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# Hybrid Control and Verification of a Pulsed Welding Process

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**Abstract.** Currently systems, which are desired to control, are becoming more and more complex and classical control theory objectives, such as stability or sensitivity, are often not sufficient to cover the control objectives of the systems.

In this paper it is shown how the dynamics of a pulsed welding process can be reformulated into a timed automaton hybrid setting and subsequently properties such as reachability and deadlock absence is verified by the simulation and verification tool UPPAAL.

## 1 Introduction

The lack of analytical methods for design of hybrid control systems can often result in excessive testing and validation, which is time consuming and even then might not guarantee that the system will meet the control objectives under all operating conditions. To overcome the design and implementation problems which may result from the deficient use of an analytical approach, a notation of hybrid automaton has been introduced in [1].

Most algorithmic verification and synthesis tools for hybrid systems today are limited to systems exhibiting simple continuous dynamics, such as piecewise-affine hybrid systems[2, 3] or timed automata[4–6]. One of the main objective in [7] was to enlarge this class of systems to all linear controllable systems, which is continued in this paper by showing how this theory apply in practice to the Gas Metal Arc Welding (GMAW) process. By restricting the observations for the system to a finite set of partitions, enables a bisimulation of the system to be modeled using simple timed automata.

With a bisimilar model of the system built with the use of timed automata, it is possible to use a verification tools such as UPPAAL to simulate and verify different system properties. Especially questions such as reachability, liveness and possibilities of deadlocks are new questions, which are of great interest to the designer of the supervisory system and which previously needed to be guessed at by simulations or ad-hoc methods.

## 1.1 Gas Metal Arc Welding

In the GMAW process the electrode is consumable and is fed continuously at a certain rate by the pistol to the welding pool. The weld is protected from the surrounding air by a gas which is also fed by the pistol. Normally argon or argon/CO<sub>2</sub> is used as shielding gas. The current between the workpiece (cathode) and the welding pistol (anode) causes an arc and an electromagnetic field. The strong current makes the electrode melt and drop into the welding pool.

The GMAW process can be divided into three modes; short arc mode, spray mode and a mixed mode of the two, of which only the spray mode will be considered in this paper. In spray mode the electrode should never touch the workpiece in order to obtain the best weld quality.

The melting process can be described by two contributions, *anode heating* and *ohmic heating*. When the current rises, the temperature of the arc rises and the tip of the electrode is heated up. The energy from the arc, which contributes to melting the electrode, is known as *anode heating*. The second contribution to the melting process is the *ohmic heating*, which is the heat energy developed as a result of the ohmic resistance in the electrode.

The high current also creates a higher electromagnetic field which contributes, together with the gravitational force, to detachment of the drop. When the tip is melting, a liquid drop of metal is formed. This drop is detached from the tip of the electrode when the surface tension on the drop, is too small to resist the gravitational- and the electromagnetic forces. Also the aerodynamic drag force from the shielding gas, contributes to the detachment of the drop. After detachment, a small liquid drop is left at the tip of the electrode and the process starts over again.

A submode of the spray mode is the pulsed GMAW method, which is similar to spray mode, but in addition to the steady current between the cathode and the anode, current pulses causes the drop to detach in intervals. The advantage of using pulsed GMAW is a lower heat development in the weld pool. Furthermore the current pulses makes it possible to control the drop detachment [8].

## 1.2 Weld Quality Criteria

As described in the introduction, one of the objectives of this paper is to integrate a control structure for the GMAW process into a hybrid framework. The nature of the GMAW process makes classic control theory specifications, such as stability, inadequate. Instead control objectives focusing on obtaining the best weld quality is desirable. The quality of a weld depends on several factors, which will be discussed in the following.

Basically a high-quality weld is characterized by a good penetration, which is essential for a strong weld, as it allows a larger area of the workpiece edges to join.

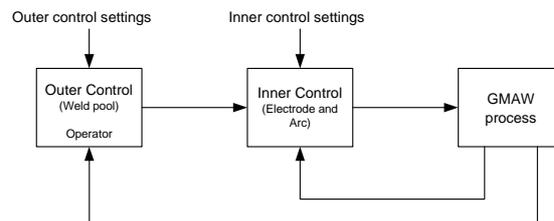
A good penetration is a necessary, but not a sufficient, condition for a good weld. If the work piece becomes too hot and cools down too quickly, the material

can lose some of its characterizing properties, e.g. heat-treated metals or metal alloys, such as stainless steel, can lose its characterizing properties. [9, ch. 5] The facts described in the latter are related to the weld pool and are the overall basic criteria, which must be fulfilled to obtain a high-quality weld and is defined as *direct weld quality influencing factors*. More indirectly an additional number of factors influences the quality of a weld. The following quality influencing factors will be referred to as *indirect quality influencing factors*. Specific for pulsed GMAW welding, the quality of the weld is influenced by the control of the drop detachment. Meaning that the current pulses should ideally detach one drop per pulse to obtain the best weld possible. It is also desirable to obtain a uniform drop size, in order to achieve a homogeneous weld. An additional control objective is to keep a short arc length, since it is easier for the operator to work with. Moreover the energy input into the workpiece should be minimized.

The indirect quality influencing factors are related to the control of the electrode and the arc.

### 1.3 Delimitation of Control Tasks

As described in the latter the control can be separated into weld pool control, and arc and electrode control. As it is only hand held welding which will be the focus on this paper, the weld pool control is handled by the operator. Figure 1 describes the control structure [8]. The outer control loop is handled by the operator and the inner loop is handled by the welding machine. The rest of this



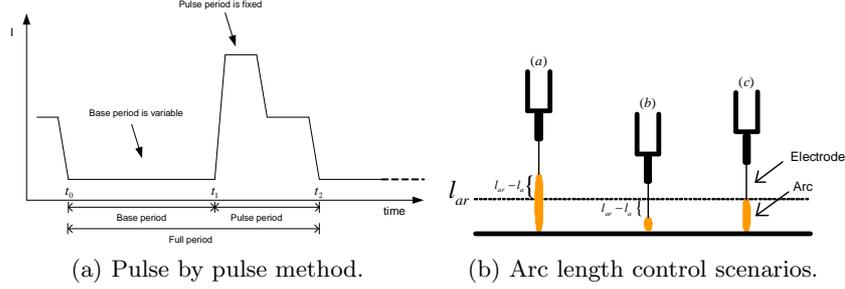
**Fig. 1.** Control structure for the GMAW process.

paper will concentrate on controlling the indirect quality influencing factors in the inner loop. Keeping a minimum arc length and minimizing the energy input into the workpiece is essential the same if the electrode velocity  $v_e$  is fixed.

## 2 System Dynamics

The pulsed GMAW process is governed by the pulsing current, which is seen in figure 2(a). In order to control the pulsing the base period, which is where the electrode is melted, is variable, thus it becomes possible to control the amount

of melt detached in each pulse. If the arc length between the work piece and the electrode becomes too big or too small, as shown in figure 2(b), then it is likewise possible to adjust the arc length, i.e. if the arc has become too small then by decreasing the base period, thus increasing the amount of electrode consumed the arc length will become longer. This is however done on the cost of a smaller drop size and is only possible within a small distance, the main part is still controlled manually.



**Fig. 2.** Left figure: The pulse by pulse method - the base current is fixed but the base period is variable. Right figure: The different arc length control scenarios - (a) the arc is too long (b) the arc is too short (c) the arc has the desired length  $l_{ar}$

The pulse condition can then be described as; a pulse should occur if the arc length is below the reference and the drop size is above minimum or if the arc length is longer than the reference and the drop size is bigger than the maximum, which can be written as:

Pulse if:

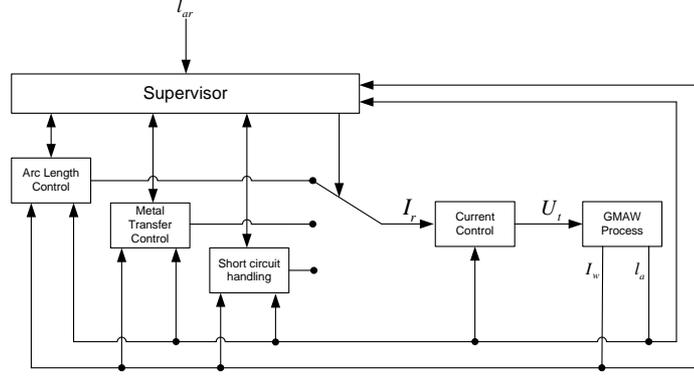
$$(l_a < l_{ar} \wedge x_{mb} \geq x_{mb\_min}) \vee (l_a > l_{ar} \wedge x_{mb} \geq x_{mb\_max}) \quad (1)$$

Where  $l_a$  is the arc length,  $l_{ar}$  is the arc length reference and  $x_{mb}$  is the current drop size with the indices *min* and *max* providing the bound on the desired drop size. The values of the bound can be regarded as weighting parameters for the controller design.

The weld process controller can thus be depicted as in figure 3, where an additional mode: *Short Circuit Handling* is shown, but which will not be discussed further in this paper. Following the overall control strategy the dynamics of the underlying process will be presented. The model used in this paper, is derived in [8].

**Arc Length Dynamics** The governing equation for the arc length dynamics can be seen in (2).

$$\dot{l}_a = k_1 I + k_2 I^2 \cdot (l_c - l_a) - v_e \quad (2)$$



**Fig. 3.** Supervisory system for the GMAW process.

where  $k_1$  and  $k_2$  are constants,  $l_c$  is the length is the distance from the contact tip to the workpiece and  $l_a$  is the length of the arc and  $v_e$  is the velocity of the electrode.

Equation (3) shows the dynamics for the current.

$$\dot{I} = -\frac{1}{\tau_i}I + \frac{1}{\tau_i}I_r \quad (3)$$

where  $\tau_i$  is a constant that characterizes the dynamics.  $I$  is the welding current and  $I_r$  is the current reference.

**Drop Dynamics** The drop dynamics, or in other words the drop growth, can be expressed as the length of melted electrode:

$$x_m = \int_{t_0}^{t_1} v_m(I, l_s) dt \quad (4)$$

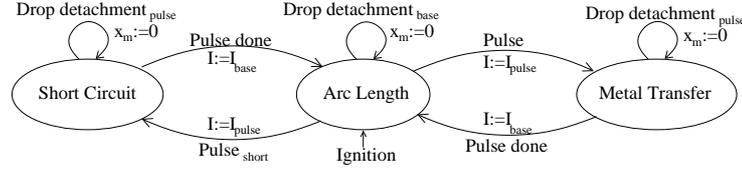
where,  $v_m$  is the velocity of melted electrode:

$$v_m = k_1 I + k_2 I^2 l_s \quad (5)$$

where,  $l_s = 0.0115$ ,  $k_1 = 3.6733 \cdot 10^{-4}$  and  $k_2 = 6.6463 \cdot 10^{-4}$  for the considered welding application.

### 3 Hybrid system modeling

The GMAW system, as described in the previous two sections can be formulated as the following hybrid automaton using a commonly used formalism for hybrid systems, as presented in [1, 10]: With the dynamics in each state as described in



**Fig. 4.** Hybrid automaton for the controlled GMAW process. Divided into the three control modes: Arc length, Metal transfer and Short circuit

the previous section. All transitions have a label, which is used for synchronization and a reset map, which in the “Drop detachment” case is the amount of melted wire which is set to  $x_m := 0$ , and in the “Pulse” and “Pulse done” case it is the current, which is set to the pulse and base current respectively.

As previously mentioned, then the goal of this paper is to reformulate the hybrid system into a network of timed automaton in order to expand the possibilities of verifying the systems properties using an automated verification tool, such as UPPAAL[11]. This is essentially done because even though the system is exhibiting a nice and stable performance in each state, then it is possible by the right combination of switching to render the system unstable, of which a classical example can be seen in [12].

### 3.1 Shift register form

In order to rewrite the dynamics of the system into shift register form it first needs to be put on Brunovsky normal form[7], for which a controllable linearized form of the system is needed.

By combining (2) and (3) into a state space formulation, linearizing and discretizing it the system give by (6) appears, which yields a controllable system, which is the condition for transforming the system into Brunovsky normal form.

$$\begin{bmatrix} \dot{l}_a \\ \dot{I} \end{bmatrix} = \begin{bmatrix} 0.3659 & 0.0027 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} l_a \\ I \end{bmatrix} + \begin{bmatrix} 6.9729 \\ 1.000 \end{bmatrix} u \quad (6)$$

Following the method described in [13] the system is transformed into the normal form shown in (7) through the state transformation,  $x = Tz$ :

$$\begin{aligned} z(t+1) &= T^{-1}ATz(t) + T^{-1}Bu(t) \Leftrightarrow \\ z(t+1) &= \tilde{A}_z z(t) + \tilde{B}_z u_z(t) \end{aligned} \quad (7)$$

where

$$\tilde{A}_z = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \tilde{B}_z = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, u_z = [F \ 1] \begin{bmatrix} z \\ u \end{bmatrix}, F = \begin{bmatrix} 0 \\ 0.366 \end{bmatrix}^T, T = \begin{bmatrix} 291.6 & 143.3 \\ -2.73 & 0 \end{bmatrix}$$

### 3.2 State Space Partitioning

A discrete state space  $\mathbb{Z}^2$  of  $\mathbb{R}^2$  is now introduced, as described in [7], which is used to form the space in which the shift-register form system will operate. The 3 domains in which the system operates is, with reference to figure 4, the Arc Length Control ( $q_1$ ), the Metal Transfer Control ( $q_2$ ) and the Short Circuit Control ( $q_3$ ).

- Domains

$$Dom(q_1) = \{(l_a, I) \in \mathbb{R}^2 \mid 0 \leq l_a \leq 0.01 \quad \wedge \quad 40 \leq I \leq 60\}$$

$$Dom(q_2) = \{(l_a, I) \in \mathbb{R}^2 \mid 0 \leq l_a \leq 0.01 \quad \wedge \quad 290 \leq I \leq 310\}$$

$$Dom(q_3) = \{(l_a, I) \in \mathbb{R}^2 \mid 0 \leq l_a \leq 0.01 \quad \wedge \quad 290 \leq I \leq 310\}$$

Where  $l_a$  [m] and  $I$  [A]. The values are specified from the normal operation of a GMAW welding machine.

These domains are then transformed into shift register form by  $[z_1 \ z_2]^T = \mathbf{T} [l_a \ I]^T$  which gives the new domains

$$Dom(q_1) = \{(z_1, z_2) \in \mathbb{R}^2 \mid 0 \leq z_1 \leq 8596 \wedge 0 \leq z_2 \leq 0.0643\}$$

$$Dom(q_2) = \{(z_1, z_2) \in \mathbb{R}^2 \mid 4.18 \cdot 10^4 \leq z_1 \leq 4.44 \cdot 10^4 \wedge 0.307 \leq z_2 \leq 0.328\}$$

$$Dom(q_3) = \{(z_1, z_2) \in \mathbb{R}^2 \mid 4.18 \cdot 10^4 \leq z_1 \leq 4.44 \cdot 10^4 \wedge 0.307 \leq z_2 \leq 0.328\}$$

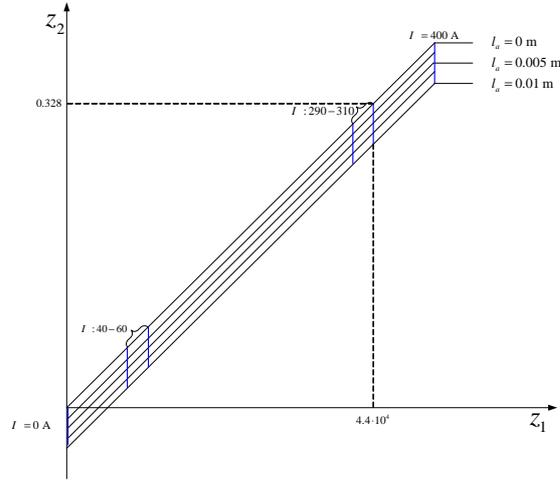
How these new space looks like compared to the original one can be seen from figure 5, where the two regions of interests are marked, one being to the left in  $I \in [40-60]$ , which is the base period, and the region to the right,  $I \in [290-310]$ , being the pulse period. As it is seen from the figure then the domains of interest are no longer square. This deficiency is however remedied by relaxing the arc length constraint, which again makes the spaces of interest squares.

As described in [7] the partitioning needs to be equidistant, which would seem rather cumbersome for these domains due to the large ratio between  $z_1$  and  $z_2$ , thus a scaling transformation is introduced,  $S_i$ , which transform each domains into a sufficiently equiproportional domain. In this case it is only desirable to divided the spaces into 3 by 3 grid to prove the concept, thus a transformation that scales the 3 domains into squares are used.

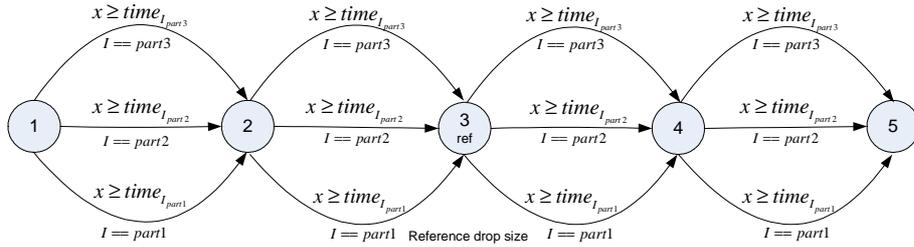
Following this the drop forming dynamics is modelled as a 5 stage timed automaton as shown in figure 6. The shifting time between the drop sizes are only dependent on the current. Estimated shifting values for different current intervals are shown in table 1. The drop formation always starts in stage 1 and will propagate through the stages over time. Stage 3 is the reference stage, i.e. the stage in which it is desirable to do a drop detachment.

### 3.3 Control System Imposed on $\mathbb{Z}^2$

In section 3.2 a new state space  $\mathbb{Z}^2$  was introduced. Utilizing that the system is in shift register form, insures a well defined controlled dynamics between the



**Fig. 5.** Plot of state transformation:  $[l_a I]^T \mapsto z$



**Fig. 6.** The automaton structure for the drop dynamics is the same in each domain  $q_1$  and  $q_2$ .

Current Partition	Current Interval [A]	Time Between Partitions [s]	Current Interval [A]	Time Between Partitions [s]
1	40.0 - 46.6	$9.9 \cdot 10^{-3}$	290 - 296.6	$3.9 \cdot 10^{-4}$
2	46.6 - 53.3	$8.0 \cdot 10^{-3}$	296.6 - 303.3	$3.7 \cdot 10^{-4}$
3	53.3 - 60.0	$6.6 \cdot 10^{-3}$	303.3 - 310	$3.5 \cdot 10^{-4}$

**Table 1.** Estimated time between drop size partitions in  $dom(q_1)$  to the left and  $dom(q_2)$  and  $dom(q_3)$  to the right.

partition blocks. This means that under appropriate inputs the blocks will move into other partition of equal division. To insure such *appropriate* inputs, a control law is needed.

The control law is constructed as described in [7] by starting with (7) and realizing that from a given position  $(z_1, z_2)=(p, q)$  the reachable set in one step is  $(z_1, z_2)=(q, r)$ , where  $r \in \mathbb{Z}$  is dependent on the input, thus it can be seen that the control law only has to ensure that  $z_2$  will be within a control section of height  $\delta$ , which is ensured by the control law:

$$u_z(k) = z_2(k) + \delta y(k) , \quad y \in \mathbb{Z} \quad (8)$$

which inserted into (7) results in the following system:

$$\begin{aligned} z_1(k+1) &= z_2(k) \\ z_2(k+1) &= z_2(k) + \delta y(k) \end{aligned} \quad (9)$$

which is not on shift register form any longer. This is however easily remedied by introducing the control law:

$$\epsilon(k) = z_2(k) + \delta y(k) \quad (10)$$

Which results in the system given by

$$\begin{bmatrix} z_{\epsilon_1}(k+1) \\ z_{\epsilon_2}(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \epsilon(k) \quad (11)$$

## 4 Example of implementation in UPPAAL

UPPAAL is a validation- and verification tool for real time systems, which has been developed in cooperation between Uppsala University and Aalborg University[11, 14]. The idea is to model a system using timed automata, simulate it and then verify the system properties on it. The tool consists of two main parts, a graphical user interface and a model checker engine.

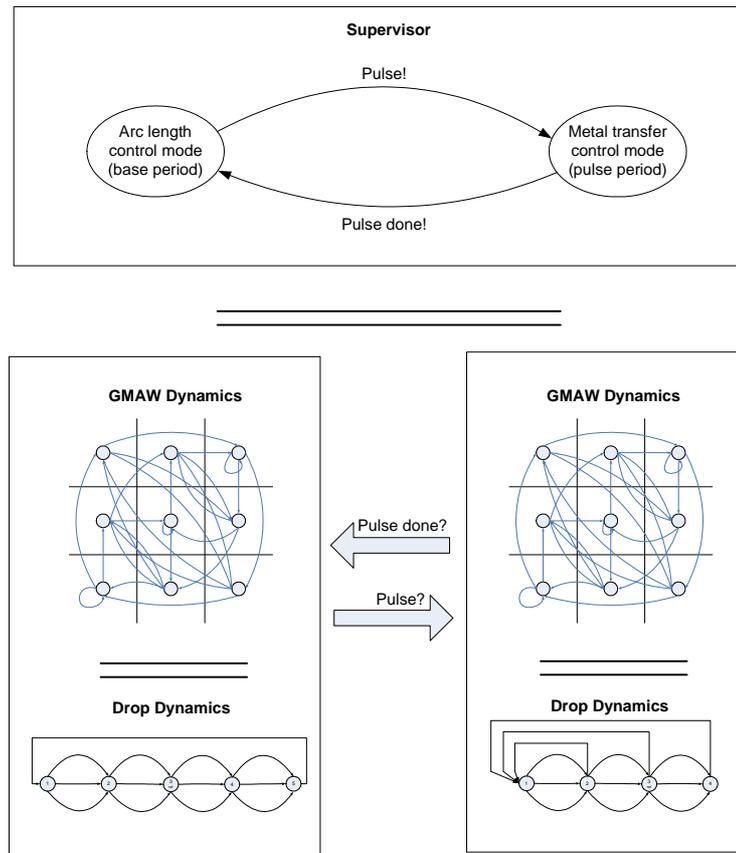
A system consists of a network of automata which are running in parallel. It is possible to step through the system, in order to check if the system behaves as intended and the system can be checked by the verifier by asking different questions such as if a certain state is reachable or if there is any deadlocks in the system. More generally speaking, the verifier can check all possible dynamical behaviors of the system[14].

### 4.1 The Controlled GMAW Process in UPPAAL

An overview of the implemented system is shown in figure 7, where the supervisor automaton controls the underlying automata; the drop dynamics automata and the GMAW dynamics automata. The supervisor decides by its two transitions which control mode the GMAW process should be in by a parallel composition with a shared label space. This is implemented in UPPAAL as events, where the

enabling transition throws an event, designated by a *!*-prefix and is received by a listening transitions, designated by a *?*-prefix. When a transition occurs between the arc length control mode and the metal transfer control mode, a synchronization *pulse!* is enabled. The *pulse!* synchronization enables the GMAW dynamics and the drop dynamics to jump from the base period to the pulse period with the synchronization *pulse?* The *pulse done!* synchronization enables the GMAW- and drop dynamics to jump back to the base period with the synchronization *pulse\_done?*

As pointed out in the previous section then the GMAW dynamics is only parti-

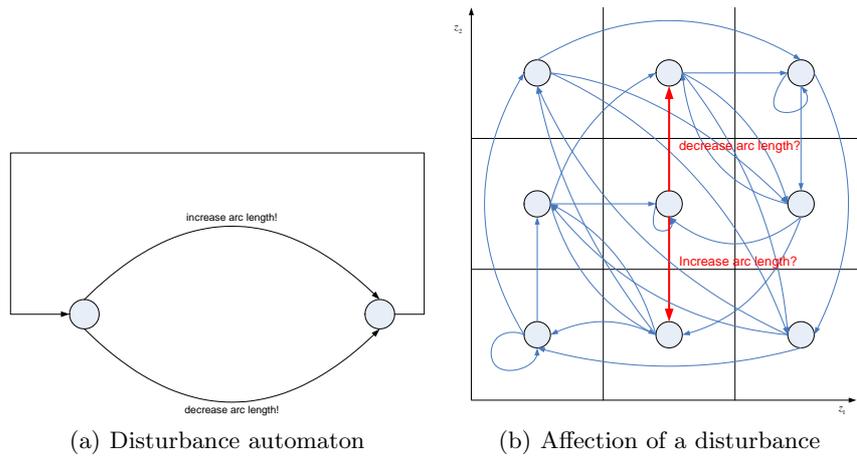


**Fig. 7.** The figure illustrates it is possible to be in two different control modes; arc length control and metal transfer control, which is controlled by the supervisor. In each control mode the processes are running parallel.

tioned into 9 parts. This leads to a timed automaton for the arc length dynamics as shown in the middle of figure 7 with some of the possible transitions displayed.

The automaton consists of 9 states, where each state represents a partition of the state space. The transitions between the states are decided by the shift register form, which is derived in the previous section.

In order to include disturbances into the model a disturbance automaton is included as shown in figure 8(a). It is designed to give a disturbance in the arc length in the base period. If the disturbance automation enables a disturbance (increase/decrease the arc length), it will affect the GMAW dynamics automaton as shown in figure 8(b).



**Fig. 8.** (a) The disturbance automaton. (b) The two thick arrows shows the affect of a disturbance from the disturbance automaton to the GMAW dynamics.

## 4.2 Model Checking

It is possible in UPPAAL to use the model checker to get answers on specific questions, e.g. to check if there are deadlocks in the system. The deadlock check can be seen as a basic check of the systems behavior. By checking reachability and liveness properties the performance of a supervisor or controller can be analyzed. In UPPAAL the query language used is a simplified version of Computation Tree Logic (CTL)[15].

In the following specific questions regarding the GMAW process will be discussed.

**Do deadlocks exists in the system?**

**Query:**

A[] not deadlock

Numerous factors can result in a deadlock in the system; A supervisor design flaw, faulty implementation etc.

**Answer:**

The property is satisfied.

**Do the supervisor continuously cycle between the base period and the pulse period?**

**Query:**

Supervisor.Arc\_length --> Supervisor.Metal\_transfer

Supervisor.Metal\_transfer --> Supervisor.Arc\_length

To guarantee the basic operation of the supervisor, a continuously cycle between the base period and the pulse period should take place. The first expression checks if the path between the states *Supervisor.Arc\_length* and *Supervisor.Metal\_transfer* will eventually be taken. The second expression checks if the path back from the state *Supervisor.Metal\_transfer* to the state *Supervisor.Arc\_length* will eventually be taken.

**Answer:**

The question is satisfied

**Is the duration of the pulse period as specified?**

**Query:**

A[] Supervisor.Metal\_transfer imply x<=600

The duration in the pulse period is set to 600 clock cycles. This question checks if it is possible for the supervisor to jump from metal transfer control to arc length control before the specified time.

**Answer:**

The question is satisfied

The first question checks if there is some states from which the system cannot switch away from, which it is found that there are not. Secondly the liveness of the supervisor is tested. This test can be seen as a check of the supervisor shown in figure 3. It is further interesting to verify if the system is staying too long in the different stages, which is tested in the third query, where the time spend in the metal transfer stage is tested. Similarly to the third query it could be tested if the supervisor is switching too fast between the different stages, which will reveal if there is a possibility for Zeno behavior in the system.

## 5 Discussion

The objective of this paper was to show that it is possible to apply the theories developed in [7] to a given process, in this case the Pulsed GMAW process.

As seen from section 3 then it is possible to formulate the GMAW welding process as a network of timed automata, which directly can be implemented in the simulation and verification tool, UPPAAL, thus giving the possibility of

posing such questions as; is this state always reachable from this state or if it possible to end up in a deadlock. Questions, which is impossible to answer with classical control theory.

## 6 Acknowledgements

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