

Domains of pseudo-differential operators: a case for the Triebel-Lizorkin spaces

The main result is that every pseudo-differential operator of type 1,1 and order d is continuous from the Triebel-Lizorkin space $F_{p,1}^d$ to L_p , $1 \leq p < \infty$, and that this is optimal within the Besov and Triebel-Lizorkin scales. The proof also leads to the known continuity for $s > d$, while for all real s the sufficiency of Hörmander's condition on the twisted diagonal is carried over to the Besov and Triebel-Lizorkin framework. To obtain this, type 1,1-operators are extended to distributions with compact spectrum, and Fourier transformed operators of this type are on such distributions proved to satisfy a support rule, generalising the rule for convolutions. Thereby the use of reduced symbols, as introduced by Coifman and Meyer, is replaced by direct application of the paradifferential methods. A few flaws in the literature have been detected and corrected.