

Domination and Leaf Density in Graphs

The domination number $\gamma(G)$ of a graph G is the minimum cardinality of a subset D of $V(G)$ with the property that each vertex of $V(G) - D$ is adjacent to at least one vertex of D . For a graph G with n vertices we define $\epsilon(G)$ to be the number of leaves in G minus the number of stems in G , and we define the leaf density $\zeta(G)$ to equal $\epsilon(G)/n$. We prove that for any graph G with no isolated vertex, $\gamma(G) \leq n(1 - \zeta(G))/2$ and we characterize the extremal graphs for this bound. Similar results are obtained for the total domination number. The 2-partition domination number $\gamma(G, \pi_2)$ of a graph G and a 2-partition $\pi_2 = \{V_1, V_2\}$ of $V(G)$ is defined by the sum $\gamma(G) + \gamma_G(V_1) + \gamma_G(V_2)$. We prove that for any graph G with no isolated vertex and any 2-partition π_2 of $V(G)$, $\gamma(G, \pi_2) \leq 3n(1 - \zeta(G))/2$ and we characterize the extremal graphs. For graphs with leaf density $\zeta > 1/6$, this new bound is an improvement of the bound given by B. Hartnell and P. D. Vestergaard [J. Combin. Math. Combin. Comput. 2003, Aug., 46].