \qquad \mathbf{p}(r,\mathscr{E}) = [(\mathbf{B}-\mathbf{A}(1+r))]^{-1}\,\mathbf{L}\,[\eta'[(\mathbf{B}-\mathbf{A}(1+r))]^{-1}\,\mathbf{L}]^{-1}$$

The solutions of the system of equations $\mathbf{Ap}(1+r) + \mathbf{L}w = \mathbf{Bp}$ and of the system $\mathbf{XAp}(1+r) + \mathbf{XL}w = \mathbf{XBp}$ are the same, i.e., independent of any re-scaling $\mathbf{X}$[7].
The calculation of the value of the net national product, Y_v, associated with a given realization $(\mathbf{XB}, \mathbf{XA}, \mathbf{XL},\eta)$ is given by:

$$(5) \qquad Y_v(\mathbf{p}(r,\mathscr{E}),\mathbf{X}) = \mathbf{e}'\mathbf{X}(\mathbf{B}-\mathbf{A})\,\mathbf{p}(r,\mathscr{E})$$

The value of the means of production (circulating capital), K_v, is given by:

$$(6) \qquad K_v(\mathbf{p}(r,\mathscr{E}),\mathbf{X}) = \mathbf{e}'\mathbf{XAp}(r,\mathscr{E})$$

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[6]The emphasis posed on the re-proportioning diagonal matrix $\mathbf{X}$ has several reasons. Here and in the sequel we will not focus our attention on a particular consumption-demand vector. Nevertheless it may be useful to recall that with a proper transformation $\mathbf{X}$ any consumption demand vector can be satisfied. For a given demand vector, $\mathbf{c}^d$, composed of different quantities of the n commodities in which the system is made, there exists a re-proportioning $\mathbf{X}^d$ of the original system $(\mathbf{B},\mathbf{A},\mathbf{L},\eta)$ such that $\mathbf{e}'\,\mathbf{X}^d\,(\mathbf{B}-\mathbf{A}) \geq \mathbf{c}^{d\prime}$. If there are no constraints in terms of primary commodities, in this case labour, any demand vector can be satisfied. Moreover a re-proportioning matrix $\mathbf{X}$ will be used in the sequel to compute, throughout a proper transformation of the original system, specific aggregate values important for the present analysis.

[7]This property, i.e., that relative prices for a given system $\mathscr{E} = \{\mathbf{B},\mathbf{A},\mathbf{L},\eta\}$ are independent of the actual production or demand vector, is also known as the non-substitution theorem (see Arrow, 1951, Koopmans, 1951 and Samuelson, 1951).

Any given constant value of the net national product, let us call it isovalue, $Y_v^{isovalue}$, can be generated by a virtual re-proportioning **X**. There exist an infinite number of re-proportioning matrices that would allow the generation of the same value of the aggregate net national product. Among them there is at least one re-proportioning matrix for which the value of capital is minimized, let's call it $\mathbf{X}_{min}(r, \mathscr{E}, Y_v^{isovalue})$, in the sequel, in the sequel $\mathbf{X}_{min}(\cdot)$. This matrix is such that:

(7) $$Y_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min}(\cdot)) = \mathbf{e}' \mathbf{X}_{min}(\cdot)(\mathbf{B}-\mathbf{A})\mathbf{p}(r, \mathscr{E}) = Y_v^{isovalue}$$

and the associated aggregate value of capital

(8) $$K_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min}(\cdot) | Y_v^{isovalue}) = \mathbf{e}' \mathbf{X}_{min}(\cdot)\mathbf{A}\, \mathbf{p}(r, \mathscr{E})$$

is minimized.

The magnitude $K_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min}(\cdot) | Y_v^{isovalue})$ can be interpreted as the value of the (aggregate) capital necessary for the production of the (aggregate) net national product, $Y_v^{isovalue}$. Particularly useful is the capital-labour ratio associated to the production of the same value of the aggregated net national product, i.e., an isoquant, which is given by:

(9) $$k_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min}(\cdot) | Y_v^{isovalue}) = \mathbf{e}' \mathbf{X}_{min}(\cdot)\mathbf{A}\, \mathbf{p}(r, \mathscr{E})/(\mathbf{e}' \mathbf{X}_{min}(\cdot)\mathbf{L})$$

Given a set of systems $\{\mathscr{E}_\alpha, \mathscr{E}_\beta, ..., \mathscr{E}_m\} = \mathbf{E}$, called an economy, and the corresponding set of wage functions $\{w(r, \mathscr{E}_\alpha), w(r, \mathscr{E}_\beta), ..., w(r, \mathscr{E}_m)\}$ the factor price frontier (**FPF**) of the economy is defined as the maximum wage rate associated to any given r.

(10) $$w_{FPF}(r, \mathbf{E}) = \max \{w(r, \mathscr{E}_\alpha), w(r, \mathscr{E}_\beta), ..., w(r, \mathscr{E}_m)\}$$

As the profit rate, r, changes, also the value of capital changes regardless of the state in which the system is in. For any couple (r, $w_{FPF}(r, \mathbf{E})$) the minimum value of the capital-labour ratio associated to the production of the constant aggregated net national product, $Y_v^{isovalue}$, notationally defined as $k_{vFPF}(r, \mathbf{E})$, is computed.

The above definitions specify a heterogeneous productive system in which different commodities are produced and at the same time link the dis-aggregated system with aggregate magnitudes. In relation (10) is imbedded an efficiency criterion which states that for any given rate of profit the wage rate per unit of labour is maximum. Relation (9) and (10) provide another efficiency criteria: for any given profit rate, r, an associated constant value of the net national product $Y_v^{isovalue}$ may be produced by employing the minimum quantity of the value of capital.

Given the above definitions the neoclassical case would be the one in which the generation of the same value of the net national product would require that the value of capital decreases as the profit rate increases. The **(Sato)Postulate A**, or 'Wicksell effect', requires that capital, $K_v(\mathbf{p}(r,\mathscr{E}), \mathbf{X}_{min}(\cdot) \mid Y_v^{isovalue})$, is negatively sloped with respect to the profit rate. In terms of the capital-labour ratio this requires that $\Delta k_v(\mathbf{p}(r,\mathscr{E}), \mathbf{X}_{min}(\cdot) \mid Y_v^{isovalue})/\Delta r \leq 0$. This last relation can be interpreted as a 'surrogate' isoquant relation consistent with modern aggregate and dis-aggregate neoclassical models.

The **(Sato)Postulate B**, or 'No-Reswitching', would require that for a given economy $\mathbf{E}=\{\mathscr{E}_\alpha, \mathscr{E}_\beta, ..., \mathscr{E}_m\}$ there is an ordering of connected intervals $[0,r_1[, [r_1, r_2[, ..., [r_{s-1}, r_s[$ and of the associated production systems $\{\mathscr{E}_{(1)}, \mathscr{E}_{(2)}, ..., \mathscr{E}_{(s)}\}$ such that for any z =1,2, ..., s and for any $r \in [r_{z-1}, r_z[$ it follows that $w(r,\mathscr{E}_{(z)})=\max\{w(r, \mathscr{E}_\alpha), w(r, \mathscr{E}_\beta), ..., w(r, \mathscr{E}_m)\}$ and for any $r \notin [r_{z-1}, r_z[$ it follows that $w(r, \mathscr{E}_{(z)}) \neq \max\{w(r, \mathscr{E}_\alpha), w(r, \mathscr{E}_\beta), ..., w(r, \mathscr{E}_m)\}$. This postulate captures the fact that once an economic system of production has been the most efficient in one interval it may not also be the most efficient in another disconnected interval[8].

The Samuelson (1962) 'surrogate' production function would assume the above postulates to hold, which, as we have discussed in the previous section, are not in general justified from the analytical point of view. In the past the question of whether the above postulates would hold in reality has been reduced to a simple matter of beliefs which should be confirmed by empirical

[8]Note that the above postulates do not depend on the specific production techniques (fixed versus flexible coefficients, constant or increasing returns to scale and so on). When we observe a state $\mathscr{E} = \{\mathbf{B},\mathbf{A},\mathbf{L},\eta\}$ nothing is assumed with respect to the production functions, but that quantities **XB**, **XA**, **XL**, are actually involved in the production process. The situation in which a certain degree of substitutability between the means of production is desirable, although not necessary for our purposes, can be tackled in two ways. The first one assumes inter industry substitutability: the whole composition between means of production and produced commodities might be conceptually changed by a re-proportioning, $\mathbf{X}_\omega$, of industries' shares. In this case the intra industry proportions between the means of production would not change. The second one gives the possibility of infra industry substitutability which is obtained by including additional systems to the original set of systems $\{\{\mathscr{E}_\alpha, \mathscr{E}_\beta, ..., \mathscr{E}_m\} \cap \{\mathscr{E}_a, \mathscr{E}_b, ..., \mathscr{E}_p\}\}$. Here the case of very small substitutability between the factors of production, which is the case of the traditional neoclassical aggregate production function of the Ramsey-Solow-Inada type, can be conceptually obtained through the 'ad libitum' augmentation of the set of economies. But this should normally not be necessary, since the production of units of commodities may occur only through the use of units of inputs. One should not forget that in the case of the traditional neoclassical aggregate production functions the infinitesimal substitutability between the factors of production is made for analytical convenience and it is itself an approximation of actual economic processes.

investigations[9]. Unfortunately, these empirical investigations have not been conducted so that Samuelson (1980, p.576) had to admit that "*The science of political economy has not yet the empirical knowledge to decide whether the real world is nearer to the idealised polar cases represented by (a) the neoclassical parable or (b) the simple reswitching paradigm*".

3 Simulating the economy.

The logical possibility that the neoclassical postulates do not hold does not exclude the possibility that they would hold in most cases, and this seems to be the most accepted view. In the following, with the purpose of investigating the likelihood that the neoclassical postulates would hold, an experimental approach is employed. The method is that of generating artificial economies and to compute the statistics of the results.

3.1 Description of the experiment.

Step 1. *Definition of the parameters.*

Number of commodities to be produced.

Seed for the random number generator.

Number of the methods of production.

Step 2. *Generation of a virtual economic production structure*

For each commodity i it is assumed that there are t_i ways to produce it. The collection of these methods of production will be called the technological set, T (the Joan Robinson book of blue prints). This technological set is generated using a random number generator and simple rules to guarantee that some systems of production are productive and that the generated physical surplus is bounded. Given the technological set it follows that the n commodities may be produced in $m=t_1t_2...t_i..t_n$ ways. Each of these ways would characterize one single system. The set of m systems, the economy $\mathbf{E} = \{\mathscr{E}_\alpha, \mathscr{E}_\beta, ..., \mathscr{E}_m\}$, is generated in this way.

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[9]Several authors and at different times have called for empirical investigations to help economist to make so-called learned guesses on whether the neo-classical postulates would be robust (See for example Samuelson, 1962, 1966, Fergusson, 1969, Sato, 1974, Cohen, 1993).

Step 3. *Computation of production prices and aggregated functions.*

For each system $\mathscr{E}_i$ the following values are computed: $w(r, \mathscr{E}_i)$, $\mathbf{p}(r, \mathscr{E}_i)$, $K_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\, Y_v^{isovalue})$, $k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\, Y_v^{isovalue})$.

Step 4. *Classification in accordance with neoclassical (Sato)Postulate A.*

Each individual system *i* is classified. It is defined to be 'neoclassical' and to hold for **(Sato)Postulate A** if the capital-labour ratio is constant or negatively related to the profit rate, i.e., if $dk_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\, Y_v^{isovalue})/dr \leq 0$ holds.

Step 5. *Classification in accordance with neoclassical (Sato)Postulate B.*

The wage functions are compared for all the possible pair wise combinations and the number of reswitches are recorded. If no-reswitching occurs, the systems do not violate **(Sato)Postulate B** and this result is recorded.

Step 6. *Factor price frontier.*

For the given set of systems, $\mathbf{E} = \{\mathscr{E}_\alpha, \mathscr{E}_\beta, ..., \mathscr{E}_m\}$, the factor price frontier $w_{FPF}(r, \mathbf{E})$ and the associated capital-labour ratio $k_{vFPF}(r, \mathbf{E})$ are computed.

Step 7. *Check on the 'neoclassical' properties for the whole economy.*

The whole economy **E**, in which all the combinations of the technological set *T* are considered, is defined as neoclassical if the frontier capital-labour ratio is non increasing with respect to the profit rate, i.e., when $\Delta k_{vFPF}(r, \mathbf{E})/\Delta r \leq 0$ for the whole range of *r*[10].

Step 8. *Collection of statistical information.*

The results associated with the above artificial economy are recorded. Return to step 1 will be repeated until a satisfying number of observations are collected.

[10]This condition is also known as the non capital reversing case. Note that the function $k_{vFPF}(r, \mathbf{E})$ may fail to be differentiable - and continuous - through the whole range of definition of *r*. It would be differentiable if the factor price frontier is. This would be the case in which there are no switch points, but this would be a restrictive case (Garegnani, 1970, pp.412-414). This explains the use of the standard difference operator, Δ. In practice the economy would still be defined as being neoclassical also at non differentiable or discontinuous points as long as $(k_{vFPF}(r + \Delta r, \mathbf{E}) - k_{vFPF}(r, \mathbf{E})) / \Delta r \leq 0$ through the whole range of *r*.

FIGURE 1. Factor price frontier: wage-profit curves

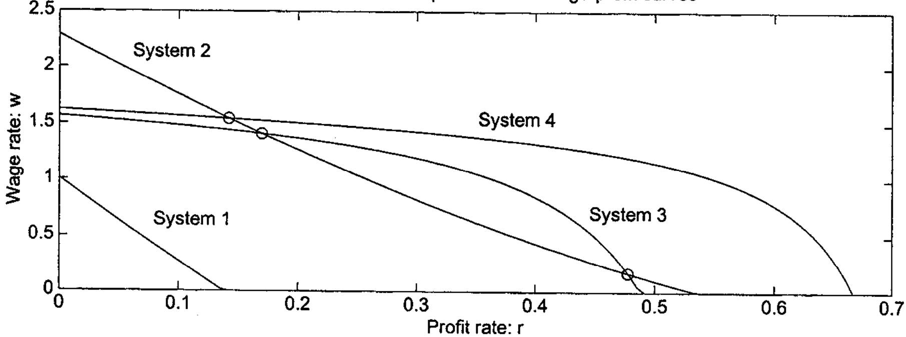
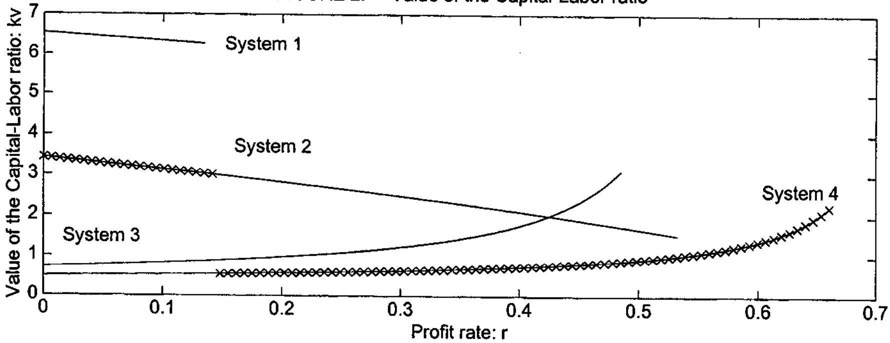


FIGURE 2. Value of the Capital-Labor ratio



3.2 **An example.**

Let's consider a system producing three commodities.

One hundred units of commodity 1 can be produced by 2 different production methods:

	good1	good2	good3	labour		output
method 1	35	15	25	10	→	100
method 2	15	1	3	50	→	100

One hundred units of commodity 2 can be produced by 2 different production methods:

	good1	good2	good3	labour		output
method 1	70	50	15	20	→	100
method 2	0	55	3	20	→	100

One hundred units of commodity 3 can be produced by one production method:

	good1	good2	good3	labour		output
method 1	10	10	55	20	→	100

The set of the above methods is the technological set T. Combining the above methods of production and a given, *numeraire* - for this example $\eta=[1, 0, 0]'$ - we have 4 possible production systems[11].

System 1: $\mathscr{E}_1 = \{\mathbf{A}_{(1,1,1)}, \mathbf{L}_{(1,1,1)}, \mathbf{B}_{(1,1,1)}, \eta\}$

	$A_{(1,1,1)}$			$L_{(1,1,1)}$	$B_{(1,1,1)}$	$X_{min,1}$
Industry 1	35	15	25	10	100	1.6538
Industry 2	70	50	15	20	100	1.1067
Industry 3	10	10	55	20	100	3.0577

System 2: $\mathscr{E}_2 = \{\mathbf{A}_{(1,2,1)}, \mathbf{L}_{(1,2,1)}, \mathbf{B}_{(1,2,1)}, \eta\}$

	$A_{(1,2,1)}$			$L_{(1,2,1)}$	$B_{(1,2,1)}$	$X_{min,2}$
Industry 1	35	15	25	10	100	0.0211
Industry 2	0	55	3	20	100	2.0340
Industry 3	10	10	55	20	100	0.1433

[11]The encoding (1,1,1) means that the first number identifies the metod used in the production of commodity 1 the second number the method used for the production of good 2 and so on. Therefore (1,2,1) means that method 1 is used for the production of good 1, method 2 for the production of good 2 and method 1 for the production of good 3.

System 3: $\mathscr{E}_3 = \{\mathbf{A}_{(2,1,1)}, \mathbf{L}_{(2,1,1)}, \mathbf{B}_{(2,1,1)}, \eta\}$

	$A_{(2,1,1)}$			$L_{(2,1,1)}$	$B_{(2,1,1)}$	$X_{min, 3}$
Industry 1	15	1	3	50	100	1.2252
Industry 2	70	50	15	20	100	0.0414
Industry 3	10	10	55	20	100	0.0946

System 4: $\mathscr{E}_4 = \{\mathbf{A}_{(2,2,1)}, \mathbf{L}_{(2,2,1)}, \mathbf{B}_{(2,2,1)}, \eta\}$

	$A_{(2,2,1)}$			$L_{(2,2,1)}$	$B_{(2,2,1)}$	$X_{min, 4}$
Industry 1	15	1	3	50	100	1.1883
Industry 2	0	55	03	20	100	0.0417
Industry 3	10	10	55	20	100	0.0791

The last column is the re-proportioning of each industry which is necessary for the production of a constant value of the net national product, $Y_v^{isovalue}$[12]. The above systems define the economy $\mathbf{E} = \{\mathscr{E}_1, \mathscr{E}_2, \mathscr{E}_3, \mathscr{E}_4\}$,

Having chosen a *numerairé*, η, and a value for the net national product, $Y_v^{isovalue}$, we are a position to compute the relevant functions associated to each system, i.e, $w(r, \mathscr{E}_i)$, $\mathbf{p}(r, \mathscr{E}_i)$, $K_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \mid Y_v^{isovalue})$, $k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \mid Y_v^{isovalue})$, where i=1, 2, 3, 4.

In Fig.1 the factor price frontier, $w(r, \mathscr{E}_i)$, associated to each system $\mathscr{E}_i$ is plotted. It is clear that reswitching takes place between system 2, $\mathscr{E}_2$, and system 3, $\mathscr{E}_3$. This case contradicts the **(Sato)Postulate B**, or 'No-Reswitching' case.

In Fig.2, the capital-labour ratios, $k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \mid Y_v^{isovalue})$, associated with each system $\mathscr{E}_i$, producing the net national product $Y_v^{isovalue}$, are reported. Clearly systems 1 and system 2 do not violate **(Sato)Postulate A**, or 'Wicksell effect' and are therefore consistent with the neoclassical theory of production. The other two systems are clearly non neoclassical because as the capital labor ratio increases, so does the profit rate. This result implies, moreover, that the violation of the neoclassical theory of production occurs also at the industry (firm) level: an upward-sloping capital-labor ratio for the whole system is possible only if at least one industry exhibits such behaviour.

The above four systems exhaust the technological possibilities. Some of the systems are inefficient. The wage rate of both systems 1 and 3 are dominated, for any profit rate r, by the

[12]The re-proportioning values are those associated to the zero profit rate. As the profit rate changes the re-proportioning necessary to produce the same constant value of the net national product changes (with the only exception occurring when the *numerairé* is the standard commodity, see Sraffa, 1960), but they do not change the relative production intensities. For this example the specific value of the constant net national product, $Y_v^{isovalue}$, is 100.

wage rates of either system 2 or 4, or both. It is therefore natural to evaluate the whole economy factor price frontier, $w_{FPF}(r, \mathbf{E})$ and the associated capital-labor ratio $k_{vFPF}(r, \mathbf{E})$. From figure 2 we see that the capital-labour ratio for the whole economy, the bolded line, is downward sloping as long as system 2 is dominant, but it is upward sloping when the system 4 is dominating. Therefore this economy clearly violates the neoclassical (SATO)postulate A and it would be classified in the experiments as a non neoclassical economy.

Clearly, with a different technological set and/or a different *numerairé*[13] we would have a different economy and possibly different qualitative results. It is for this reason that many artificial economies have been made and the results are reported below.

3.3 **Results.**

The computations reported above were made for different economies, each characterized by a different number of commodities and a different set of methods of production for each commodity (a total of 18000 economies, equivalent to 2268000 systems, was generated). The Appendix describes the results of the experiments in more detail.

After having constructed these artificial economies we are in a position to pose the following research questions and to show some results:

Research question a): **(Sato)Postulate A**, or 'Wicksell effect' - What is the likelihood that for a given artificially generated production system $\mathscr{E} = \{\mathbf{B},\mathbf{A},\mathbf{L},\eta\}$, the capital-labour ratio, $k_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min} \mid Y_v^{isovalue})$, is negatively sloped with respect to the profit rate?

Result a) The **(Sato)Postulate A** holds for 50% of the productive systems (see TABLE 2).

Research question b): **(Sato)Postulate A** for the whole economy - Given a set of systems, i.e., an economy **E**, what is the likelihood that the capital-labour ratio associated with the efficient frontier, $k_{vFPF}(r, \mathbf{E})$ is negatively sloped with respect to the profit rate?

Result b) The number of times in which a set of techniques is defined for

[13]Note that the results reported in this example, and in the simulations, depend on the specific choice of the *numerairé*. If we take a different *numerairé*, say $\eta=[1, 1, 1]'$, we would have different results. In this specific example the switch between system 2 and system 4 remains, but the reswitching between system 3 and system 2 disappears. Moreover each of the four systems exhibit neoclassical properties, i.e., $\Delta k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \mid Y_v^{isovalue})/\Delta r \leq 0$ for the whole domain and consequently the new economy is defined to be, contrary to the result reported in the text, as neoclassical.

being efficiently neoclassical, i.e., the number of times in which $\Delta k_{vFPF}(r, \mathbf{E})/\Delta r \leq 0$ for the whole domain, is below 40% (see TABLE 3).

Research question c): **(Sato)Postulate B**, or 'No-Reswitching' - What is the likelihood that reswitching does not takes place between one system, $\mathscr{E}_i$ and any other system belonging to the same economy?

Result c) The likelihood that reswitching does not occurs between the wage-profit frontier of system i, $w(r, \mathscr{E}_i)$, and any other alternative wage-profit frontier belonging to the same economy is above 97% (see TABLE5a).

Research question d): **(Sato)Postulate B**, or 'No-Reswitching' for the whole economy. What is the likelihood that an economic system will not generate a reswitching phenomenon?

Result d) The likelihood that a reswitching phenomenon is not generated at all by an economy is around 64% (see TABLE 5b). But as the number of the possible combinations of the methods of production increase the likelihood that the economy will not generate a re-switching phenomenon decreases substantially.

4. **Conclusion**

It is common practice to assume that the production level of an economic system may be described by an aggregate neoclassical production function. Technically, its theoretical existence requires stringent assumptions on the individual production functions. These assumptions, due to frequent repetition, have become widely accepted, but are supported neither by empirical nor by experimental investigations.

The present article attempts to assess the likelihood that the neoclassical postulates hold with respect to an artificially generated world has been made. For the artificial economies described here, the results indicate that there is a not-too-small world for which the neoclassical postulates, in particular the postulate A, do not hold. The mere existence of the re-switching phenomena raises a logical problem, but its occurrence might be, if not negligible (see results c and d above), then sporadic. A real difficulty for the tenability of the neoclassical system emerges when we consider that the likelihood for the value of the capital labor ratio to be negatively related with respect to the interest rate is only 40% (results b above).

A further difficulty emerges when it is realized that whether an economy can be defined to be neoclassic or not depends on the choice of the *numerairé*. One of the experiments shows that once an economy has been classified as neoclassical under a given *numerairé* it has only a little above 40% likelihood to be classified neoclassical again when a different *numerairé* is chosen (see TABLE 6.). This result shows that the slope of the capital-labor ratio associated to the efficient factor price frontier of the whole economy is not unambiguously determined, but relies on the specific choice of the observer.

It could be objected that the technological set, being originated randomly, lacks a realistic structure and hence is not relevant. Neoclassical and non neoclassical results could obviously be obtained through an *ad hoc* tailoring of an appropriate technological set[14]. This would amount to an *a priori* choice of postulates, but it is precisely this choice that has been questioned during the capital controversy and after.

Whether these simulations are satisfactory is clearly an open question, but the implications of the results for the neoclassical production functions should be taken seriously because *"it may well be that the (neo)classical theory represents the way in which we should like our economy to behave. But to assume that it actually does so is to assume our difficulties away (*Keynes, 1936, p.34*)"*.

[14] Champernowne (1953-4) and Samuelson (1962) showed, for example, how proper assumptions on the individual commodity production functions could lead towards a 'well-behaved' neoclassical production function..

APPENDIX Simulation results.

In this appendix a detailed description of the simulation and of the results is provided.

- For the simulations the number of commodities produced range from 3 to 8.
- The *numerairé* η is given by units of the first good.
- For each of the commodities, except the first, it is assumed that they can be produced in only 2 alternative ways. The first commodity can be produced from a minimum of 2 to a maximum of 4 different ways (techniques).
- For a given number of commodities constituting the system (from 3 to 8) and a given number of alternative techniques for commodity 1 (from 2 to 4) 1000 different set of techniques T have been generated (i.e, 18000 technological sets T or economies, **E**, and 2268000 systems of production $\mathscr{E}$ = {**B**,**A**,**L**,η}). The means of production used in the production of each commodity and the labour inputs are generated with a random value drawn from a uniform probability distribution on the unit interval. In order to assure that at least one of the randomly generated systems is self-replacing (and able to generate a surplus) the output is randomly determined to be greater than the means of production used by the whole system.
- Once the set of techniques T has been generated all the possible combinations of different techniques are combined to generate the tripples (**B**,**A**,**L**). (In the case in which only three commodities and 2 alternative techniques for commodity one are defined the number of possible triples is $2^3=8$ and in the case in which the commodities are 8 the number of possible triples is $2^8=256$. In the case in which the alternatives for the first techniques are 4 the maximum number of possible triples is $4*2^7=512$).
- Each triple (**B**,**A**,**L**) is re-proportioned so that the amount of labour used for the whole system for production is equal to 1 and the aggregate value of capital used for production is minimized.

  This means that for any production system $\mathscr{E}$ = {**B**,**A**,**L**,η} a re-proportioning matrix $\mathbf{X}_{min}$ is found so that $K_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min})$ is minimized under the condition that X>0 and **e'XL**=1. The algorithm used for the determination of the re-proportioning matrix is similar to the stochastic approximation procedure. First the potential contraction or expansion of each individual industry is computed, given the existent surplus structure of the whole economy. The system is randomly re-proportioned inside these feasible bands of contraction and expansion. Subsequently a re-scaling of the system is made to assure that **e'XL**=1. The new re-proportioning matrix is kept if the new capital value is less than the previous one. This procedure is repeated until the expected reduction of the value of capital is less than a certain (very low) threshold value.
- For each new re-proportioned triple, defining the ith production system $\mathscr{E}_i$ the following values are computed: $w(r, \mathscr{E}_i)$, $\mathbf{p}(r, \mathscr{E}_i)$, $K_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\, Y_v^{isovalue})$, $k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\, Y_v^{isovalue})$.
- Each individual economy is subsequently classified. It is defined as 'neoclassical' and as holding for **(Sato)Postulate A** if the capital-labour ratio is constant or negatively related to the profit rate, i.e., if $dk_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min} \,|\, Y_v^{isovalue})/dr \leq 0$ holds.
- The wage-profit functions are compared for all the possible pair-wise combinations of economies and the number of re-switches are recorded. If no-reswitching occurs, the two economies do not violate

(Sato)Postulate B and this result is recorded.

- For the given the set of production systems the factor price frontier $w_{FPF}(r, \mathbf{E})$ and the associated capital-labour ratio $k_{vFPF}(\mathrm{r}, \mathbf{E})$ are computed. The technological set T is defined as neoclassical, that is capital reversing does not occur, if the frontier capital-labour ratio is non increasing with respect to the profit rate (i.e., when $dk_{vFPF}(\mathrm{r}, \mathbf{E})/dr \leq 0$).

The results.

In the above simulation a total of 18000 technological sets T, economies **E**, have been generated. A technological set is characterized by the collection of alternative ways in which a commodity may be produced. It is assumed that the systems are characterized by the production of commodities from 3 to 8 and that each commodity can be produced in two alternative ways, but good one may be produced in 2, 3 or 4 alternative ways. The table below reports the number of all possible combinations of these alternative ways that lead to the definition of different alternative productive systems.

The columns define the number of commodities, while the rows the number of alternative ways in which good 1 can be produced.

Number of commodities.

Alternative methods of production for commodity 1	**3**	**4**	**5**	**6**	**7**	**8**
2	*8*	*16*	*32*	*64*	*128*	*254*
3	*12*	*24*	*48*	*96*	*192*	*384*
4	*16*	*32*	*64*	*128*	*256*	*512*

Total number of systems associated to each seed for the random number generator is **2268**. The values for 1000 different seeds have been computed for a total of **2268000** possible economies.

It is obvious that as the number of commodities increases (or the number of alternative techniques increase) the number of possible productive systems grows exponentially. The parameters for our simulations are the random seed, the number of alternatives ways in which good one is produced and the number of commodities.

To give an example. The case of 3 alternative ways and 7 commodities means that 192 possible production systems define the economy. For each of these 192 systems the following values, relevant for our purposes, are computed: the wage-profit function $w(r, \mathscr{E}_i)$; the re-proportioning matrix, $\mathbf{X}_{min}$; the value of capital-labor ratio as a function of the profit rate, $k_v(\mathbf{p}(r, \mathscr{E}_i), \mathbf{X}_{min} \,|\mathrm{Y}_v^{isovalue})$. Of these 192 economies not all of them are productive (vital). For the productive systems the number of times for which $dk_v(\mathbf{p}(r, \mathscr{E}), \mathbf{X}_{min} \,|\mathrm{Y}_v^{isovalue})/dr \leq 0$ is computed.

Moreover the 192 economies are compared pair-wise to see whether switching or re-switching takes place. This means that 18336 comparisons of wage-profit functions are made (i.e., 192*(192-1)/2).

For the whole economy, the factor price frontier , $w_{FPF}(r, \mathbf{E})$, and the associated capital value $k_{vFPF}(r, \mathbf{E})$ able to generate a constant net national product ($Y_v^{isovalue}$) are computed.

Below, the average over 1000 different simulations is reported.

Table 1. Vital systems.

The ratio of the number of vital system over the totally generated ones.

Number of commodities:	3	4	5	6	7	8
2	0.7150	0.7288	0.7419	0.7817	0.8116	0.8394
3	0.6789	0.7001	0.7275	0.7623	0.7983	0.8300
4	0.6650	0.6878	0.7167	0.7555	0.7924	0.8264

Total average: 0.7533

Table 2. Neoclassical productive systems: **(Sato)postulate A**

Ratio of the number of times in which $dk_v(p(r, \mathcal{E}), X_{min}/Y_v^{isovalue})/dr \leq 0$ over the total number of productive systems.

Number of commodities:	3	4	5	6	7	8
2	0.5034	0.4999	0.5198	0.5088	0.4979	0.5051
3	0.4988	0.4982	0.5134	0.5052	0.4984	0.4983
4	0.5014	0.4997	0.5110	0.5012	0.5010	0.4943

Average: 0.5031

Table 3. Aggregate behaviour. Neoclassical economies: **(Sato)postulate A.**

Ratio between the number of economies for which a negative relationship between the value of capital, $\Delta k_{vFPF}(r, E)/\Delta r \leq 0$, and the profit rate does hold and the number of economies.

Number of commodities:	3	4	5	6	7	8
2	0.4330	0.4200	0.4400	0.4480	0.3980	0.4080
3	0.3750	0.3920	0.4040	0.3930	0.3800	0.3810
4	0.3400	0.3770	0.3720	0.3910	0.3690	0.3640

Average: 0.3936

Table 4. Switches.

Ratio of the number of times in which the wage function of a productive system, $w(r, \mathscr{E}_i)$, intersects the wage function of another productive system belonging to the same economy over the total number of productive systems.

Number of commodities:	3	4	5	6	7	8
2	0.3317	0.4784	0.6045	0.7022	0.7658	0.8126
3	0.4150	0.5382	0.6393	0.7128	0.7687	0.8111
4	0.4594	0.5717	0.6529	0.7182	0.7694	0.8109

Average: 0.6424

Table 5a. Re-switches: **(Sato)postulate B.**

Ratio of the number of times in which the wage function of a productive system, $w(r, \mathscr{E}_i)$, intersects twice the wage function of another productive system belonging to the same economy over the total number of productive systems.

Number of commodities:	3	4	5	6	7	8
2	0.9988	0.9978	0.9960	0.9919	0.9822	0.9628
3	0.9984	0.9950	0.9912	0.9828	0.9606	0.9329
4	0.9959	0.9927	0.9853	0.9711	0.9444	0.9031

Average: 0.9768

Table 5b. Re-switches: **(Sato)postulate B.**

Ratio of the number of economies in which the re-switching phenomenon does not appear over total number of economies.

Number of commodities:	3	4	5	6	7	8
2	0.9950	0.9850	0.9450	0.8290	0.6310	0.3540
3	0.9910	0.9450	0.8340	0.6140	0.2920	0.0920
4	0.9720	0.9000	0.7100	0.3870	0.1200	0.0240

Average: 0.6456

Table 6. Changes of the Numerairé: Aggregate neoclassical economies: **(Sato)poastulate A.**

The experiments have been conducted also with a different numerairé (the unit vector). In this table the ratio between the economies that could be classified neoclassical in both systems over the economies previously classified neoclassical is reported (see TABLE 3.).

Number of commodities:	**3**	**4**	**5**	**6**	**7**	**8**
	=====	=====	=====	=====	=====	=====
2	0.5497	0.4881	0.4659	0.4375	0.4146	0.3652
3	0.5120	0.4643	0.4134	0.4224	0.3816	0.3858
4	0.4794	0.4668	0.4167	0.3836	0.3794	0.4066

Average: 0.4352

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