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Automated Frequency Domain Decomposition for Operational Modal Analysis

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Nomenclature

t, τ	Time
f	Frequency
$\mathbf{y}(t)$	System response
N_m	Number of measurement channels
$\boldsymbol{\varphi}, \boldsymbol{\Phi}$	Mode shape, mode shape matrix
$\mathbf{C}$	Covariance matrix
$\mathbf{G}$	Spectral density matrix
$\mathbf{u}, \mathbf{U}$	Singular vector, Matrix of singular vectors
$[s_i]$	Diagonal matrix of singular values
d_1, d_2	Discriminator functions
Ω_1, Ω_2	Threshold levels
μ, σ, γ	Mean, standard deviation, kurtosis

Abstract

The Frequency Domain Decomposition (FDD) technique is known as one of the most user friendly and powerful techniques for operational modal analysis of structures. However, the classical implementation of the technique requires some user interaction. The present paper describes an algorithm for automated FDD, thus a version of FDD where no user interaction is required. Such algorithm can be used for obtaining a default estimate of modal parameters in commercial software for operational modal analysis - or even more important – it can be used as the modal information engine in a system for structural health monitoring.

Introduction

Frequency domain techniques have always been popular. Even among people who will state that they only use time domain techniques for modal identification, as soon as they get new data in their hands, the first thing they will normally do is to take a look at some frequency domain functions. For Operational Modal Analysis frequency domain techniques are based on spectral density functions, Bendat and Piersol [1].

Working directly with spectral density function has been popular and is still used a lot; see for instance Felber [2]. One of the problems working directly with the spectral density functions is the amount of data the user has to work with simultaneously. For instance in a case with $N_m = 8$ channels of measurements, the user has to deal with $(8+8^2)/2 = 36$ different spectral density functions. Further, even though the spectral densities by directly depicting the modal peaks and thus gives a direct indication of the presence of modes, the spectral density function in itself does not provide the user with modal information since the spectral density function is linear combination of the modal responses. Therefore working directly with spectral density functions will limit modal identification to cases with well separated modes.

The Frequency Domain Decomposition technique is a way to solve these two problems, Brincker et al [3], [4]. The technique simplifies the user interaction because the user has only to consider one frequency domain function - the singular value plot of the spectral density matrix. This plot concentrates information from all spectral density functions. Further, if some simple assumptions are fulfilled, the technique directly provides a modal decomposition of the vibration information, and the modal information for each mode – even in the case of closely spaced modes and noise – can be extracted easily and accurately.

The principle in the Frequency domain Decomposition (FDD) techniques is easiest illustrated by realizing that any response can by written in modal co-ordinates

$$\mathbf{y}(t) = \boldsymbol{\varphi}_1 q_1(t) + \boldsymbol{\varphi}_1 q_2(t) + ... = \boldsymbol{\Phi}\mathbf{q}(t) \tag{1}$$

Now obtaining the covariance matrix of the responses

$$\mathbf{C}_{yy}(\tau) = E\left\{\mathbf{y}(t+\tau)\mathbf{y}(t)^T\right\} \tag{2}$$

and using equation (1) leads to

$$\begin{aligned}\mathbf{C}_{yy}(\tau) &= E\left\{\boldsymbol{\Phi}\mathbf{q}(t+\tau)\mathbf{q}(t)^T\boldsymbol{\Phi}^T\right\} \\ &= \boldsymbol{\Phi}\mathbf{C}_{qq}(\tau)\boldsymbol{\Phi}^T\end{aligned} \tag{3}$$

Then by taking the Fourier transform

$$\mathbf{G}_{yy}(f) = \boldsymbol{\Phi}\mathbf{G}_{qq}(f)\boldsymbol{\Phi}^T \tag{4}$$

Thus if the modal co-ordinates are un-correlated, the power spectral density matrix $\mathbf{S}_{qq}(f)$ of the modal co-ordinates is diagonal, and thus, if the mode shapes are orthogonal, then Eq. (4) is a singular value decomposition (SVD) of the response spectral matrix.

Therefore, FDD is based on taking the SVD of the spectral density matrix

$$\mathbf{G}_{yy}(f) = \mathbf{U}(f)[s_i]\mathbf{U}(f)^T \tag{5}$$

The matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \cdots]$ is a matrix of singular vectors and the matrix $[s_i]$ is a diagonal matrix of singular values. As it appears from this explanation, plotting the singular values of the spectral density matrix will provide an overlaid plot of the auto spectral densities of the modal coordinates. Note here that the singular matrix $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \cdots]$ is a function of frequency because of the sorting process that is taking place as a part of the SVD algorithm. A mode is identified by looking at where the first singular value has a peak, let us say at the

frequency f_0. This defines in the simplest form of the FDD technique - the peak picking version of FDD - the modal frequency. The corresponding mode shape is obtained as the corresponding first singular vector $\mathbf{u}_1$ in $\mathbf{U}$.

$$\boldsymbol{\varphi} = \mathbf{u}_1(f_0) \tag{6}$$

Introducing modal discrimination

The process of findings peaks on a function is actually easy to automate. However, we need to define indicators that can help us distinguishing between different modes and between modes and noise.

Let us say that we have identified a peak in the first singular value. The question is now if this is a liable modal peak or is if it just a noise peak. Calculating the correlation between the first singular vector at the peak – the mode shape vector at that point - and the first singular vector at neighboring points defines the discriminator function called the modal coherence

$$d_1(f_0) = \mathbf{u}_1(f)^T \mathbf{u}_1(f_0) \tag{7}$$

If the modal coherence is close to unity, then the first singular value at the neighboring point correspond to the same modal coordinate, and therefore, the same mode is dominating. This function is helpful in discriminating between points dominated by modal information and points dominated by noise. If the components of each of the vectors in Eq. (6) are random, then

$$E\left\{\mathbf{u}_1(f_0)^T \mathbf{u}_1(f)\right\} = 0 \tag{8}$$

and since the length is unity

$$Var\left\{\mathbf{u}_1(f_0)^T \mathbf{u}_1(f)\right\} = 1/N_m \tag{9}$$

Thus the more measurement channels we have the closer two points with random (non-physical) information will get to zero. A reasonable criterion for accepting the neighboring point as a point with similar physical information, and thus accepting the presence of physical information at that frequency, could be by introducing a threshold level Ω_1 and the requirement

$$d_1 \geq \Omega_1 \tag{10}$$

setting the limit Ω_1 equal to a number n times the standard deviation of the correlation for random vectors as given by Eq. (9)

$$\Omega_1 = n/\sqrt{N_m} \tag{11}$$

where n could be chosen in the region 3-5. This criterion is strongly dependent upon the numbers of measured channels N_m, if we choose $n = 3$ and $N_m = 16$, then $\Omega_1 = 0.75$, if $N_m = 10$, then $\Omega_1 = 0.95$. Thus for channel counts lower than say 16, the criterion becomes of less value using only correlation between two points. In this case several points on each side of the peak can be combined to calculate the correlation between the considered peak point and a set of neighboring points increasing the effective value of N_m correspondingly.

Once a peak has been accepted as representing modal information, another discriminator function can be helpful in discriminating between different modes. In this case the discriminator function is defined as

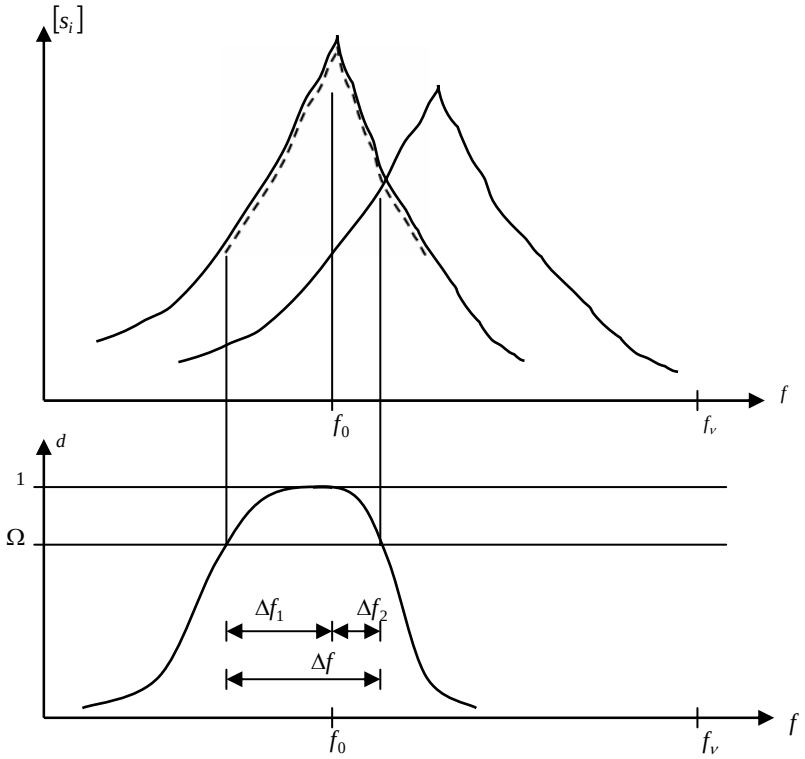

Figure 1. *Illustration of the definition of the modal domain for of a considered mode. The top picture shows the modal decomposition using the SVD of the spectral density matrix. Bottom picture shows how the modal domain is defined by the part of the discriminator function* $d_2 \geq \Omega_2$.

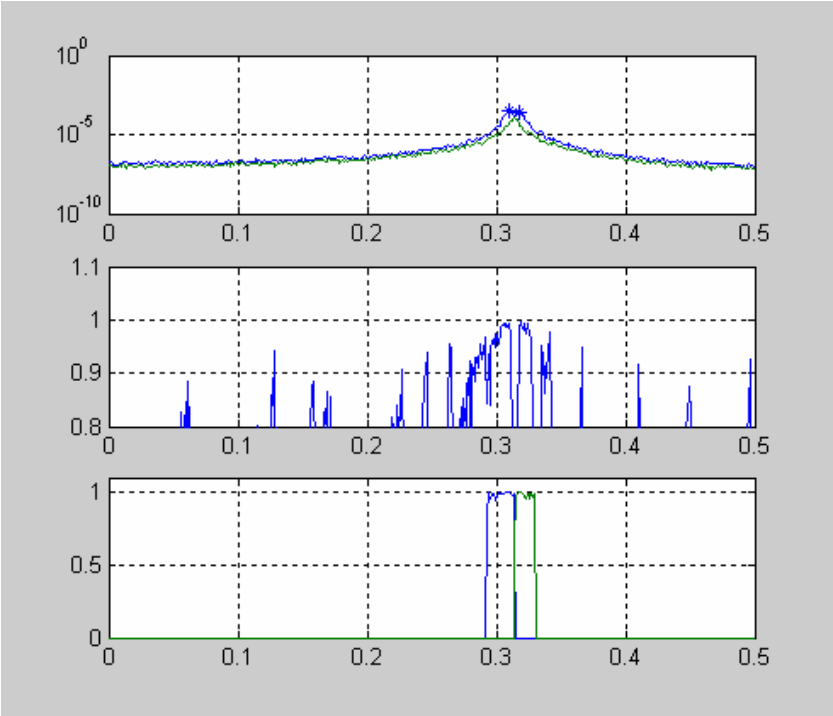

Figure 2. *Example of modal discrimination. Top: SVD of two closely spaced modes measured in two channels. Middle: modal coherence function* d_1, *bottom: modal domain function* d_2

$$d_2(f) = \mathbf{u}_1(f)^T \mathbf{u}_1(f_0) \tag{12}$$

Thus this discriminator function is not a function of the initial point given by the frequency f_0, but is a function of the frequency f of the considered neighboring point. If a high correlation is present over a certain frequency range around the considered peak it means that over that frequency range only that mode is dominating and introducing a similar criterion

$$d_2 \geq \Omega_2 \tag{13}$$

defines a frequency range $[f_0 - \Delta f_1; f_0 - \Delta f_2]$ around each peak of modal dominance called the modal domain, see Figure 1. The lower the value Ω_2, the larger the size $\Delta f = \Delta f_1 + \Delta f_2$ of the corresponding modal domain. An example of discriminator functions are shown in Figure 2.

Introducing harmonic discrimination

One important problem often arising in practice is when harmonics are present in the signal. A harmonic is easily confused with a modal peak if not special measures are taken to avoid mistakes. The reason is that a harmonic appear as a narrow peak in the spectral density functions, thus the peak will also be present in the singular values.

An efficient way to discriminate harmonics is by the statistical characteristics of the response in a narrow frequency band around a harmonic peak. It is well known that the statistical properties of a harmonic are different from the properties of a stochastic response. Due to the central limit theorem, and the fact that in practice a structure is loaded by many stochastically independent forces, the stochastic distribution of a modal response will be close to Gaussian. Further, the distribution of a harmonic is different from Gaussian since it has two distinctive peaks where the distribution goes to infinity, see Bendat and Piersol [1], see Figure 3. This difference between stochastic and harmonic response was proposed as a basis for harmonic discrimination in Brincker et al [5].

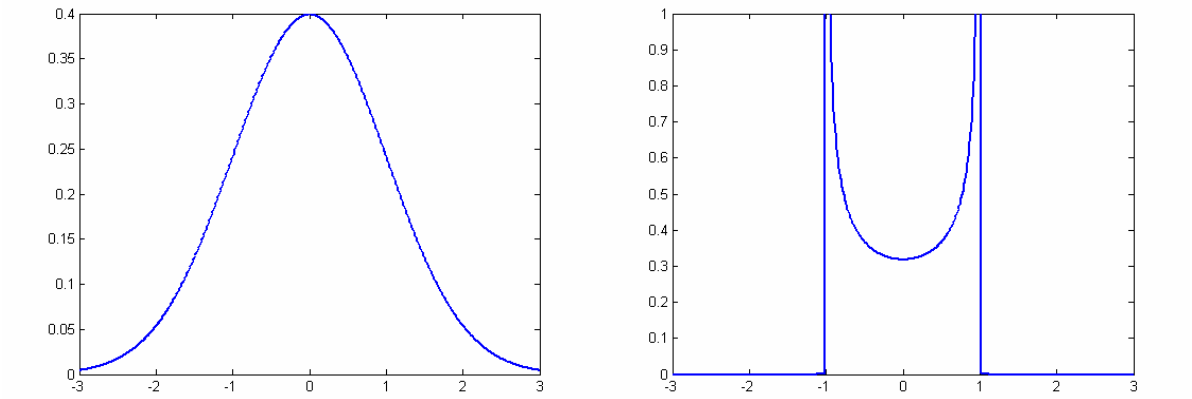

Figure 3. *Normalized PDF of the response of a pure structural mode (left) and pure harmonic component (right)*

In Jacobsen et al [6] it is shown how to use the kurtosis to discriminate between modal peaks and harmonic peaks. The kurtosis γ of a stochastic variable x provides a way of expressing how peaked or how flat the probability density function of x is. The kurtosis is defined as the fourth central moment of the stochastic variable x normalized with respect to the standard deviation σ as follows

$$\gamma\left(x|\mu,\sigma\right)=\frac{E\left\{\left(x-\mu\right)^4\right\}}{\sigma^4} \tag{14}$$

Often the number 3 is subtracted from equation (1) as this gives a kurtosis of zero, when x follows as normal distribution

$$\gamma^*\left(x|\mu,\sigma\right)=\gamma\left(x|\mu,\sigma\right)-3 \tag{15}$$

Using Eq. (15), a PDF with a positive kurtosis is said to be leptokurtic. If its kurtosis is negative, it is said to be platykurtic. A PDF with kurtosis equal to zero is called mesokurtic. Leptokurtosis is associated with PDFs that are simultaneously "peaked" and have "fat tails." Platykurtosis is associated with PDFs that are simultaneously less peaked and have thinner tails. For the response of a pure structural mode, the PDF will be normally distributed, and hence the kurtosis $\gamma^* = 0$ (mesokurtic). For a sinusoidal component $\gamma^* = -1½$. This fact is used in the harmonic detection technique described further in Jacobsen et al [6]. Further in this paper it is described how the influence of harmonics can be removed from the measured response when the FDD is used for identification.

FDD automated

The search set includes all points on the first singular value plot that is within a predefined frequency band (as a special case the total frequency band of the vibration data) and is above a predefined excitation level. A procedure can be the following

1. Identify a peak on the first SVD representing a maximum
2. check if the peak is likely to be physical
3. If so, establish the modal domain
4. If not define a noise domain around the peak
5. Exclude the modal domain or noise domain from the search set
6. Continue until the search set is empty, the peak is below the predefined excitation level, or a specified number of modes has been estimated

The key point of the algorithm is point 2). As described earlier, it is essential at this point to include a criterion concerning the correlation between neighboring points as described by the modal coherence function d_1. Also it is essential to be able to distinguish between a harmonic peak and a modal peak. Additional criteria can be based on for instance the size of the modal domain being larger than a certain value or the damping estimate being below a certain value. Calculating the damping it might be useful to isolate the modal coordinate by using the a modal filter as proposed by Zhang [7], thus the auto spectral density for the modal coordinate is calculated by

$$G_{qq}(f)=\mathrm{Re}\left\{\boldsymbol{\varphi}^T\mathbf{G}_{yy}(f)\boldsymbol{\varphi}\right\} \tag{16}$$

And then the damping is extracted from the decay of the corresponding auto correlation function.

When doing automated identification quite often we have a'priori information about what mode we are looking for. In this case we simply use the a'priori mode shape $\boldsymbol{\varphi}$ in stead of $\mathbf{u}_1(f_0)$ when calculating the modal coherence function d_1.

If we are looking for a certain number of modes, we can take the modes that have the largest modal domain, or could take the modes that have the largest excitation level.

For peak picking FDD we are satisfied when the peak is identified as a modal peak, and the corresponding mode shape vector is estimated.

For enhanced FDD the following step is to estimate the auto correlation function for the modal coordinate as described above, see more information in Brincker et al [3],[4]. The correlation level Ω used for estimation of the

part of single degree of freedom bell function that is going to be used in the inverse Fourier transform can be equal to Ω_2, or a different value for Ω can be used. Identifying the damping and natural frequency from the corresponding correlation function is easily automated, since a robust algorithm is based on excluding some points in the beginning of the correlation function and using only the function down to a certain decay level, for instance using from 0.95 down to 30 % of the envelope.

Conclusions

The proposed algorithms have been tested on different data. The conclusion is that the proposed technique is useful and robust and in many cases provides information similar to what can be achieved by manual interaction.

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