Stochastic Analysis of the Multi-dimensional Effect of Chloride Ingress into Reinforced Concrete

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Stochastic analysis of the multi-dimensional effect of chloride ingress into reinforced concrete

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ABSTRACT: For many reinforced concrete structures corrosion of the reinforcement is an important problem since it can result in expensive maintenance and repair actions. One mode of corrosion initiation occurs when the chloride content around the reinforcement bars exceeds a critical threshold value, which is the subject of the present paper. Typically, the chloride concentration is obtained by solving Fick's diffusion partial differential equation assuming chloride flow into an infinite half space. However, when the concrete structure is relatively thin or when a rebar is situated at a corner of the structure, the assumption seems to be inaccurate. In this study, comparisons are made between analytical models based on infinite domains and series expansion solutions as well as numerical models based on FEM (Finite Element Method). As the parameters governing the problem are random in nature, MCS (Monte Carlo Simulation) is applied to find the probability distributions for the time for initialization of corrosion using the different methods. Comparisons are made between treating the random quantities as random variables or as spatial and temporal distributed random fields. The random fields are modeled by the EOLE (Expansion Optimum Linear Estimation) approach. A bridge pier in a marine environment is considered to exemplify the results.

1 INTRODUCTION

The present work address chloride-ingress into reinforced concrete structures which is one of the most common destructive mechanisms. The most typical type of chloride initiated corrosion is pitting corrosion which may locally cause a substantial reduction of the cross-sectional area and cause maintenance and repair actions which can be very costly.

The owners of the structures, therefore, have to plan and perform measurements which can be used as a basis for an assessment of the structures and as a basis for deciding which repair and maintenance strategy should be applied. The optimal decisions about the experimental plan and the maintenance methods can be made on the basis of modern structural reliability theory together with economic decision theory, see e.g. Raiffa & Schlaifer (1961), Sørensen & Engelund (1999) and Madsen et al. (1993). This paper will address the structural reliability part of the problem.

The chloride ingress into the structure can be modeled by Fick's law of diffusion, see Crank (1975). In many applications, the diffusion problem is only considered in one spatial dimension, i.e. it is assumed that the rebar is positioned at a given distance from the surface of an infinite half space and an analytical expression for the chloride concentration is available, see Crank (1975). However, in real life structures, the rebar is typical positioned in beam or column cross sections, where not only the flow of chloride is two- or even three dimensional but the cross sections are so small that the flow of chloride from surfaces positioned opposite from each other may be significant.

The multidirectional effect of chloride flow can be investigated by applying both analytical and numerical modeling, which is the subject of the present paper.

Four different analytical solutions to the diffusion problem are presented. The solution based on an infinite half space together with its extension to the situation where a corner in an infinite domain is investigated is presented. These solutions are appropriate for relatively thick structures where the flow of chloride from neighboring surfaces is insignificant. However, when this effect is investigated, the flow of chloride can be modeled by series expansion solutions, based on the technique of separation of variables, se e.g. Edwards & Penney (1993). The series expansion solution can be applied both in the case of a one dimensional slab with two neighboring parallel surfaces and for the two dimensional case with a rectangular domain. Thus, the series solutions can be regarded as exact, if the parameters governing the problem, i.e. diffusion coefficient and chloride concentration are constant both in time and space.

If this is not the case, one might turn to numerical modeling, which can also be applied for structures with irregular geometry, e.g. T-beams. In this work, FEM (the Finite Element Method) is used to find the distribution of chloride over time, see Cook et al. (1989).

In order to exemplify the differences between the five methods, the chloride distributions within a thin and a thick quadratic concrete cross section are considered, assuming that the diffusion coefficient and the boundary chloride concentration are deterministic and constant in both time and space.

However, the chloride boundary concentration, the diffusion coefficient, etc. are all varying randomly, and failure can be defined when the chloride concentration at the rebar reaches a critical value corresponding to the time when the rebar begin to corrode. Probabilistic analysis of the time to initiation of corrosion in concrete structures has been treated by a number of researchers; see e.g. Engelund & Sørensen (1998) and Estes & Frangopol (2000). Typically, the closed form analytical solution based on an infinite half space is applied in a random variable model to obtain the probability of exceeding the critical concentration of chloride, by representing the input properties as random variables.

In this work, the Monte Carlo simulation technique is used to generate statistical distribution functions for the time of exceeding the critical chloride concentration, see Hammersley & Handscomb (1964). A bridge pier in a marine environment is investigated, and the probability distributions for the time for initialization of corrosion are obtained based on the different analytical solutions to the diffusion problem. The diffusion coefficient, the chloride surface concentration, the critical level of chloride concentration and the position of the reinforcement bars are all modeled as Log-normal random variables. Log-normal distributed parameters are used to avoid unphysical negative realizations of the random properties. Results are obtained and compared for rebars positioned at the corner of the cross section and in the middle of a surface side, together with the distribution corresponding to the first of a total of eight rebars reaching initialization of corrosion.

However, in real life, the governing parameters are varying both in time and space, suggesting that they should be modeled as multidimensional correlated random fields. As noted in Stewart (2004), correlation of pitting corrosion at different locations highly influences the resulting probability of corrosion and thus the ultimate structural reliability. As suggested in Stewart & Faber (2003) stochastic field modeling should be employed to better model the spatial variability of the corrosion problem.

In an earlier work by the authors, the diffusion coefficient and the chloride boundary concentration were modeled as Gaussian random fields, see Frier & Sørensen (2005). However, in this work, the random field approach is compared with the simpler random variable solutions in order to investigate the necessity of applying a random field model. The random fields are modeled by the EOLE (Expansion Optimum Linear Expansion) method, Li & Der Kiureghian (1993).

As the analytical solutions to the diffusion problem are derived assuming constant diffusion coefficient and boundary chloride concentration they can not be applied in cases where these properties are varying within the domain and one have to turn to numerical modeling, e.g. in the random field approach only the FEM solution is considered.

Both spatial and temporal correlation of the random fields is considered. In the case of spatial correlation, the diffusion coefficient is modeled as a two dimensional spatial distributed random field, whereas the boundary chloride concentration is modeled as a one-dimensional distributed random field over the surface parameter of the cross-section. In the temporal correlated case, both fields are modeled as one-dimensional fields over time.

The random fields are considered to be Log-normal distributed. The remaining parameters are modeled as Log-normal distributed random variables, i.e. the critical chloride concentration and position of the reinforcement bars. The case study with the concrete pier in a marine environment is repeated for the random field approach.

Thus, the paper examines and compares the different types of stochastic and deterministic modeling of the diffusion process, e.g. address the question if it is sufficient to use a random variable approach together with an analytical solution to the diffusion problem, or if a more elaborate numerical model should be used together with a random field model.

2 DIFFUSION PROBLEM

The diffusion problem can be formulated by the following partial differential equation: (Engelund 1997):

$$\dot{c} = D\nabla^2 c \quad \text{in V} \tag{1}$$

with boundary conditions:

$$\mathbf{q}^T\mathbf{n} = q_n \quad \text{on } S_q \tag{2}$$

$$c = c_b \quad \text{on } S_c \tag{3}$$

where D is the diffusion coefficient, $c(x_1, x_2, x_3, t)$ is the chloride concentration, $\dot{c}(x_1, x_2, x_3, t)$ is the time derivative of the concentration, ∇^2 is the Laplacian with respect to spatial coordinates, x_1, x_2 and x_3, $\mathbf{q}$ is the flow(flux) vector, V the volume, $\mathbf{n}$ a unit normal to the flux based boundary, S_q, q_n the size of the flow on the flux based boundary normal to the boundary, and c_b is the concentration on the concentration based boundary S_c.

3 ANALYTICAL SOLUTIONS TO THE DIFFUSION PROBLEM

Assuming that the flow of chloride is one dimensional, the differential Equation (1) can be rewritten as:

$$\frac{dc}{dt} = D\frac{\partial^2 c}{\partial x_1^2} \tag{4}$$

It the thickness of the structure in consideration is infinite, an analytical solution to the differential Equation (4) is given by, see Crank (1975):

$$c(x_1,t) = c_b\left[1 - erf\left(\frac{x_1}{2\sqrt{Dt}}\right)\right] \tag{5}$$

where $erf(x)$ is the error function, x is the depth and D is the diffusion coefficient, which is here considered to be a constant even though it seems more realistic to model it as a time varying function, see Frier & Sørensen (2005).

However, when a thin slab or structural element is considered, the influence of chloride flow from the opposite boundary of the structure might be significant. In this case, a series expansion solution to Equation (4) can be obtained as, see Edwards & Penney (1993):

$$c(x_1,t) = c_b\left[1 + \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{(-1)^m - 1}{m}\exp\left(-D\frac{m^2\pi^2}{a^2}t\right)\sin\left(\frac{m\pi x_1}{a}\right)\right] \tag{6}$$

where a is the depth of the structural element.

In some cases, e.g. when a rebar is positioned at the corner of a structural element, the flow of chloride is two dimensional. In this case, the differential equation, Equation (1), is written as:

$$\frac{dc}{dt} = D\left(\frac{\partial^2 c}{\partial x_1^2} + \frac{\partial^2 c}{\partial x_2^2}\right) \tag{7}$$

If the corner of a infinite domain is considered, corresponding to a relative thick structural element, the two-dimensional solution can be obtained as the product solution of two one-dimensional solutions, and Equation (7) can be solved to obtain, (Crank 1975):

$$c(x_1,x_2,t) = c_b\left[1 - erf\left(\frac{x_1}{2\sqrt{Dt}}\right)erf\left(\frac{x_2}{2\sqrt{Dt}}\right)\right] \tag{8}$$

When the structural element is thin, a series solution to Equation (1), obtained by the method of separation of variables, is given by (Edwards & Penney 1993):

$$c(x_1,x_2,t) = c_b\left[1 - \frac{4}{\pi^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{[(-1)^m - 1][(-1)^n - 1]}{mn}\cdot \exp\left[-D\left(\frac{m^2\pi^2}{a^2} + \frac{n^2\pi^2}{b^2}\right)t\right]\sin\left(\frac{m\pi x_1}{a}\right)\sin\left(\frac{n\pi x_2}{b}\right)\right] \tag{9}$$

where a, b are the dimensions of the rectangular structural element.

Three dimensional flow of chloride could also be considered, but as the rebars are relatively long, the effect of three dimensional flow of chloride will only be significant at the very end of the rebars, resulting to a very localized effect of corrosion with a very little influence on the overall safety of the structure.

4 FINITE ELEMENT SOLUTION

The differential equations, Equation (1), can also be solved by the Finite element method, resulting in the following matrix equation (Frier & Sørensen 2005):

$$\mathbf{C}\dot{\mathbf{c}} + \mathbf{K}\mathbf{c} = \mathbf{f} \tag{10}$$

where $\mathbf{c}$ are nodal concentrations, $\dot{\mathbf{c}}$, is the time-derivative of the nodal concentrations, $\mathbf{f}$, is the boundary load vector, $\mathbf{C}$ is the capacity matrix and $\mathbf{K}$ is the conductivity matrix.

The finite element equations (10) are solved numerically. Using the central difference technique, see Edwards & Penney (1993), the differential equations are discretized into time steps and the time derivative is approximated by:

$$\dot{\mathbf{c}} \approx \frac{1}{\Delta t}(\mathbf{c} - \mathbf{c}_0) \tag{11}$$

where $\mathbf{c}_0$ is the solution for the nodal concentrations at the previous time step and Δt is the time increment between the previous time step and the new time step.

When using the backward Euler method, the FE-equations are formulated for the end of each load step, resulting in the following linear equation system, which can be solved at each time step:

$$(\mathbf{C}+\Delta t\,\mathbf{K})\,\mathbf{c}=\Delta t\,\mathbf{f}+\mathbf{C}\,\mathbf{c}_0 \tag{12}$$

Four node linear elements with four Gauss point are used. A linear variation of the diffusion coefficient within the element is assumed.

5 DETERMINISTIC ANALYSIS

In order to investigate the accuracy of the different solutions to the diffusion problem, a quadratic concrete cross section is analyzed, see Figure 1. The distribution of chloride concentration is investigated at a distance 50 mm from the surface as indicated in the figure, in order to examine the chloride concentration at the position of main reinforcement bars, sensitive to corrosion, situated near the surface of the cross section.

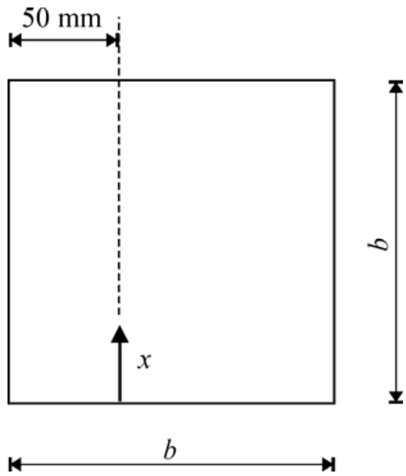

Figure 1. Quadratic concrete specimen with cross section where chloride concentration is investigated.

Figure 2 shows the distribution of chloride within cross-sections with side lengths of b = 100 mm and 400 mm, respectively, after 30 years of exposure to the environment, modeled by the different analytical solutions and the finite element method. The diffusion coefficient is constant D = 10 mm^2/year and the boundary chloride concentration is constant c_b = 1 % on all four sides of the cross-section. The finite element solution is based on a mesh of 40x40 finite elements and 50 time increments. The series solutions are based on a large number of terms insuring convergence of the result.

It can be observed, that there exist a significant difference between the 1D and the 2D solutions, as the overall level of chloride is higher for the 2D solutions caused by the multidirectional flow of chloride. It can also be seen, that the solutions based on infinite domains are sufficient accurate for the structure 400 mm thick, whereas the neighboring surfaces show some significant influences on the structure 100 mm thick. However, as the structure of 100 mm is very thin, the use of the infinite boundary solutions seems appropriate for many realistic structures. The FEM-solution should be equal to the exact 2D series solution.

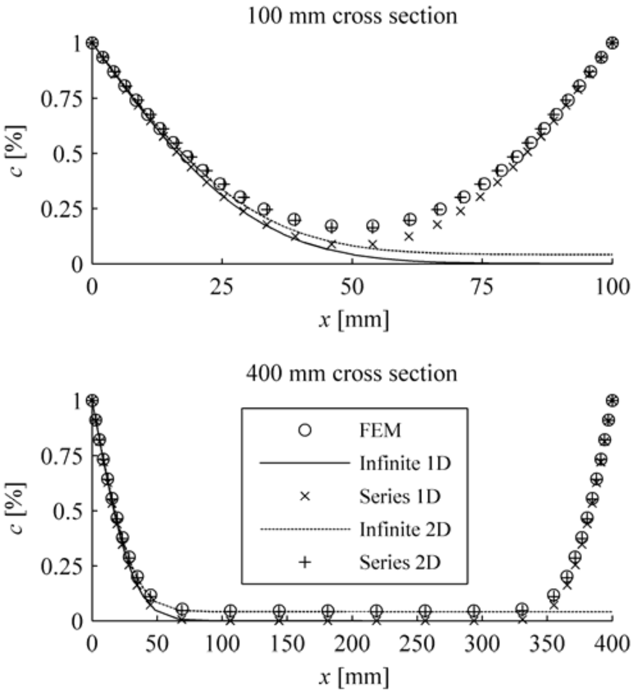

Figure 2. Distribution of chloride within a quadratic domain after 30 years of exposure, 50 mm from the surface with a side length of 100 mm (upper) and 400 mm (lower).

Using the different models, the time for initialization of corrosion, t_{ini}, can be obtained by solving:

$$c(\mathbf{x}_{bar}, t_{ini}) = c_{cr} \tag{13}$$

where c_{cr} is the critical value of the chloride concentration when the rebar starts to corrode and $\mathbf{x}_{bar}$ is the location of the rebar within the structure.

The value of the chloride concentration at the onset of corrosion is about 0.05-0.1 %, see Engelund (1997) so if the rebars are situated 50 mm from the surface, corrosion is about to start for the cases shown in Figure 2 after 30 years.

6 RANDOM VARIABLE APPROACH

The parameters governing the problem are uncertain in nature and are in this section modeled by random variables. The stochastic model in Table 1 is adapted for the different parameters partly based on values from Engelund, (1997). The position of the rebars and the critical concentration of chloride are assumed to be independent random variables for each rebar.

A concrete pier in a marine environment is investigated as shown in Figure 3. Initialization of corrosion of the main reinforcement bars is assumed to be critical.

Distributions for the time for initialization of corrosion, t_{ini}, , is obtained by Monte Carlo Simulation by solving Equation (13), based on a large number of realizations of the random variables governing the problem, see Hammersley & Handscomb (1964).

Table 1. Stochastic model for chloride diffusion problem.

Parameter	Distribution	Mean	Std.
D [mm^2/year]	Log-normal	10	3
c_b [%]	Log-normal	1.0	0.3
c_{cr} [%]	Log-normal	0.075	0.0022 5
dx_1 [mm] *	Log-normal	50/200/350	2.5
dx_2 [mm] *	Log-normal	50/200/350	2.5

*) Dislocation of rebar from mean.

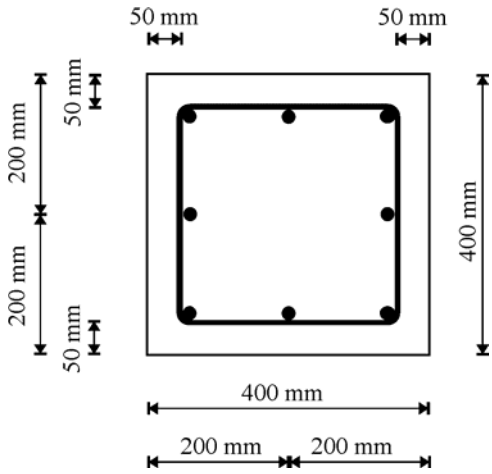

Figure 3. Concrete pier in a marine environment.

Figure 4 shows the resulting probability distributions of the time to initialization of corrosion based on both the four analytical methods and FEM. In the finite element approach, constant values for the diffusion coefficient and boundary chloride concentration is used in each simulation. A mesh of 40x40 finite elements and 50 time increments is used in the simulation.

Three different cases are presented in Figure 4, corresponding to initialization of corrosion of a corner rebar, a rebar situated on the side of the surface, and the first rebar to reach initialization of corrosion of all eight rebars, respectively.

The two-dimensional series expansion solution can be regarded as the ‘exact’ solution to the diffusion problem in the present case. It can be observed that the solutions based on infinite domains gives nearly identical results as the solutions based on series expansion. Thus, the thickness of the concrete pier is so large that the more simple infinite solutions can be applied without error in the present case.

It can be observed, that for the corner rebar, the one-dimensional models where the effect of the two-dimensional flow of chloride is ignored gives very poor results, whereas for the middle rebar, the effect of two-dimensional flow is insignificant and all analytical models gives identical results. The difference between the FEM solution and the two-dimensional analytical models is due to the discretization of the FE-mesh.

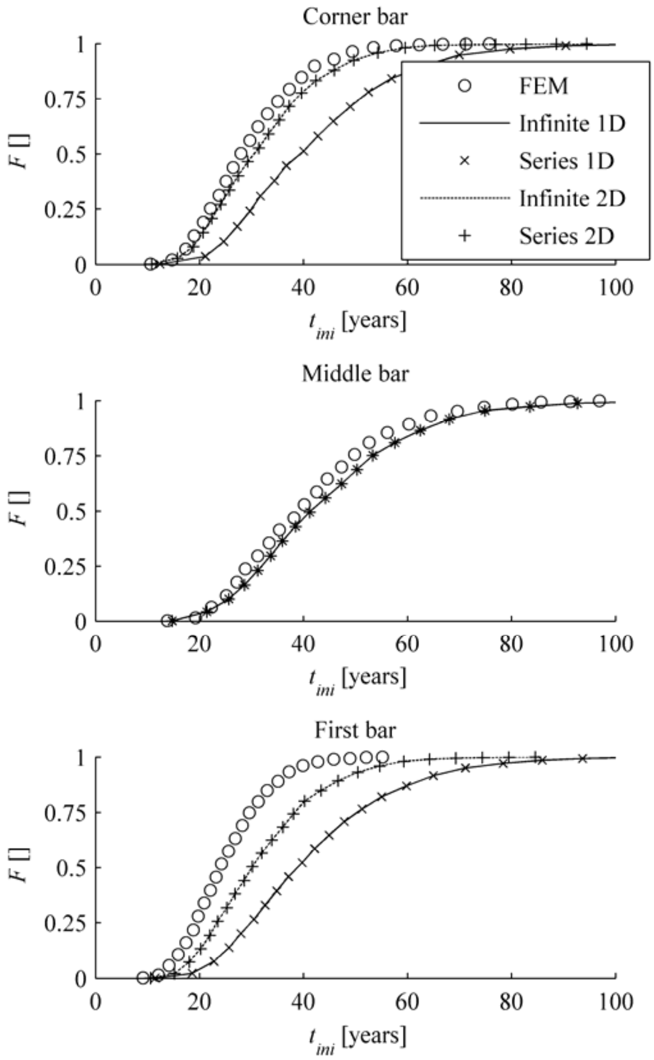

Figure 4. Probability distributions for the time for initialization of corrosion using the four analytical models and FEM.

The distributions corresponding to initialization of corrosion of the first rebar are very close to the distributions corresponding to the outer rebar for the analytical models, thus in this case, initialization of corrosion can be modeled with reasonable accuracy by only considering the outer rebar.

7 RANDOM FIELD APPROACH

It might not be sufficient to adapt a random variable approach as some of the parameters defining the diffusion problem might vary both spatial and in time, in that case, a random field model can be used, see Vanmarcke (1988).

A random field, $W(\mathbf{x})$, is a multi-dimensional continuously varying random process. The vector, $\mathbf{x}$, of dimension n, is describing the spatial or temporal location within the field

In order to use a random field for practical numerical calculations, the field is discretized using a vector of standard normal random variables, **U,** i.e. independent Gaussian random variables with zero expected value and unit standard deviation. Having expressed the field in this way, it is possible to use standard Monte Carlo simulation and reliability theory tools to find the probability of exceeding the critical chloride concentration at the reinforcement bars.

The EOLE (Expansion Optimum Linear Expansion) method is used here to discretize a random property, W, into a Gaussian random field by Li & Der Kiureghian (1993):

$$W(\mathbf{x}) = \mu_W + \sigma_W \sum_{i=1}^{M} \frac{U_i}{\sqrt{\lambda_i}} \boldsymbol{\varphi}_i^T \mathbf{C}_{\mathbf{X},\mathbf{x}_i} \tag{14}$$

where μ_W is the mean and σ_W is the standard deviation of the field. U_i, $i = 1,\ldots, m$ are components of a vector of standard normal variables, and $\mathbf{C}_{\mathbf{X},\mathbf{x}_i} = \{\rho_{WW}(\mathbf{x}-\mathbf{x}_i), i = 1,\ldots,M\}$ is the i'th column of the correlation coefficient matrix, given by:

$$\mathbf{C}_{\mathbf{X},\mathbf{X}}(i, j) = \rho_{WW}(\mathbf{x}_i - \mathbf{x}_j) \tag{15}$$

(λ_i, $\boldsymbol{\phi}_i$) is the solution of the following eigenvalue problem:

$$\mathbf{C}_{\mathbf{X},\mathbf{X}} \boldsymbol{\varphi}_i = \lambda_i \boldsymbol{\varphi}_i \tag{16}$$

with $\boldsymbol{\phi}_i$ being the unit eigenvector corresponding to eigenvalue, λ_i.

The Gaussian auto-correlation function is considered in this work:

$$\rho_{WW}(\mathbf{x}_i - \mathbf{x}_j) = \exp\left[-\sum_{k=1}^{n}\left(\frac{x_{ki} - x_{kj}}{a_{Wk}}\right)^2\right] \tag{17}$$

where a_{Wk} is the correlation length of the field.

In the present work, the fields are assumed to be Log-normal, in over to avoid negative realizations of the parameters during the simulation. A Log-normal random field, $V(\mathbf{x})$, is modeled by noting that $W(\mathbf{x}) = \log(V(\mathbf{x}))$ is normally distributed.

In the present work, $W(\mathbf{x})$ is either representing the diffusion coefficient in the Finite element nodes into which the problem is discretized, or the boundary chloride concentrations in the FEM-nodes along the boundary.

The eigenvalue problem is most conveniently solved by the subspace iteration method, where only the M highest eigenvalues of the correlation coefficient matrix are obtained, see Lapack (2001). The advantage of the EOLE method is that relatively few eigenvalues are needed to discretize a given random field.

Figure 5 shows realizations of the diffusion coefficient and boundary chloride concentration modeled as spatially correlated Log-normal random fields. The diffusion coefficient field is a two dimensional field within quadratic spatial domain with side length 400 mm, with mean value and standard deviation as given in Table 1, and a correlation length of 350 mm, as suggested in Engelund (1997). The boundary concentration field is as a one dimensional field with length corresponding to the 1600 mm boundary of the structure, also with statistics as given in Table 1 and a correlation length of 350 mm. The correlation coefficient between different spatial points in the boundary concentration field is obtained based on the minimum distance between points on the surface, thus the field is closed-looped. The mesh/points on the figure indicate the discretization applied for obtaining the correlation coefficient matrix, Equation (17).

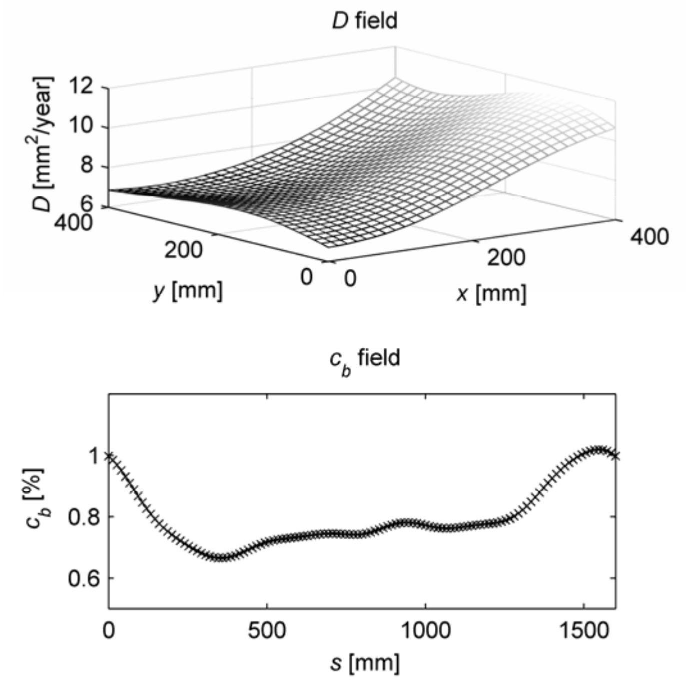

Figure 5. Realizations of diffusion coefficient and boundary concentration fields, modeled as spatially distributed random fields, both with a correlation length of 350 mm.

The fields can also be assumed to be temporally correlated. Figure 6 shows the diffusion coefficient and chloride boundary concentration both modeled as temporally correlated Log-normal distributed random fields. No information is available regarding the correlation lengths of the field, but a value of the correlation length of 25 years is used in the following. The time for initialization of corrosion has been simulated using the random field approach and FEM. The analytical models can not be used for the random field approach as the analytical solutions for the chloride concentration are derived assuming constant values of the diffusion coefficient and boundary chloride concentration. However, as the solution is obtained numerically using an incremental approach, both temporally and spatially correlated parameters can be assigned using the FEM-approach. The results are shown in Figure 7.

The result for the random variable approach is shown together with the two types of random field modeling, assuming spatial and temporal correlation, for the same cases as for the random variable approach shown in Figure 4.

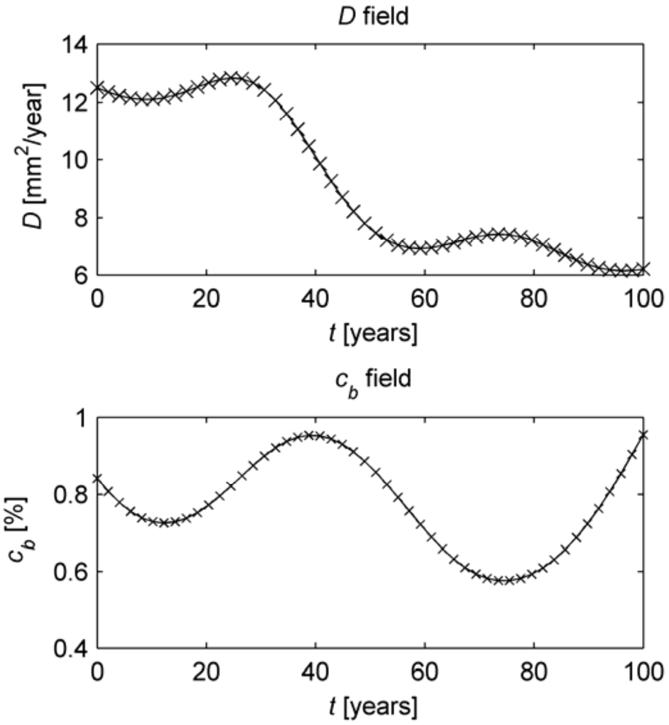

Figure 6. Realizations of diffusion coefficient and boundary concentration fields, modeled as temporally distributed random fields, both with a correlation length of 25 years.

It can be observed from Figure 7 that the effect of treating the diffusion coefficient and boundary chloride concentration as random fields is insignificant, when only a single rebar is considered. However, in the case where several rebars are considered at the same time, when defining initialization of corrosion as the first rebar reaching the critical chloride concentration, the effect of considering spatially distributed random fields is significant, whereas the effect of using temporally distributed fields is insignificant. The reason is that the conditions at the different rebars are not fully correlated.

8 CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

For many reinforced concrete structures the corrosion of the reinforcement is a major problem. To this end, modern reliability theory together with economic decision theory can be used. The present paper addresses the structural reliability part of the problem, i.e. how to obtain a probability distribution function of the time to initialization of corrosion.

Typically, a simple analytical solution to the diffusion problem is applied, assuming one dimensional flow of chloride into an infinite domain. However, in reality, the flow of chloride is two or even three dimensional when considering a real life structural component. Another issue is that the parameters governing the problem are often modeled as random variables even though they are varying both spatially and temporally, suggesting that they should be modeled as random fields instead. Thus, the paper examines and compares analytical and numerical modeling of the diffusion problem and examines if a random variable or random field approach should be used when modeling the problem.

Four different analytical models are considered; the one-dimensional infinite analytical solution and its extension to the two-dimensional case, considering the flow of chloride at the corner of an infinite domain. The concentration of chloride in a finite domain is obtained both in the case where the domain is one-dimensional with two parallel neighboring surfaces, and is a two-dimensional rectangular domain, using a series expansion solution.

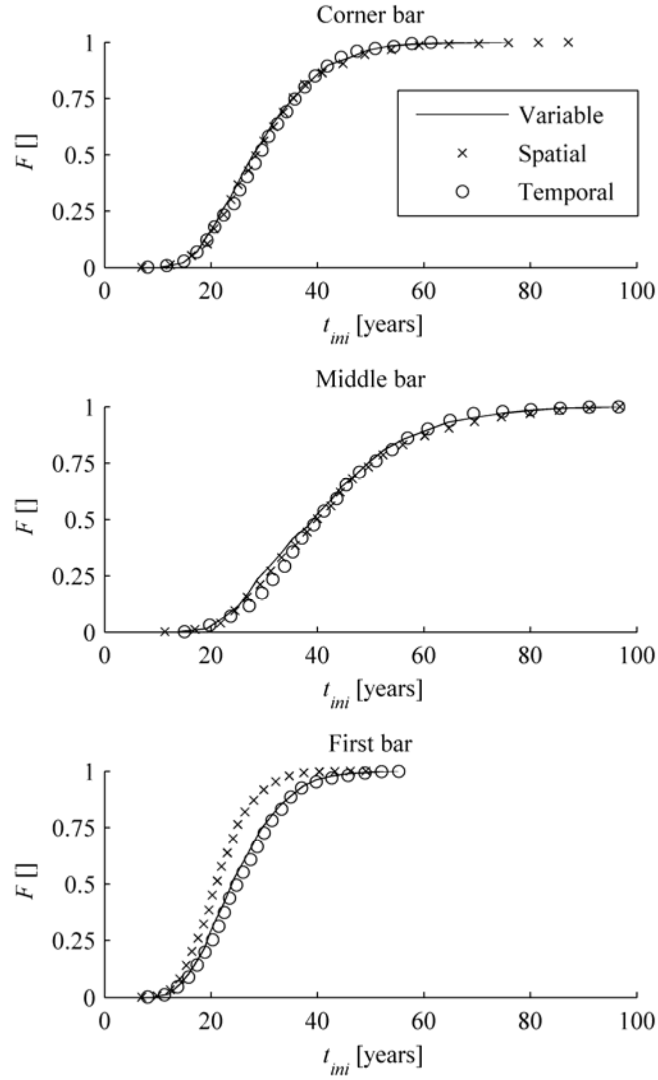

Figure 7. Probability distributions for the time to initialization of corrosion, random field approach.

The four different analytical solutions and a numerical FEM-solution have been compared for quadratic concrete components 100 mm and 400 mm thick, respectively. First, all parameters governing the problem are assumed to be deterministic. The results show that the effect of two dimensional chloride flow at locations near to the corner of the section is significant, in which case the one-dimensional model is inadequate. However, the cross-section must be relatively thin, before the effect of the neighboring surfaces becomes significant, thus the two-dimensional model based on the infinite domain seems to be adequate for many cases.

As a case study, a bridge pier in a maritime environment is examined, and the probability distribution functions for the time to initialization of corrosion are estimated using Monte Carlo simulation. A quadratic cross section with side length 400 mm is considered and the distribution of time for initialization of corrosion is examined at a corner rebar, a middle side rebar and corresponding to the first of the eight reinforcement bars reaching corrosion. Again, it is found that it is very important to include the two-dimensional effect of the chloride flow into the structure, when i.e. a corner rebar is considered, as the distribution functions for a corner and a mid side rebar differs significantly.

When considering initialization of chloride at the first of all the rebars, the results show that it is important to include system behavior, when no corrosion is accepted in a structure with multiple reinforcement bars. The result also implies that it may be important to model the spatial correlation of the parameters. Therefore, the diffusion coefficient and boundary concentration of chloride have also been modeled by spatial and temporal correlated random fields, in which case the concentration of chloride is obtained using FEM. The same case study with the concrete pier is examined using the random field approach. The random fields are discretized into vectors of random variables by means of the Expansion Optimum Linear Estimation method (EOLE). Again Monte Carlo Simulation (MCS) is used to obtain the probability distributions of the time to initialization of corrosion.

The random variable and the random field approach, assuming spatially correlated random fields, differ mostly in the case where the distribution corresponding to corrosion of the first rebar is investigated, whereas the random variable and random field approach gives almost identical results, when corrosion of a single rebar is examined. The results for temporally correlated random fields give almost the same results as for the random variable approach.

However, as noted in the beginning, the diffusion coefficient could be modeled as a function of time, in which case it seems necessary to turn to random field modeling.

In this work, only the cross-section of the pier is investigated, corresponding to the case where the splash zone is relatively thin over the height of the pier. However, for a case where the distribution of chloride over the length of the element, where the entire length is exposed to chloride, i.e. along the reinforcement bars, the influence of treating the parameters as random fields could be more significant.

The parameters prescribing the random variables/fields have primary been selected according to the literature, but of cause, these parameters have to be measured for realistic real life applications of the method, i.e. by measurements on chloride profiles obtained from concrete samples. Important future work in the area will partly be to develop methods for determining the values of the parameters describing the random quantities, e.g. by analysis of chloride profiles.

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