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Load Estimation by Frequency Domain Decomposition

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LOAD ESTIMATION
BY
FREQUENCY DOMAIN DECOMPOSITION

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Abstract

When performing operational modal analysis the dynamic loading is unknown, however, once the modal properties of the structure have been estimated, the transfer matrix can be obtained, and the loading can be estimated by inverse filtering. In this paper loads in frequency domain are estimated by analysis of simulated responses of a 4 DOF system, for which the exact modal parameters are known. This estimation approach entails modal identification of the natural eigenfrequencies, mode shapes and damping ratios by the frequency domain decomposition technique. Scaled mode shapes are determined by use of the mass change method. The problem of inverting the often singular or nearly singular transfer function matrix is solved by the singular value decomposition technique using a limited number of singular values. The dependence of the eigenfrequencies on the accuracy of the scaling factors is investigated and the errors on the estimated loads are determined.

Nomenclature

Estimated natural eigenfrequency	$\hat{f}$	Estimated damping ratio	$\hat{\zeta}$
Estimated unscaled mode shape matrix	$\hat{\mathbf{\Phi}}$	Scaling factor diagonal matrix	$\hat{\boldsymbol{\alpha}}$
Estimated scaled mode shape matrix	$\hat{\mathbf{\Psi}}$	Frequency response function (FRF)	$\hat{\mathbf{H}}(\omega)$
Simulated response	$\mathbf{Y}(\omega)$	Re-estimated load	$\hat{\mathbf{X}}(\omega)$
Correlation coefficient function	ρ_{YY}	Sampling frequency	f_S
Frequency resolution	Δf	Power spectral density	$S_{XX}(\omega)$
Mean value	μ	Coefficient of variation	δ

1 Introduction

Operational Modal Analysis (OMA) is mainly concentrated on analysis of measured responses. The input load is thus in far the most cases unknown. This unmeasured input is, however, of great interest since valuable information on the magnitude and distribution of such ambient excitations as wind, wave and traffic loads etc. can be estimated. The process of estimating input loads from measured responses involves at first identification of the modal parameters ω, $\mathbf{\Phi}$ and ζ, from which the frequency response function (FRF) can be formulated, see (2) and (3). Scaled mode shapes $\mathbf{\Psi}$ are in this relationship needed in order to estimate the FRF correctly, see (4).

The modal parameters are identified with the frequency domain decomposition (FDD) technique, see [1], while the scaled mode shapes are determined by use of the mass change method, see [2]. The governing equations are:

$$\hat{\mathbf{X}}(\omega) = \hat{\mathbf{H}}(\omega)^{-1}\,\mathbf{Y}(\omega) \tag{1}$$

where the FRF is calculated by:

$$\hat{\mathbf{H}}(\omega) = \begin{bmatrix} \underset{n\times 1}{\hat{\psi}^{(1)}} & \cdots & \underset{n\times 1}{\hat{\psi}^{(m)}} \end{bmatrix} \begin{bmatrix} H_1(\omega) & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & H_m(\omega) \end{bmatrix} \begin{bmatrix} \underset{n\times 1}{\hat{\psi}^{(1)}} & \cdots & \underset{n\times 1}{\hat{\psi}^{(m)}} \end{bmatrix}^H \tag{2}$$

$$H_k(\omega) = \frac{1}{-\omega^2 + 2i\hat{\zeta}_k\hat{\omega}_k\omega + \hat{\omega}_k^{\,2}}, \quad k \in [1;m] \tag{3}$$

The scaling factor α, which here is formulated on diagonal matrix form, is defined by [2]:

$$\hat{\alpha}_{kk} = \sqrt{\frac{\left(\hat{\omega}_k^0\right)^2 - \left(\hat{\omega}_k^1\right)^2}{\left(\hat{\omega}_k^1\right)^2\left(\hat{\varphi}_k^0\right)\Delta\mathbf{M}\left(\hat{\varphi}_k^1\right)^H}} \Rightarrow \hat{\mathbf{\Psi}} = \hat{\mathbf{\Phi}}\hat{\boldsymbol{\alpha}} \tag{4}$$

The mass change method is based on repeated eigenvibration tests performed prior and post to structural modifications of the structure being considered. The structural modification is performed by applying additional masses to the structure, which result in changed eigenfrequencies, and possibly changed mode shapes. The superscript 0 indicates modal parameters of the unmodified structure, while superscript 1 indicates modal parameters of the modified structure. $\Delta\mathbf{M}$ is the deviation mass matrix between the unmodified and modified structure. [2] suggests a homogenous mass change of 5% of the total mass.

The load estimation procedure involves furthermore an inversion of the FRF, see (1). Problems with this inversion are solved by use of the singular value decomposition (SVD) technique, since the FRF matrix is often singular or close to singular. In this case the inversion is performed by the following actions [3]:

$$\hat{\mathbf{H}}(\omega) = \mathbf{U}\,\mathbf{S}(\omega)\,\mathbf{V}^H \tag{5}$$

$$\hat{\mathbf{H}}(\omega)^{-1} = \mathbf{V}\begin{bmatrix} \mathbf{D}(\omega)^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\mathbf{U}^H \tag{6}$$

where **U**, **S** and **V** are matrices obtained from the singular value decomposition of the FRF. **D** is the non-singular part of **S**. (6) is the also referred to as the Moore-Penrose pseudo inverse.

The results presented in this paper are based on simulated responses, for which the input loads are known. In this way the re-estimated loads can be compared with the exact input loads. The modal parameters are identified from the simulated responses, and by use of the scaling factors and the modal parameters, the FRF is estimated, see (2) and (3). An estimation overview is given in Figure 1.

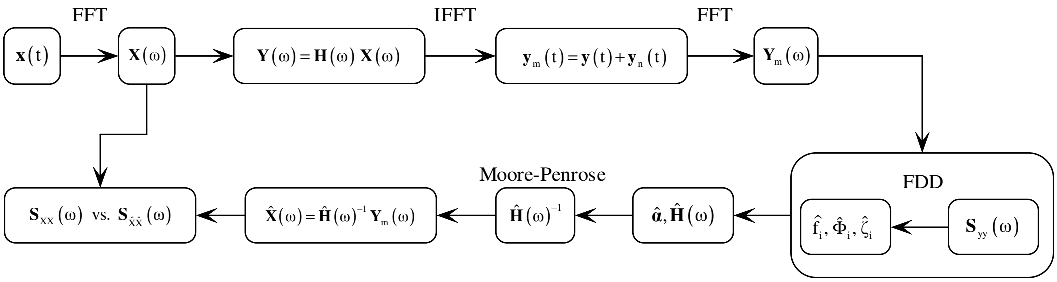

Figure 1. Estimation overview. y_n is additional noise added to the generated response.

The system on which the responses are simulated is a 4 DOF system, which is shown in Figure 2.

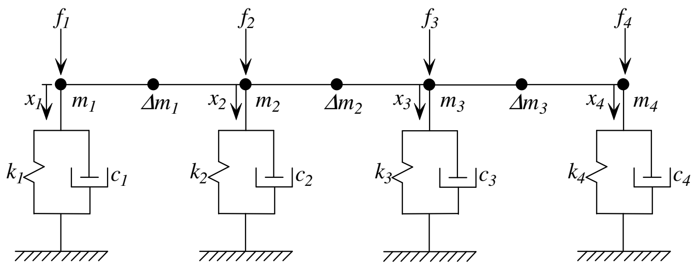

Figure 2. 4 DOF system for load estimation analysis.

Sample records of two different lengths are simulated; the second one ten times longer than the first one. The simulated responses are split up in 100 and 1000 data segments, respectively (no windowing has been used). The input load is a Gaussian white noise load generated segment wise in MATLAB. This eliminates the effect of leakage, which otherwise would occur when no windowing is used. The sampling frequency f_s is chosen to 0.512 Hz. while the frequency resolution Δf is equal to 0.001 Hz. This gives 512 frequency lines in frequency domain.

The following load estimation analysis is partly concentrated on the influence of the estimated eigenfrequencies on the accuracy of the scaling factors, and partly on the error on the re-estimated loads compared to the exact loads (distinctions are made between the exact loads, the realized loads and the re-estimated loads). For this purpose 100 simulations are performed for each single calculation in order to obtain a representative sample quantity. The load comparison is made upon the spectral densities in a bandwidth of a 1/6 decade with 1/3 of the Nyquist frequency f_v as the centre frequency. The error ε is thus defined by:

$$\varepsilon\left(S_{XX}^{exact}(\omega),S_{\hat{X}\hat{X}}(\omega)\right)=100\cdot\left(\frac{\left|\int_{\frac{\frac{1}{3}f_\nu}{\beta}}^{\frac{1}{3}f_\nu\beta} S_{XX}^{exact}(\omega)\,d\omega-\int_{\frac{\frac{1}{3}f_\nu}{\beta}}^{\frac{1}{3}f_\nu\beta} S_{\hat{X}\hat{X}}(\omega)\,d\omega\right|}{\int_{\frac{\frac{1}{3}f_\nu}{\beta}}^{\frac{1}{3}f_\nu\beta} S_{XX}^{exact}(\omega)\,d\omega}\right)[\%]\ ,\qquad \begin{array}{l}\beta=10^{\frac{1}{12}}\\ f_\nu=\frac{f_S}{2}\end{array} \tag{7}$$

In Figure 4, Figure 5 and Figure 6 the centre frequency and the band width are shown, respectively. The centre frequency is chosen in the centre of the most modal dominated area, as it is seen from Figure 4.

2 Enhanced Eigenfrequency Estimation

From preliminary studies it has been concluded that the accuracy of the scaling factor is strongly dependent on the accuracy on the estimated eigenfrequencies, and to a lesser extent on the estimated mode shapes. The accuracy of the peak picked eigenfrequencies is dependent on the frequency resolution Δf, which means that long measurements might be required in order to obtain a satisfactory resolution of the frequency axis. The estimated auto correlation coefficient function ρ_{YY}, which is also used for damping estimation [4], may alternatively be used to perform an enhanced eigenfrequency estimation without increasing the frequency resolution.

At first, all positive and negative extreme values of ρ_{YY} are identified, as it is also the case when estimating damping ratios [4]. Next, all crossings with the time decay axis are identified. A linear regression of all the identified points of time provides an estimate of the damped eigenperiod T_d, see Figure 3. The enhanced estimate of the eigenfrequency is finally determined upon the following well-known relationship:

$$\hat{\omega}_0=\frac{2\pi}{\hat{T}_d\sqrt{1-\hat{\zeta}^2}} \tag{8}$$

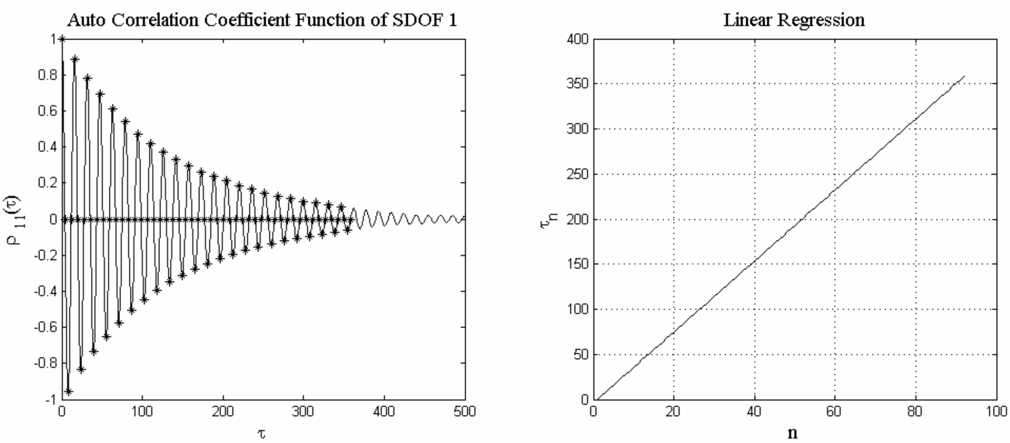

Figure 3. Extreme values and zero crossings of the estimated auto correlation coefficient function for enhanced eigenfrequency estimation.

For the 4 DOF system in Figure 2 the following exact, peak picked and enhanced eigenfrequencies are determined. For the enhanced eigenfrequencies it is the mean values μ and coefficients of variation δ over 100 simulations that are shown. Mean values and coefficients of variation of the errors on the corresponding scaling factors are shown in Table 2.

Table 1. Exact, peak picked and enhanced eigenfrequencies. μ = mean value of enhanced eigenfrequency, δ = coefficient of variation of enhanced eigenfrequency [%].

μ and δ **[%]** on estimated and enhanced $\hat{f}_i$ [Hz]				
f_i / $\hat{f}_i$	Exact f_i [Hz]	Peak picked $\hat{f}_i$ [Hz]	Enhanced $\hat{f}_i$ [Hz] (100 seg.)	Enhanced $\hat{f}_i$ [Hz] (1000 seg.)
i = 1	0.06357	0.06400	μ = 0.06359 / δ = 0.3	μ = 0.06358 / δ = 0.1
i = 2	0.07503	0.07500	μ = 0.07504 / δ = 0.3	μ = 0.07502 / δ = 0.1
i = 3	0.09629	0.09600	μ = 0.09632 / δ = 0.3	μ = 0.09628 / δ = 0.1
i = 4	0.11254	0.11200	μ = 0.11250 / δ = 0.3	μ = 0.11255 / δ = 0.1

Table 2. Mean values and coefficients of variation of the errors on the estimated scaling factors in accordance with the peak picked and enhanced eigenfrequencies, respectively.

μ and δ **[%]** on error of $\hat{\alpha}_{ii}\left(\hat{f}_i,\hat{\varphi}_i\right)$ [$mass^{-\frac{1}{2}}$] for mass changes of 2% and 5%, respectively.							
$\hat{\varphi}_i$		Peak picked $\hat{f}_i$ [Hz]		Enhanced $\hat{f}_i$ [Hz] (100 seg.)		Enhanced $\hat{f}_i$ [Hz] (1000 seg.)	
	α_{exact}	$\hat{\alpha}_{2\%\Delta M}$	$\hat{\alpha}_{5\%\Delta M}$	$\hat{\alpha}_{2\%\Delta M}$	$\hat{\alpha}_{5\%\Delta M}$	$\hat{\alpha}_{2\%\Delta M}$	$\hat{\alpha}_{5\%\Delta M}$
i = 1	0.3994	μ = 55.9 δ = 64.7	μ = 16.5 δ = 14.3	μ = 14.5 δ = 95.6	μ = 6.0 δ = 83.3	μ = 6.0 δ = 62.0	μ = 2.1 δ = 74.7
i = 2	0.4714	μ = 28.8 δ = 101.6	μ = 8.2 δ = 87.1	μ = 16.1 δ = 76.6	μ = 5.6 δ = 82.5	μ = 5.0 δ = 70.7	μ = 1.9 δ = 81.5
i = 3	0.6050	μ = 46.9 δ = 100.1	μ = 14.5 δ = 89.5	μ = 17.8 δ = 70.1	μ = 6.7 δ = 78.3	μ = 5.0 δ = 70.5	μ = 2.1 δ = 73.3
i = 4	0.7071	μ = 57.2 δ = 81.4	μ = 16.0 δ = 75.3	μ = 17.9 δ = 86.0	μ = 6.7 δ = 70.9	μ = 4.5 δ = 77.7	μ = 2.6 δ = 73.9

The accuracy of the eigenfrequencies is very decisive for the error on the scaling factor, which is seen from Table 1 and Table 2. In the end, the magnitude of the FRF is very dependent on the scaling factor, which again is decisive for the best possible re-estimate of the load. It is furthermore seen from Table 2 that the best results are obtained with a mass change of 5% and 1000 segments in preference to 100 segments.

3 Load Estimation

In the overall perspective, three different cases are present in reference to (2). In the first case the response is known in all four degrees of freedom and four mode shapes are known. This means that m = n in (2). In practice, however, one may often have either more responses than mode shapes (m < n), or more mode shapes than responses (m > n). This load estimation analysis is however limited to the first case, i.e. 4 responses and 4 mode shapes.

The load estimation analyses are performed with scaling factors determined with 5% mass changes upon the conclusions drawn from Table 2. Calculations are performed with no additional noise, 1% noise and 5% noise added to the generated response, respectively. The results, presented in the form of the errors defined by (7), are shown in Table 3 and Table 4.

Table 3. Mean values and coefficients of variation of the errors on the spectral densities of the re-estimated loads.

μ and δ **[%]** on error of $S_{\hat{X}\hat{X}}(\omega)$ [N^2/Hz] (100 seg.)					
DOF i	$S_{XX}^{exact}(\omega)$	$S_{XX}^{realized}(\omega)$	Without noise	1% noise	5% noise
i = 1	4882.8	4881.1	μ = 14.5 δ = 97.1	μ = 17.0 δ = 91.3	μ = 18.9 δ = 107.9
i = 2	4882.8	4871.0	μ = 17.2 δ = 83.5	μ = 17.6 δ = 96.4	μ = 20.5 δ = 106.8
i = 3	4882.8	4871.7	μ = 16.3 δ = 86.1	μ = 18.0 δ = 96.0	μ = 20.8 δ = 104.6
i = 4	4882.8	4887.6	μ = 14.5 δ = 93.7	μ = 16.5 δ =92.7	μ = 20.4 δ = 107.8

Table 4. Mean values and coefficients of variation of the errors on the spectral densities of the re-estimated loads.

μ and δ **[%]** on error of $S_{\hat{X}\hat{X}}(\omega)$ [N^2/Hz] (1000 seg.)					
DOF i	$S_{XX}^{exact}(\omega)$	$S_{XX}^{realized}(\omega)$	Without noise	1% noise	5% noise
i = 1	4882.8	4884.1	μ = 4.7 δ = 77.7	μ = 4.8 δ = 76.7	μ = 5.8 δ = 76.0
i = 2	4882.8	4883.5	μ = 4.8 δ = 72.0	μ = 5.1 δ = 79.7	μ = 5.6 δ = 78.6
i = 3	4882.8	4883.5	μ = 4.9 δ = 74.2	μ = 5.3 δ = 77.2	μ = 5.6 δ = 82.0
i = 4	4882.8	4884.5	μ = 5.2 δ = 70.6	μ = 5.1 δ = 77.5	μ = 5.7 δ = 79.3

The difference between the exact spectral densities and the realized spectral densities is less than 0.5%. This error is due to the fact that the measurement series have finite lengths, and the error must thus be conceived as a random error. The errors on the estimated spectral densities are conceived as random errors too, since the estimated spectral densities converge towards the realized spectral densities as the number of data segments increases.

In Figure 5 and Figure 6 examples of the realized and re-estimated loads are shown for the two cases (100 segments / 1000 segments).

4 Conclusion

It is seen from Table 2 that the error on the scaling factor spans from 1.9% to 57.2%. The best result is obtained by splitting up of the long measurement in 1000 segments, enhanced eigenfrequencies and a homogeneous mass change of 5%, while the most inaccurate results is obtained with the peak picked eigenfrequencies and a mass change of 2%. Generally seen, the best results are obtained with a 5% mass change and by use of the response averaged over 1000 segments.

The errors on the spectral densities of the loads span from 4.8% to 20.8%. Again, the best result is obtained with 1000 segments. The additional noise has only a little influence on the end result, even with 5% noise added. Since the centre frequency for the investigated frequency band is centred in the modal dominated area the additional added noise constitute a limited part of the total response. The errors on the estimated spectral densities are larger in the areas, where the noise is somewhat more dominating compared to the original generated response.

The errors on the estimated spectral densities seem to converge towards zero as the number of data segments increases. Whether the estimate is biased or unbiased can, however, not be answered without performing calculations with longer measurements and more data segments. When estimating the loads using the exact modal parameters, the errors are equal to zero, which indicates that the estimate is unbiased.

5 References

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[4] Brincker, R., Ventura, C. E., and Andersen, P., ”Damping Estimation by Frequency Domain Decomposition”, Proc. of the 19th International Modal Analysis Conference

6 Graphics

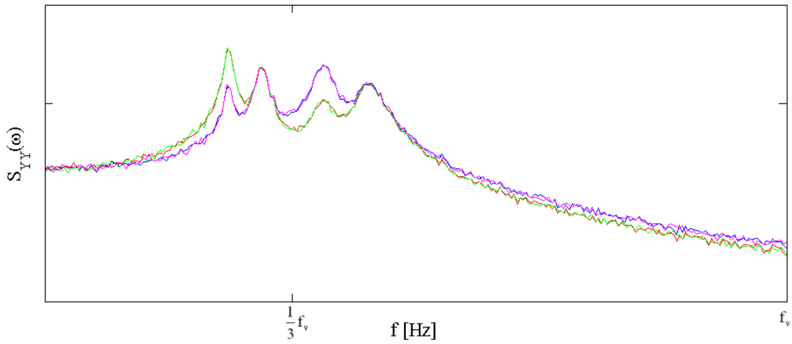

Figure 4. Spectral density $S_{YY}(\omega)$ of response. Note the centre frequency 1/3 f_v.

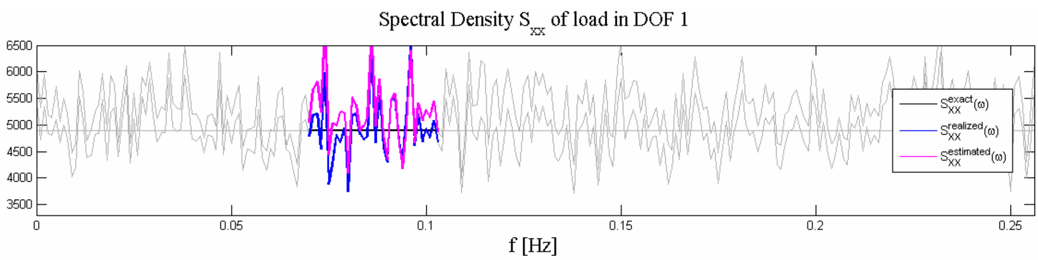

Figure 5. Example of realized and estimated spectral density $S_{XX}(\omega)$ of load for 100 segments.

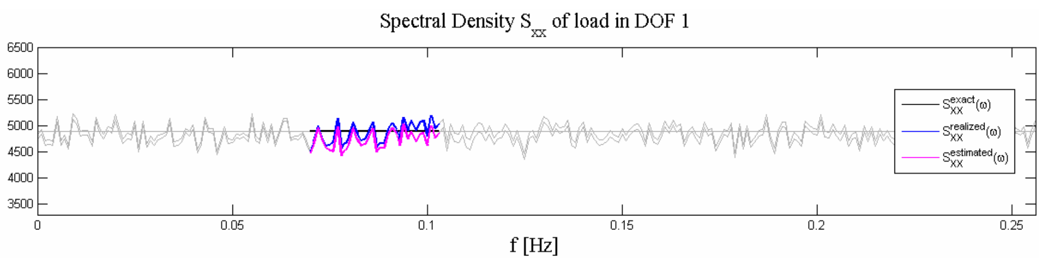

Figure 6. Example of realized and estimated spectral density $S_{XX}(\omega)$ of load for 1000 segments.