

We characterize the approximation spaces associated with the best  $n$ -term approximation in  $L_p(\mathbb{R})$  by elements from a tight wavelet frame associated with a spline scaling function. The approximation spaces are shown to be interpolation spaces between  $L_p$  and classical Besov spaces, and the result coincide with the result for nonlinear approximation with an orthonormal wavelet with the same smoothness as the spline scaling function. We also show that, under certain conditions, the Besov smoothness can be measured in terms of the sparsity of expansions in the wavelet frame, just like the nonredundant wavelet case. However, the characterization now holds even for wavelet frame systems that do not have the usually required number of vanishing moments, e.g. for systems built through the Unitary Extension Principle, which can have no more than one vanishing moment. Using these results, we describe a fast algorithm that takes as input any function and provides a near sparsest expansion of it in the framelet system as well as approximants that reach the optimal rate of nonlinear approximation. Together with the existence of a fast algorithm, the absence of need for vanishing moments may have an important qualitative impact for applications to signal compression, as high vanishing moments usually introduce Gibbs-type phenomenon (or "ringing" artifacts) in the approximants.