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Modal Indicators for Operational Modal Identification

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Abstract

Modal validation is of paramount importance for all two-stage time domain modal identification algorithms. However, due to a higher noise/signal ratio in operational/ambient modal analysis, being able to determine the right model order and to distinguish between structural modes and computational modes become more significant than in traditional modal analysis. The two major modal indicators, i.e. Modal Confidence Factor (MCF) and Modal Amplitude Coherence (MAmC) are extended to two-stage time domain modal identification algorithms, together with a newly developed indicator, named as Modal Participation Indicator (MPI). The application of the three indicators is illustrated on different cases of operational/ambient modal identification. Three major time domain modal identification algorithms are used, the Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD), Eigensystem Realization Algorithm (ERA). The three identification algorithms are implemented from a unified point-of-view with the modal indicators. Numerical simulations are conducted on a two-story building structure and on an aircraft model and it is investigated how the modal indicators work to distinguish the physical modes from the computational modes.

Introduction

Operational modal identification has attracted great attention in civil, aerospace and mechanical engineering in recent years. Compared to traditional modal analysis, which is normally conducted in the lab environment making use of both input-output data, operational/ambient modal analysis has many advantages:

- No artificial excitation needed and no boundary condition simulation required;
- Dynamic characteristics of the whole system, instead of component, can be obtained;
- For all or part of measurement coordinates can be used as references, the operational modal identification is always Multi-Input (Reference)-Multi-Output MIMO algorithm. The closed-spaced or even repeated modes can easily be handled, and, therefore, suitable for real world complex structures;
- The model identified under real loading will be linearized, due to broad band ambient/random excitation, for much more representative working points;
- Operational modal identification can not only be utilized for structural dynamics analysis and design, but also In-situ vibration based structural health monitoring and damage identification.

Many time domain MIMO modal identification algorithms such as Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD), Eigensystem Realization Algorithm (ERA) and its extension [1]-[6], etc. have been developed in 1980's. Impulse Response Functions (IRF) is measured at first, normally via inverse FFT from FRF, and then modal parameters are identified via above-mentioned algorithms using IRF data. The 2-stage modal identification techniques have been successfully used for traditional modal analysis. However, they can also be adopted for operational modal analysis. In the 1990's a Natural Excitation Technique (NExT) was proposed [7]. NExT is based on the principle that Correlation Function (CF) measured under natural excitation (or operational/ambient condition) can be expressed as a sum of decaying sinusoids. Each decaying sinusoid has a damped natural frequency, damping ratio and mode shape coefficient that is identical to the one of the corresponding structural mode. According to this principle, all the 2-stage time domain MIMO identification techniques can be adopted for operational/ambient modal identification by using CFs instead of IRFs.

However, all the time domain (TD) modal identification algorithms have a serious problem on model order determination. When extracting physical or structural modes, the TD modal identification algorithm always generates spurious or computational modes to account for unwanted effects, such as noise, leakage, residuals and non-linearity's, etc. The computational modes fulfill an important role in that they permit more accurate modal estimation by supplying statistical DOF to absorb these effects. In the traditional modal identification IRF can be obtained via inverse FFT of Frequency Response Function (FRF), and may have less computational modes. For operational modal identification, which makes use of correlation function calculated from random response data, the model order determination and structural modes distinguishing become much more significant. Therefore, it is extremely important to determine the correct number of model order or total number of modes at first, and then to distinguish structural modes from computational ones. In order to accomplish this important task, many modal validation approaches have been developed.

Modal validation can be performed via three kind approaches: visual inspection, modal indicator and diagram. Visual inspection of mode shapes and comparing measured data with those synthesized from the estimated modal parameters are typical examples of these qualitative approaches. The second kind of approaches make use of quantitative modal indicators, such as Modal Assurance Criterion (MAC), Modal Confidence Factor (MCF), Modal Amplitude Coherence (MAmC), etc. Graphical validation involves tracking the model error, or rank of the data matrix, or estimating frequency, damping as a function of model order. The resulting Error Chart, Rank Chart or Stability Diagram is then utilized for modal validation.

In this paper two modal indicators, MCF and MAmC, are extended and a new one named as Modal Participation Indicator (MPI) is developed for major 2-stage time domain modal identification algorithms. Numerical simulations via a two-story building and an aircraft model are conducted to show the performance of the three modal indicators for operational modal identification algorithms—PRCE, EITD and ERA.

Modal Indicators

1. Modal Assurance Criterion (MAC).

Modal Scale Factor (MSF) and MAC are used widely to compare two modal vectors. The MSF gives a least squares estimate of the ratio between two vectors

$$MSF(\phi_r,\phi_s)=\frac{\phi_r^H\phi_s}{\phi_r^H\phi_r} \tag{1}$$

MAC is defined as [8]

$$MAC(\phi_r,\phi_s)=\frac{\left|\phi_r^H\phi_s\right|^2}{\left[\phi_r^H\phi_r\right]\left[\phi_s^H\phi_s\right]} \tag{2}$$

Which is actually the square of the correlation coefficient of the two modal vectors. If MAC is unity the two modal vectors are identical within modal scale factor. Therefore, the MAC can be utilized as a modal indicator for different modal estimates.

2. Modal Confidence Factor (MCF)

Ibrahim introduced the concept of MCF by generating pseudo-measurements in the ITD modal identification algorithm [9]. These pseudo-measurements are actually delayed physical time signals. MCF exploits redundant phase relationships that are satisfied by physical modes, but which are meaningless for computational modes. The MCF has been extended for the PRCE [10] For r-th mode MCF can be calculated by the following formula

$$MCF_r=\frac{\phi_r^H\overline{\phi}_r}{\phi_r^H\phi_r}e^{-\lambda_r p\Delta t} \tag{3}$$

Where λ is the eigenvalue, Δt is the sampling time interval and p is a positive integer. For a physical mode, the MCF would be unity, whereas computational modes would have a MCF of arbitrary phase and amplitude. A MCF close to one is thus a necessary, but not sufficient reason for an eigenvector to be associated with a physical mode.

It is obvious that MCF can also be used for other 2-stage TD modal identification algorithms, such as EITD, ERA, etc. MCF is a complex number. For simplicity only the norm can be used for modal indicator. The main drawback of the method is that the amount of the data is doubled.

3. Modal Amplit6ude Coherence (MAmC)

MAmC was proposed by the authors of ERA [11] for distinguishing structural modes from noise modes with ERA. We have extended the MAmC to all 2-stage TD modal identification algorithms (PRCE, EITD, ERA, etc). The basic formulation for MAmC is derived as follows.

For a linear system, the map from input to output can be described by Markov parameter (Impulse Response Function in traditional modal analysis or Covariance Functions in Ambient modal analysis) sequence

$$Y=\left[Y_0\ Y_1\ Y_2\cdots Y_{Nt-1}\right] \tag{4}$$

Where Nt is the number of the data points. In the modal coordinate the Markov parameter can be expressed as

$$Y_k=\sum_{r=1}^{n}\phi_r\lambda_r^{k-1}\gamma_r^H \tag{5}$$

Where ϕ_r, λ_r and γ_r are r-th modal vector, eigenvalue and modal participation factor, respectively. Define the sequence

$$\hat{q}_r=\left[\hat{\gamma}_r^H\ \hat{\lambda}_r\hat{\gamma}_r^H\ \hat{\lambda}_r^2\hat{\gamma}_r^H\cdots\hat{\lambda}_r^{Nt-1}\hat{\gamma}_r^H\right] \tag{6}$$

Which represents the time series reconstructed from the identified eigenvalue and modal participation factor. The Markov parameter becomes

$$\hat{Y}_k = \sum_{r=1}^{n} \hat{\phi}_r \hat{q}_r \quad (7)$$

It can be seen that the sequence q_r is associated with mode shape ϕ_r, and is called the identified Modal Amplitude time history for the r-th mode. The modal amplitude can also be calculated directly from measured Markov parameters via SVD of Hankel matrix, and denoted as q_r. With noise-polluted data and nonzero singular values truncated, the identified modal amplitude is an approximation of the one calculated directly from measured Markov parameters. The MAmC can then be defined as correlation coefficient or coherence function of the two modal amplitude vectors as

$$MAmC = \frac{\left|\hat{q}_r^H \bar{q}_r\right|}{\left(\left|\bar{q}_r^H \bar{q}_r\right|\left|\hat{q}_r^H \hat{q}_r\right|\right)^{1/2}} \quad (8)$$

4. Modal Participation Indicator (MPI)

In the ambient/operational modal analysis, Correlation or Covariance Function can be measured as Markov parameter, and expressed via eigenvalue, modal vector (mode shape) and modal participation factor:

$$Y_k = \sum_{r=1}^{n} \phi_r \lambda_r^{k-1} \gamma_r^T \quad (9)$$

Choosing all the measurement coordinates as references, the dimension of modal partition vector is then equal to corresponding mode shape. We can therefore define Modal Participation Scale (MPS) α_r as

$$\gamma_r = \alpha_r \phi \quad (10)$$

The contribution of the r-th mode to the covariance matrix can then be expressed as

$$Y_k^r = \alpha_r \phi_r \phi_r^H \lambda_r^{k-1} \quad (11)$$

MPI represents a kind of "kinetic energy" in time domain, and can be adopted as a modal indicator to distinguish structural and computational modes. MPI can be calculated via least square solution of the two vectors as the following formula

$$MPI_r = \alpha_r = \frac{\left|\phi_r^H \gamma_r\right|}{\phi_r^H \phi_r} \quad (12)$$

When implementing, r-th modal participation indicator MPI_r is normalized as the percentage of the "total energy".

Numerical Simulations for Operational Modal Identification

The MAmC, MPF and MCF are applied in major 2-stage time domain modal identification algorithms and applied to operational modal identification as modal indicators.

Three major time domain modal identification algorithms, PRCE, EITD and ERA, are implemented via unified point-of-view as follows:

- Establish Hankel matrices H_0 and H_1 from measured covariance functions;
- Calculate system matrix via least squares solution from Hankel matrices for PRCE or EITD;
- Calculate system input and measurement matrices via singular value decomposition for ERA;
- Eigenvalue solution of system matrix to obtain eigenvalues and mode shapes for EITD or eigenvalues and modal participation vectors for PRCE;
- Least squares solution to obtain modal participation vectors for EITD, and mode shapes for PRCE;
- For ERA, eigenvalue solution of the system matrix to obtain eigenvalues and mode shapes together with measurement matrix, and modal participation vectors from input matrix;

Two examples with closely spaced modes are used to show the performance of the different modal indicators.

1. Two-story Building

The first numerical example is a two-story building, which is simulated by a lumped parameter system with 6 degrees of freedom. The measurements are assumed to be taken so that the rigid body motions of the floor slaps can be estimated. The geometry and the measurement points are shown in Figure 1. This structure has two sets of close modes. The first two modes are first bending modes, and these two bending modes are close, but not very close. The third mode is a torsion mode. The fourth and fifth modes are very closed second bending modes. Figure 2 depicts first 5 modes. The response was simulated using a vector ARMA model to ensure that the simulated responses were covariance equivalent [13]. The model was loaded by white noise, and the response was analyzed using the 2-stage time domain identification techniques introduced above. The simulated time series had a length of 10,000 data points with 20 % noise added.

Computer simulations of operational modal identification were conducted using PRCE, EITD and ERA with MAmC, MPI and MCF as modal indicators. Table 1 to 3 present MAmC and MPI results via PRCE, EITD and ERA identification, respectively. Tables 4 and 5 show the results of MCF via EITD and PRCE separately for double data are needed in order to compute MCF. The main parameters to be selected in the numerical simulation are the number of total modes (n) and the number of data points. In the Tables "*" denotes the target modal frequencies. The range of damping ratio, 0-5 %, is used as the first "filter" to eliminate computational modes. It can be seen that all three modal indicators work pretty well in distinguishing structural modes from computation modes. Compared to MAmC and MCF, the newly proposed MPI has better performance.

2. GARTEUR Aircraft Model

An aircraft model called GARTEUR developed by the

Group of Aeronautical Research and Technology in EURope is adopted as the second example [14]. The model represents the dynamic characteristics of real world aircraft, and is widely used in Europe. The main requirement for the GARTEUR model is to simulate dynamic characteristics of real world aircraft. GARTEUR model has the following features: (1) A group of 3 very closely spaced modes, (2) Frequency range from 5 to 60 Hertz, (3) Special damping. Treatment via adding visco-elastic materials on the wing surface; (4) A joint at the wing/fuselage connection for transportation with model dimension of 2 by 2 meters

The Finite Element Model (FEM) of Garteur consists of 51 three-dimensional beam elements and 68 nodal points with altogether 408 DOF model. Figure 3 presents the first 6 modes of GARTEUR model. The first six natural frequencies are: 6.09Hz, 15.80Hz, 33.01Hz, 33.66Hz, 35.14Hz and 49.79 Hz.

Markov parameters are synthesized from the modal parameters calculated from FEM with 1.00% damping ration added. Altogether 24 DOFs are selected as measurement locations. To simulate the noise-pollution test data, 10% Gaussian distributed noise is added to synthesized Markov parameters. Sampling frequency is 150Hz with 1024 sampling points

As for the 2-story building case, all three modal identification algorithms are used for GARTEUR example. However, the simulation data are synthesized using 2-input 24-output measurement, therefore, only MAmC and MCF are adopted for modal indication. Tables 6 to 10 show the performance of MAmC and MCF for operational modal identification algorithms PRCE, EITD and ERA. It is observed that the two modal indicators exhibit favorable performance

Concluding Remarks

1. Two modal indicators, Modal Confidence Factor (MCF) and Modal Amplitude Coherence (MAmC) are extended to major 2-stage time domain operational modal identification algorithms;
2. A new modal indicator named as Modal Participation Indicator (MPI) is developed and implemented;
3. Three major operational/ambient modal identification algorithms, Polyreference Complex Exponential (PRCE), Extended Ibrahim Time Domain (EITD) and Eigensystem Realization Algorithm (ERA), are implemented from unified point-of-view together with three modal indicators;
4. Numerical simulations are conducted using two examples: 2-story building and an aircraft model. The results show that all three modal indicators work pretty well in distinguishing structural modes from computational ones;
5. MCF needs double data, and hence more computing intensive and time consuming; MAmC often results with the number closed to unity and some times is hardly to separate noise modes from the structural ones;
6. Newly proposed Modal Participation Indicator (MPI) can clearly indicate the structural modes in most cases, and performs better than the other two indicators;
7. The identification results are normally depending on the parameter selection for most of 2-stage time domain modal identification. To finally determine the true structural modes Stability Diagram is suggested together with modal indicators.

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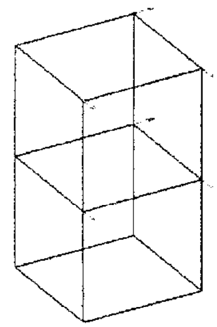
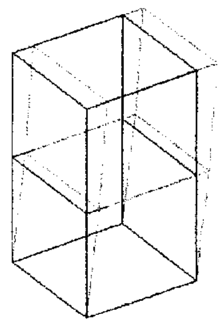
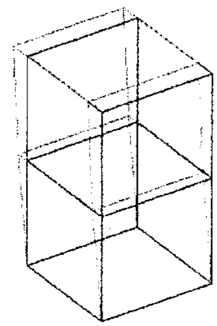


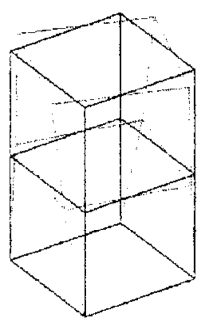
Fig.1 2-story building



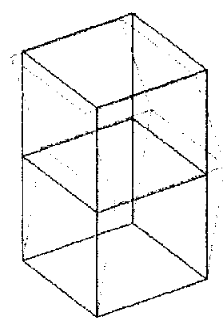
(a) f_1=18.69, ξ_1=0.0213



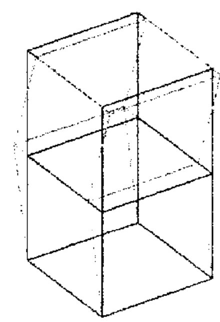
(b) f_2=21.05, ξ_2=0.0189



(c) f_3=38.17, ξ_3=0.0104



(d) f_4=55.06, ξ_4=0.0072



(e) f_5=55.12, ξ_5=0.0072

Fig.2 First Five Modes of 2-story building

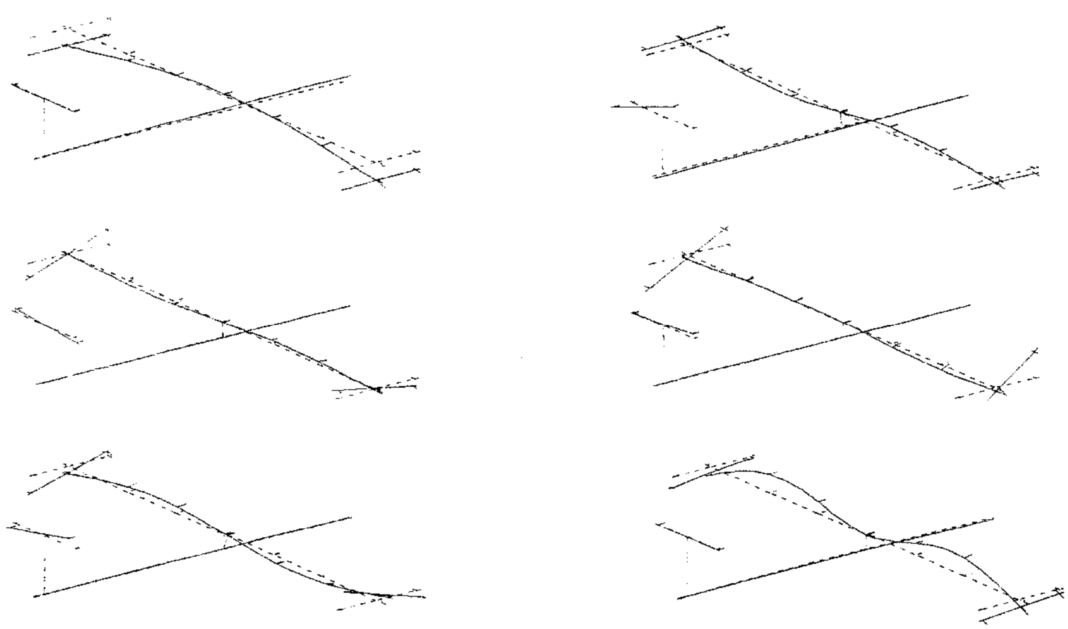


Figure 3 The First Six Modes of the GARTEUR Aircraft Model

Table 1 Results of EITD for Two-Story Building (n=12)

Mode	Freq (Hz)	Damp.(%)	**MamC**	**MPL(%)**
1	18.69*	2.46	1.00	3.02
2	20.97*	2.14	1.00	7.11
3	38.08	1.86	1.00	0.01
4	38.14*	0.89	1.00	7.64
5	55.03*	0.65	1.00	26.05
6	55.08*	0.64	1.00	55.67
7	55.13	2.30	0.99	0.11
8	66.64	4.33	0.90	0.38

Table 2. Results of PRCE for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	**MamC**	**MPL(%)**
1	18.76*	2.61	1.00	1.89
2	20.88*	1.87	1.00	4.32
3	38.14	0.79	0.98	0.60
4	38.15*	0.99	1.00	4.84
5	54.95	2.26	0.86	0.20
6	55.02*	0.61	0.81	42.68
7	55.09*	0.67	1.00	44.92
8	62.01	2.22	0.67	0.40
9	69.39	0.23	0.24	0.15

Table 3. Results of ERA for Two-Story Building (n=8)

Mode	Freq.(Hz)	Damp.(%)	**MAmC**	**MPL(%)**
1	18.68*	2.19	1.00	18.62
2	20.93*	1.88	1.00	18.56
3	38.16*	1.03	1.00	18.36
4	54.62	1.40	0.96	6.22
5	55.01*	0.55	1.00	15.59
6	55.17*	0.54	1.00	15.94
7	59.19	3.71	0.54	6.70

Table 4. Results of EITD for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	**MCF**
1	18.70*	2.15	0.98
2	20.91*	1.80	0.94
3	37.31	1.75	0.21
4	38.12*	1.32	0.53
5	38.19*	0.86	0.93
6	55.00*	0.74	0.89
7	55.03	1.11	0.71
8	55.16*	0.54	0.93
9	55.49	3.80	0.06
10	61.10	2.37	0.20

Table 5. Results of PRCE for Two-Story Building (n=15)

Mode	Freq.(Hz)	Damp.(%)	**MCF**
1	18.65*	2.20	0.92
2	21.01*	1.65	0.93
3	22.78	3.29	0.71
4	35.05	1.56	0.43
5	38.15*	1.00	0.93
6	38.38	1.22	0.92
7	53.57	0.46	0.92
8	54.98*	0.61	0.94
9	55.13*	0.57	0.94
10	55.23	1.03	0.89
11	55.37	3.06	0.72
12	58.29	0.34	0.80
13	61.22	0.02	0.33
14	62.41	4.21	0.36
15	66.52	4.32	0.72

Table 6. Results of EITD for GARTEUR.(n=48)

Mode	Freq.(Hz)	Damp.(%)	**MAmC**
1	6.11*	1.10	1.00
2	13.52	0.27	0.95
3	15.80*	0.99	1.00
4	17.64	0.77	0.75
5	19.81	4.87	0.98
6	20.58	1.05	0.89
7	26.66	0.82	0.90
8	28.16	0.31	0.92
9	29.40	0.65	0.90
10	30.16	2.45	0.97
11	33.00*	1.20	1.00
12	33.51*	1.31	1.00
13	33.84	3.78	0.99
14	35.09*	0.91	1.00
15	37.01	4.06	0.99
16	37.01	4.06	0.99

Table 7. Results of PRCE for GARTEUR.(n=20)

Mode	Freq.(Hz)	Damp.(%)	**MAmC**
1	6.09*	1.05	1.00
2	7.97	0.64	0.37
3	11.73	8.65	0.56
4	15.80*	0.76	1.00
5	17.73	0.74	0.72
6	20.39	3.24	0.99
7	24.38	2.16	0.83
8	26.65	1.17	0.55
9	30.67	3.47	0.98
10	33.00*	1.14	0.99
11	33.74*	0.93	1.00
12	35.20*	0.53	0.99
13	37.51	2.03	1.00

Table 8. Results of ERA for GARTEUR.(n=20)

Mode	Freq.(Hz)	Damp.(%)	**MamC**
1	6.09*	1.03	1.00
2	15.80*	1.02	1.00
3	17.19	28.33	0.29
4	24.26	22.65	0.22
5	32.99*	1.02	1.00
6	33.65*	1.11	1.00
7	33.71	13.05	0.47
8	35.12*	1.02	1.00
9	43.06	8.29	0.48
10	45.04	27.86	0.39
11	47.16	7.77	0.21
12	52.21	10.04	0.41
13	56.89	9.51	0.30
14	62.93	5.37	0.33
15	63.77	16.11	0.16
16	71.26	6.51	0.32

Table 9. Results of EITD for GARTEUR (n=48)

Mode	Freq.(Hz)	Damp.(%)	**MCF**
1	6.10*	0.91	0.99
2	10.24	3.53	0.64
3	11.45	1.89	0.68
4	12.63	0.58	0.68
5	15.81*	0.85	0.94
6	17.13	4.38	0.33
7	18.92	1.43	0.27
8	20.10	2.18	0.84
9	24.05	3.59	0.33
10	24.79	4.53	0.17
11	26.19	2.67	0.35
12	27.04	2.64	0.45
13	27.97	4.60	0.12
14	28.64	1.81	0.85
15	29.68	1.46	0.40
16	31.51	3.03	0.58
17	31.61	2.11	0.65
18	32.968	0.89	0.88
19	33.63*	1.00	0.94
20	34.24	3.18	0.22
21	35.13*	1.01	0.98
22	36.10	0.63	0.26

Table 10. Results of PRCE for GARTEUR (n=30)

Mode	Freq.(Hz)	Damp.(%)	**MCF**
1	6.09*	1.03	0.99
2	15.86*	0.69	0.97
3	24.10	2.14	0.79
4	29.94	0.39	0.55
5	29.94	0.39	0.55
6	32.99*	1.08	0.90
7	33.76*	0.88	0.94
8	35.06*	0.92	0.92