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Estimating Modal Parameters of Civil Engineering Structures subject to Ambient and Harmonic Excitation

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ABSTRACT

In this paper addresses the problems of separating structural modes and harmonics arising from sinusoidal excitation. Though the problem is mostly know in mechanical engineering applications such as rotating machinery, some civil engineering applications experiences the same challenges. A robust and fast harmonic detection procedure is presented and illustrated on a civil engineering case.

1 INTRODUCTION

One of the major advantages of testing civil engineering structures compared to mechanical structures is that the ambient excitation nearly always is broad-banded and multiple input. This makes the response measurements obtained from such structures extremely suitable for all popular estimation algorithms in Operational Modal Analysis. They all rely on the assumptions that the input forces are derived from Gaussian white noise and are exciting in multiple points.

Mechanical engineering structures such as engines and other structures having rational part tend to be much more difficult to handle for most algorithms, especially if the sinusoidal forces have more energy than the ambient excitation. In this case the structural modes typically are weakly excited and sometimes they are more or less drowning in the noise. To account for this the measurement systems used must a high measurement range to be able to catch the weak structural response and at the same time prevent clipping from the strong sinusoidal forces.

However, even with good measurements system it is impossible to prevent the harmonics from the sinusoidal excitation to appear in the acquired data, which means that also the modal estimation algorithms must be able to handle the presence of harmonics. Further, it turns out that the presence of harmonics not only is limited to mechanical application, there is a range of cases where civil engineers have to face the harmonics presence as well. Large structures like gravity dams

have rotating parts in terms of the turbines, production facilities in cement and mining industry have large rotating parts as well and bell towers exhibits sinusoidal excitation during ringing with the bells.

In this paper, we will present a fast algorithm for detection of harmonics originating from sinusoidal excitation. The technique consists of two steps; first a fast search for potential harmonics is performed in frequency domain. In step two a statistical assessment of the potential harmonics is made to determine which in fact are harmonics. In the following, step 2 is described first running over all discrete frequencies between DC and the Nyquist frequency. After that we describe how to optimize the algorithm by the introduction of step 1.

When the harmonics are detected, the information is fed to the modal estimation algorithm, enabling it to account for the harmonic presence. The harmonic detection approach will be demonstrated on a civil engineering case; A gravity dam.

2 TESTING FOR HARMONICS AT SPECTRAL FREQUENCIES

2.1 *The Central Limit Theorem*

According to the central limit theorem the distribution of the response of a structural system subject to multiple random inputs will tend to a Gaussian distribution as the number of independent input goes to infinity.

If the distribution of the different inputs have a bell shaped distribution indicating that most amplitudes will be close to their mean value, which is typical for wind and wave loading, then only a few number of inputs are necessary for the response to become approximately Gaussian. However, if the input on the other hand is dominates by amplitudes far from the mean value, which is the case of a sinusoidal excitation, then it takes much more inputs before the structural response will turn Gaussian, Wirsching et al. [1].

Therefore, testing of the shape of Probability Density Function (PDF) of the measured response is an effective way to detect if a few sinusoidal excitation forces are presents. Especially, if the response is examined in narrow frequency intervals, it is possible to obtain information about which intervals that are dominated by harmonics and which are not.

2.2 *Testing PDF's Shape using Kurtosis*

There are a numerous ways to test if sampled data has a specific PDF or not, like the X^2-test, Papoulis [2], most in some ways based on the sampled mean value μ and the sampled standard deviation σ. Here, we will use the fact that the Kurtosis γ of a the $n_y \times 1$ dimensional vector $y(t)$ of measured response, defined as

$$\gamma = \frac{E\left[(y(t)-\mu)^4\right]}{\sigma^4} \tag{1}$$

for Gaussian distributed data with zero mean value and unit variance, is equal to 3. The Kurtosis for a sinusoidal data with zero mean value, unit variance and a random phase is on the other hand always 1.5.

2.3 *The Basic Testing Algorithm*

In practice we have to be able to test the probability density function in several frequency intervals, characterized by their center frequency f_j, and in several measurement channels $y_i(t)$, for $i = 1$ to n_y. The output of the test algorithm should an indicator H_j, for $j = 1$ to n_f. H_j is a function of the center frequency f_j, where j is all center frequencies we like to test between DC and the Nyquist frequency setting DC to $j = 1$ and the Nyquist frequency to $j = n_f$. The indicator should be 1 at center frequencies where a harmonic is present and otherwise 0.

The algorithm used here contains the following steps:

1. Normalize each measurement channels $y_i(t)$ to zero-mean and unit variance using sampled mean and variance.
2. For all center frequencies f_j of interest, perform a narrow band-pass filtering around f_j.
3. Calculate the Kurtosis γ_{ij} for the at f_j band-pass filtered signal $y_i(t)$.
4. For each center frequency f_j calculate the median value m_j of the Kurtosis γ_j over all measurement channels $y_i(t)$. This median is a robust measure for the mean value used to account for possible outliers due to noise etc.
5. For each center frequency f_j assess if m_j deviates significantly from 3. If so set H_j equal to 1, and otherwise 0.

This algorithm has been tested on a series of real data cases and proven efficient, Jacobsen et al. [3],[4].

3 OPTIMIZING SEARCH FOR POSSIBLE HARMONICS

The major drawback of the basic algorithm for harmonic detection is that it becomes rather time consuming in case of many measurement channels n_y and when testing many frequencies n_f. It would be desirable to find ways of reducing both n_y and n_f.

3.1 *Reducing the Number of Measurement Channels*

Since the spectral density matrix $G_{yy}(f)$ of the measured response at some discrete frequency f typically consist of much more columns than there are modes participating at that frequency, many of the columns of are linear dependent upon each other resulting in a rank deficiency of the spectral matrices. For system with many measurement channels n_y there is therefore typically a substantial amount of redundancy, indicating that it might not be necessary to actually Kurtosis test on all channels. The subset of channels we will test is called the Projection Channels in the following and the number of projection channels denoted n_p.

In case of multiple testing using multiple setups, where a certain amount of sensors are kept at the same locations, while the rest are moved from one measurement setup to another, a good initial choice of projection channels is to choose them as the reference channels.

The quality of this choice can be verified by applying the Singular Value Decomposition (SVD) to the spectral densities matrices $G_{yp}(f)$,

$$USV^H = G_{yp}(f) \tag{2}$$

where index y indicate the measurement vector $y(t)$ and p the subset of channels selected as the projection channels. The matrices $G_{yp}(f)$ and S both have dimension $n_y \times n_p$. S is diagonal matrix consisting of n_p singular values.

By plotting the singular values for all the frequencies all modes in the projection channels will be revealed. If the last plotted singular value forms a horizontal line over the frequency band of interest, and if the other singular values display a good mode separation, then the choice is fine. If not then other and / or more projection channels should be included.

If more projection channels are needed, the channels to look for should contain as much new information as possible about the system compared to the channels already selected. This evaluation can be performed using a simple analysis of the correlation coefficients between the difference measurement channels.

Figure 3.1 display a poor choice of projection channels for a system, whereas figure 3.2 shows an appropriate choice.

3.2 *Reducing the Number of Frequencies Needed Check*

In figure 3.3 the singular values of the spectral densities of a system excited with a broad-banded excitation as well as a sinusoidal excitation. The natural frequency first mode appear at 354 Hz, and only the first singular value is significant larger that the rest at this frequency. This indicates that this particular mode is dominating at this particular frequency. On the other hand, at 374 Hz and even more clear at 748 Hz a distinct narrow peak appear in several of the singular values. The peak at 374 Hz indicates the rotational speed of the harmonic excitation and the 748 Hz is the first over harmonic originating from the same sinusoidal excitation source. That more than one singular value has a peak at these two frequencies indicates that several modes have been significantly excited at these frequencies compare to the surrounding frequencies.

We will make use of this phenomenon that always happens in case of sinusoidal excitation, when the ambient excitation acting on a structure is much weaker. If the sinusoidal excitation is much weaker than the broad-banded ambient excitation, the sinusoidal excitation becomes negligible and does not affect the modal analysis algorithms.

If an abrupt change happens at the same frequency at least in two singular value lines, then we have detected a potential harmonic. This potential harmonics should then be tested using the basic algorithm described in section 2.3. In this way it is possible to limit the number of times that the Kurtosis needs to be checked. In the example in figure 3.3 it reduces the number of times from 1024 to 7 times.

There are several ways to test if there is an abrupt change on a curve, see e.g. Basseville et al. [4]. Here we apply a simple approach based on a median calculation.

Given a sequence of positive and non-zero singular values $S_{i,j}$ of length n_s, where index i is the singular value number and j the discrete frequency index, we construct the following normalized sequence $X_{i,j}$

$$X_{i,j} = \begin{cases} \log_{10}\left(\dfrac{S_{i,j}}{median\,(S_{i,j-k},\ldots,S_{i,j+k})}\right) & , j>k \wedge j < n_s - k \\ 0 & , j \le k \wedge j \ge n_s - k \end{cases} \quad (3)$$

where k is a small number, say 2-5. If the median of the values $S_{i,j-k}$ to $S_{i,j+k}$ is equal to the value $S_{i,j}$ then $X_{i,j}$ is 0 and otherwise $X_{i,j}$ will be a non-zero value. Since the sequence is normalized using the median that is robust towards outliers, the result is that $X_{i,j}$ will have significant values at the locations where the singular values have significant but narrow peaks.

The algorithm used here contains the following steps:

1. For each singular value $S_{i,j}$ calculate the sequences $X_{i,j}$ for i = 1 to n_p and j = 1 to n_s.
2. Calculate the sampled standard deviations of the sequences $X_{i,j}$.
3. Check if some of the values of $X_{i,j}$ exceeds a threshold of say 2-3 times the standard deviation of $X_{i,j}$.
4. If, for a certain index j, more than one of the n_s sequences $X_{i,j}$ exceeds the threshold, then a potential harmonic has been detected at position j.
5. Apply Kurtosis check described in section 2.3 at position j.

In figure 3.4 the sequences $X_{i,j}$ are shown for 3 singular values corresponding to the example shown in figure 3.3. The number of potential harmonics to check has been decreased to only 7.

Once the harmonics have been detected they can easily be removed from any frequency domain based modal parameter estimation algorithm, see e.g. Jacobsen et al. [3],[4], Brincker et al. [6] and Andersen et al. [7].

4 EXAMPLE

In the following the complete harmonic detection algorithm is tested on measurements of a Canadian gravity dam.

4.1 *Description of the gravity dam*

In figure 4.1 two pictures display the dam from both the low and high water level sides. The dam is 130 m long and 58 m high, and built in 1930. An ambient vibration test was conducted using 20 setups of 8 channels. In setup 8 channel 8 was dead and was disabled from the analysis. A 3D accelerometer served as reference station mounted on the dam itself. Some part of the rock at both side of

the dam was also measured. In figure 4.2 all measured degrees of freedom are presented on the test geometry used by the operational modal analysis software ARTeMIS Extractor.

The measurements were conducted using an 8-channel measurement system for 819 seconds. The sensors were Kinemetrics Episensors accelerometers of the forced balanced type. Due to the turbines running the measurements are affected by harmonics at every 2 Hz.

4.2 *The Analysis*

Since setup 8 only have 7 active channels the number of projection channels used in this analysis is 7. The 7 largest singular values of the spectral densities of all 20 setups were then averaged, and the harmonic detection described in section 3.2 was applied using $k = 2$ in eq. (3). In step 3 of the algorithm, the threshold was set to 2 times the standard deviation of the sequence $X_{i,j}$.

In figure 4.3 the results of the harmonic detection analysis are shown. All harmonics at 2 Hz intervals have been detected. The algorithm has mistakenly detected two harmonics at 19.5 Hz and 39 Hz. Taking the scatter of the SVD spectrum from the poor signal to noise ratio into account, it is a quite satisfactory results.

5 CONLUSIONS

In this paper, we have presented a fast algorithm for detection of harmonics from sinusoidal excitation. A statistical assessment algorithm of potential harmonics, based on evaluation of the Kurtosis of band pass filtered measurement, has been introduced. The speed of this algorithm has been optimized by applying a search algorithm that looks for abrupt changes in more than one singular value at a certain frequency, since this is a typical phenomenon in case of harmonics. This search algorithm typically increases the performance of the statistical assessment algorithm significantly. The harmonic detection approach has been demonstrated on measurements from a gravity dam.

6. FIGURES

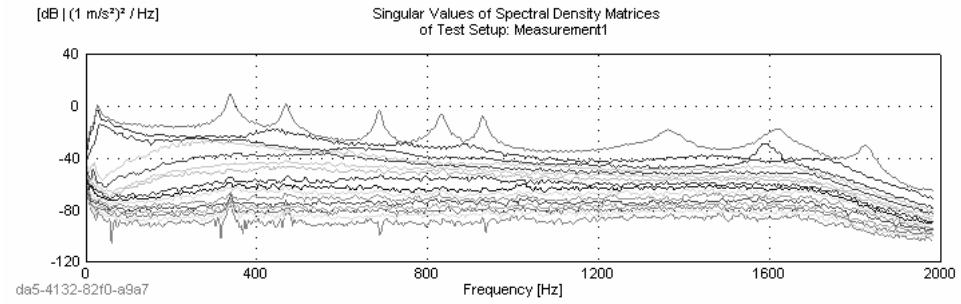

Figure 3.1: 16 projection channels were chosen resulting in 16 singular values per frequency. All the lower singular values being completely horizontal indicates a substantial amount of redundant information at all frequencies.

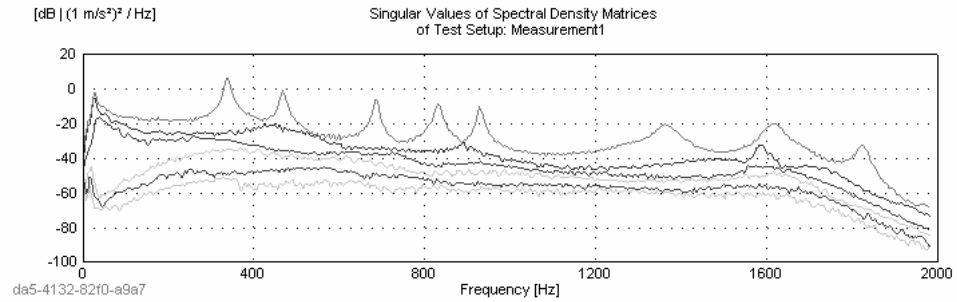

Figure 3.2: 6 projection channels were chosen resulting in only 6 singular values per frequency. Only the lowest singular values is significantly flat (horizontal) this indicates that 6 measurement channels are sufficient to contain all information about the system dynamics.

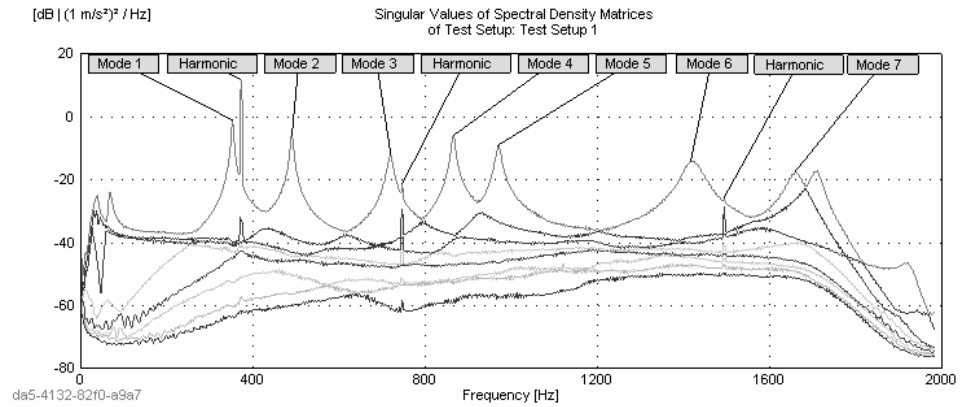

Figure 3.3: Similar structure as in figure 3.3, but now also with harmonic excitation. The excitation is a single point shaker with a sinusoidal force at 374 Hz. Number of discrete frequencies between DC and the Nyquist is 1024. First, second and forth harmonic are clearly shown at 374 Hz, 748 Hz and 1496 Hz.

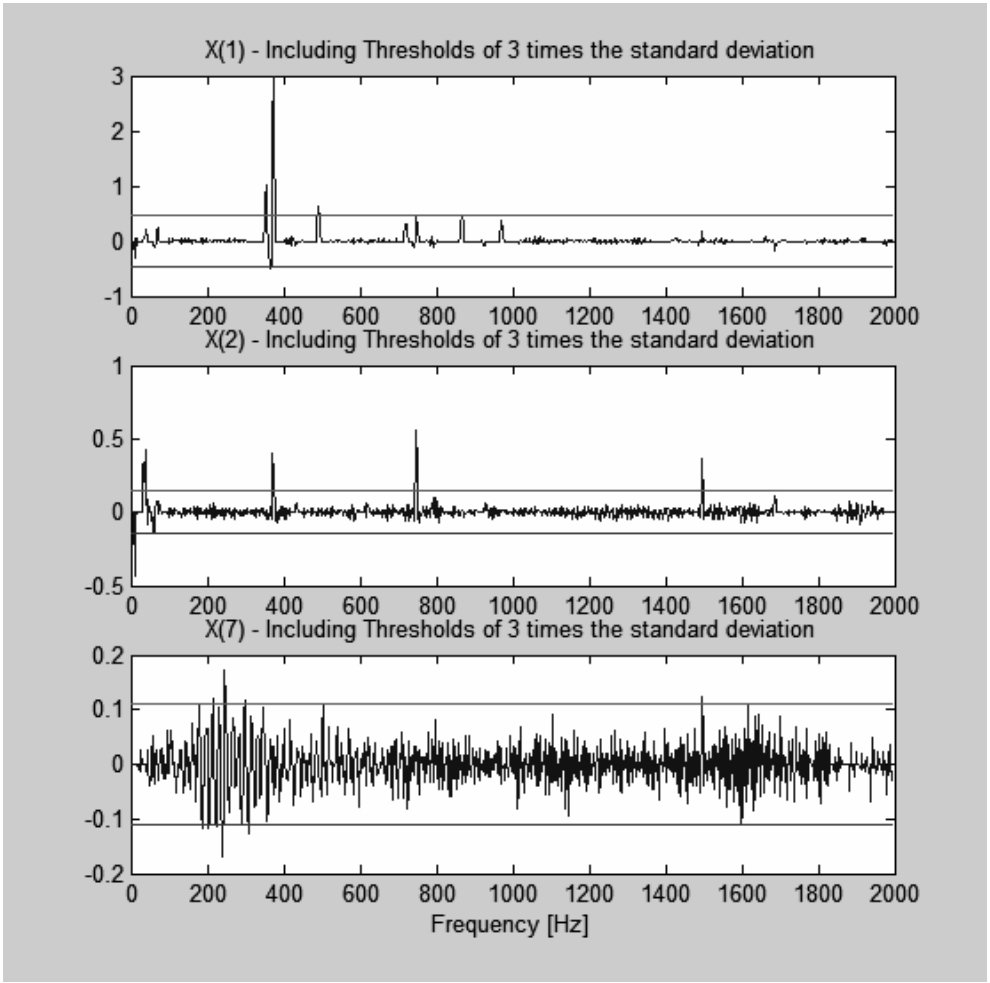

Figure 3.4: Sequences $X_{i,j}$ shown for 3 singular values (1,2 an 7) of the example shown in figure 3.3. Thresholds are exceeded in more than one singular value at frequencies 28 Hz, 372 Hz, 374 Hz, 376 Hz, 746 Hz, 748 Hz and 1496 Hz.

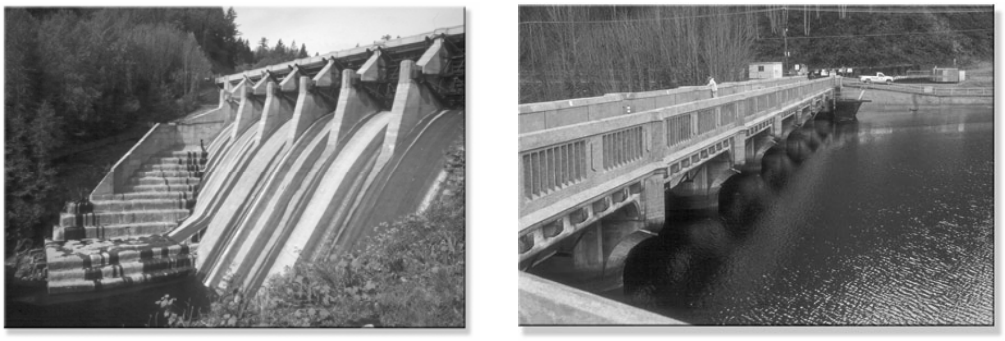

Figure 4.1: The dam seen from low and high water level.

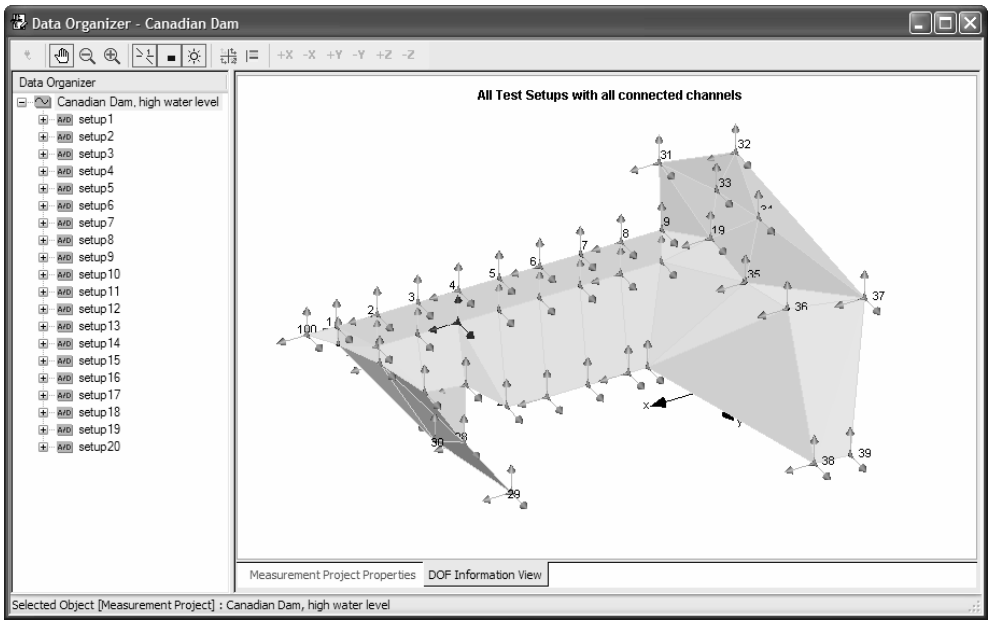

Figure 4.2: All measured degrees of freedom of all 20 setups on the dam as well as the surrounding rock on both sides. The three dark arrows below point 4 is the reference station.

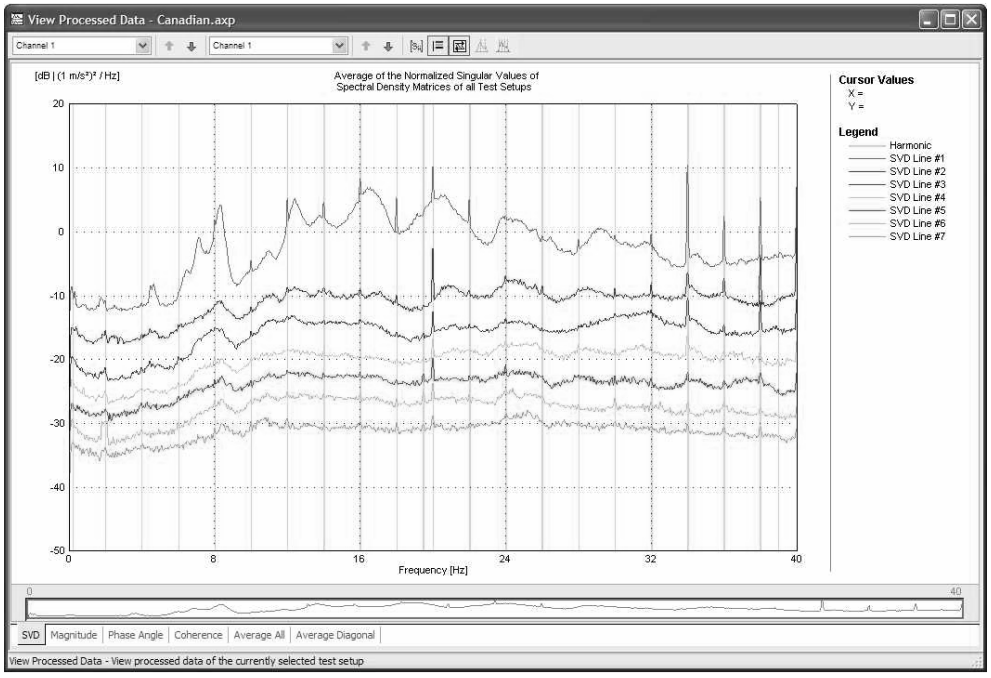

Figure 4.3: Result of the harmonic detection analysis. All harmonics at 2 Hz intervals have been detected. The algorithm has mistakenly detected harmonics at 19.5 Hz and 39 Hz. The harmonics are indicated with vertical lines at the harmonic frequencies.

7 REFERENCE LIST

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