Extremal results for rooted minor problems

In this paper, we consider the following problem:

Given four distinct vertices v_1, v_2, v_3, v_4. How many edges guarantee the existence of seven connected disjoint subgraphs X_i for $i = 1, \ldots, 7$ such that X_j contains v_j for $j = 1, 2, 3, 4$ and for $j = 1, 2, 3, 4$ X_j has a neighbour to each X_k with $k = 5, 6, 7$. This is the so-called "rooted $K_{3,4}$-minor problem".

There are only few known results on rooted minor problems, e.g. [8,3]. In this paper, we prove that a 4-connected graph with n vertices and $5n - 14$ edges has a rooted $K_{3,4}$-minor. Also, we consider the similar problems concerning rooted $K_{3,3}$-minor problem and $K_{3,2}$-problem.