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# Optimal, risk-based operation and maintenance planning for offshore wind turbines

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## Summary

For offshore wind turbines costs to operation and maintenance are substantial. This paper describes a risk-based life-cycle approach for optimal planning of operation and maintenance. The approach is based on pre-posterior Bayesian decision theory. Deterioration mechanisms such as fatigue, corrosion, wear and erosion are associated with significant uncertainty. Observations of the degree of damage can increase the reliability of predictions, especially in connection with condition-based maintenance. The approach can be used for gearboxes, generators, cracks, corrosion, etc. The paper also describes how probabilistic indicators can be used to quantify indirect information about the damage state for critical components, e.g. gearboxes.

## 1. Introduction

Costs to operation and maintenance for offshore wind turbines can be very large compared to other costs, and can be expected to increase when wind farms are placed at deeper water depths and in more harsh environments. This paper describes a risk-based life-cycle approach for optimal planning and design of offshore wind turbines. For other offshore installations such as oil & gas installations, cost-effective procedures for risk-based inspection planning have been developed during the last 10-15 years and are used at several locations world wide, see e.g. Moan [1], Faber et al. [2] and Sørensen et al. [3]. These procedures are based on pre-posterior Bayesian decision theory. This paper describes how procedures based on a similar theoretical basis can be applied for wind turbines, especially offshore wind farms. For wind turbines the main aspects related to operation and maintenance are availability, reliability and cost reductions.

Maintenance activities can be divided in corrective and preventive (time-tabled or conditioned) maintenance. Conditioned maintenance using observations from e.g. condition monitoring and inspections should optimally be based on risk and pre-posterior Bayesian decision theory. In section 2 the basic principles are described. In section 3 application to optimal maintenance planning is considered. Section 4 gives an example application related to gearboxes in wind turbines.

## 2. Optimal planning of inspection and maintenance

Figure 1 shows a decision tree related to the life cycle of an engineering structure such as a wind turbine or wind farm. The decisions taken by the decision maker (designer / owner / ...) and observations of uncertain parameters (unknown at the time of the decision) are:

- At the design stage a decision on the optimal design parameters  $\mathbf{z} = (z_1, \dots, z_N)$  is made which in principle should maximize the total expected benefits minus costs during the whole lifetime such that safety requirements are fulfilled at any time. In practice requirements from standards and actual costs of materials are used to determine the optimal design.
- During the lifetime continuous monitoring of the wind turbines and inspections of critical components / details are performed. These are indicated in the box 'repeated inspection/maintenance' in figure 1. Each box consists of:
  - a decision on times and types of inspection / monitoring for the rest of the lifetime
  - observations from inspection / monitoring

- decision on eventual maintenance / repair based on the inspection / monitoring results
- Realisation of uncertain parameters such as wind and wave climate, strengths, degradation, model uncertainties will take place during the lifetime. It is noted that these uncertainties can be divided in aleatory and epistemic uncertainties. Aleatory uncertainty is inherent variation associated with the physical system or the environment – it can be characterized as irreducible uncertainty or random uncertainty. Epistemic uncertainty is uncertainty due to lack of knowledge of the system or the environment – it can be characterized as subjective uncertainty, reducible uncertainty.
- The total cost is the sum of all costs in the remaining part of the lifetime after the decision time.

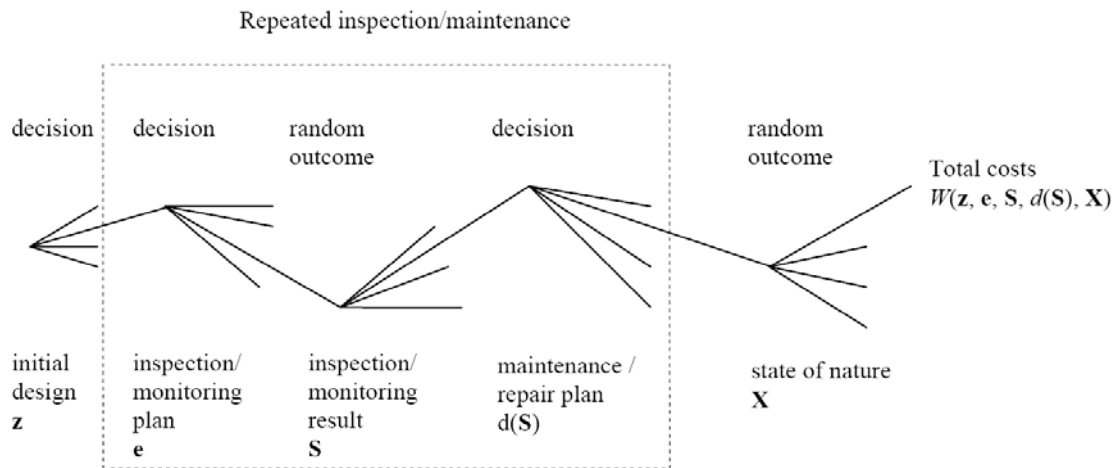


Figure 1. Decision tree for optimal maintenance planning.

The approach can be used for operation and maintenance planning related to different failure & error types in Gearbox, Generator, Rotor blades, Blade pitch mechanism, Yaw mechanism, Main shaft, Tower / support structure (fatigue cracks, corrosion), ...

Further, decisions related to operation and maintenance are related to different time scales:

- short (minutes) for decision related to e.g. parking the wind turbine,
- medium (days) for e.g. decisions on when to start offshore maintenance / repair actions depending on e.g. weather forecasts, or
- long (months / years) for e.g. preventive maintenance and inspection / monitoring planning for gear boxes.

An important step in risk-based inspection & maintenance planning is collection of data / information and probabilistic modelling of this information. Information can come from Condition Monitoring Systems (CMS), inspections or indicators. Indicators that contain indirect information on e.g. failure rates can be formulated and updated based on Bayesian statistics, see Faber & Sørensen [4].

The size of wind turbines for electricity production has increased significantly during the last decades both in production capability and in size. Compared to onshore turbines and building structures, humans spent little time in the vicinity of offshore turbines one could argue that the reliability of offshore structures can be lower than for onshore turbines. The concept of partial safety factors and characteristic values has been adopted from civil engineering. However, due to the dominance of the wind loads the level of reliability of wind turbines is somewhat lower than the average structural reliability. The current reliability level of civil engineering standards has emerged over many years of evolutionary development through which a level of reliability acceptable to the public has been reached. This level ensures that a low risk of human injury is obtained at reasonable costs. Thus the cost of avoiding human injury is implicit in current wind

turbine standards. For offshore turbines the probability of human injury during storm conditions is small. One could therefore argue that neglecting the price of preventing human injury, which is high in industrialized countries, would open a possibility for assessing a lower level of structural reliability of offshore turbines by cost-optimization, and further that optimal planning of operation and maintenance can be based on cost-benefit analyses. In the following first general formulations to assess the optimal reliability level are briefly described, and next these are extended to the situation of optimal planning of inspection and maintenance during operation.

## 2.1 Formulation of reliability-based optimisation problems for wind turbines

First, it is assumed that the wind turbines are systematically rebuild in case of failure. The main design variables are denoted  $\mathbf{z} = (z_1, \dots, z_N)$ , e.g. diameter and thickness of tower and main dimension of wings. The initial (building) costs is denoted  $C_I(\mathbf{z})$ , the direct failure costs are  $C_F$ , the benefits per year are  $b$  and the real rate of interest is  $r$ . Failure events are modeled by a Poisson process with rate  $\lambda$ . The probability of failure is  $P_F(\mathbf{z})$ .

The optimal design is determined from the following optimization problem, see e.g. Rackwitz [5]:

$$\begin{aligned} \max_{\mathbf{z}} \quad & W(\mathbf{z}) = \frac{b}{r C_0} - \frac{C_I(\mathbf{z})}{C_0} - \left( \frac{C_I(\mathbf{z})}{C_0} + \frac{C_F}{C_0} \right) \frac{\lambda P_F(\mathbf{z})}{r + \lambda P_F(\mathbf{z})} \\ \text{s.t.} \quad & z_i^l \leq z_i \leq z_i^u, \quad i = 1, \dots, N \\ & P_F(\mathbf{z}) \leq P_F^{\max} \end{aligned} \quad (1)$$

where  $\mathbf{z}^l$  and  $\mathbf{z}^u$  are lower and upper bounds on the design variables.  $C_0$  is the reference initial cost of corresponding to a reference design  $\mathbf{z}_0$ .  $P_F^{\max}$  is the maximum acceptable probability of failure e.g. with a reference time of one year. This type of constraint is typically required by regulators. The optimal design  $\mathbf{z}^*$  is determined by solution of (1). If the constraint on the maximum acceptable probability of failure is omitted, then the corresponding value  $P_F(\mathbf{z}^*)$  can be considered as the optimal probability of failure related to the failure event and the actual cost-benefit ratios used.

The failure rate  $\lambda$  and probability of failure can be estimated for the considered failure event, if a limit state equation,  $g(X_1, \dots, X_n, \mathbf{z})$  and a stochastic model for the stochastic variables,  $(X_1, \dots, X_n)$  are established. If more than one failure event is critical, then a series-parallel system model of the relevant failure modes can be used.

Next, the situation is considered where the assumptions are the same as above except that the wind turbine is assumed not to be rebuild in case of failure. The design lifetime is  $T_L$  and the probability of failure of a component or the structure in the time interval  $[0, T]$  is denoted  $P_F(T, \mathbf{z})$ . The annual probability of failure is  $\Delta P_F(T, \mathbf{z}) = P_F(T, \mathbf{z}) - P_F(T-1, \mathbf{z})$  with  $T$  in [years].

The optimal design  $\mathbf{z}^*$  is determined from the following optimization problem:

$$\begin{aligned} \max_{\mathbf{z}} \quad & W(\mathbf{z}) = \sum_{t=1}^{T_L} \frac{b}{C_0} (1 - P_F(t, \mathbf{z})) \frac{1}{(1+r)^t} - \frac{C_I(\mathbf{z})}{C_0} - \sum_{t=1}^{T_L} \frac{C_F}{C_0} \Delta P_F(t, \mathbf{z}) \frac{1}{(1+r)^t} \\ \text{s.t.} \quad & z_i^l \leq z_i \leq z_i^u, \quad i = 1, \dots, N \\ & \Delta P_F(t, \mathbf{z}) \leq \Delta P_F^{\max}, \quad 0 \leq t \leq T_L \end{aligned} \quad (2)$$

where  $\Delta P_F^{\max}$  is the maximum acceptable annual probability of failure.

## 2.2 Inspection / Maintenance Included

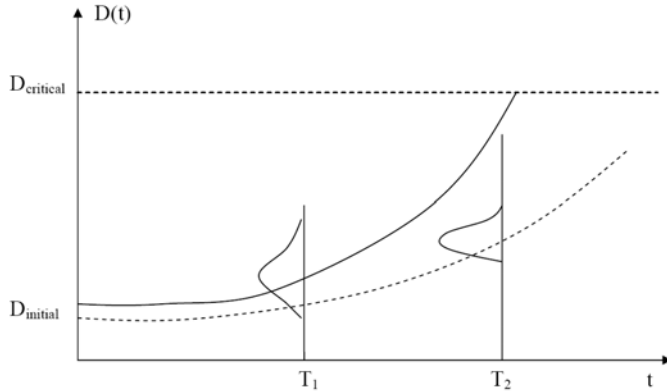


Figure 2.

Deterioration mechanisms such as fatigue, corrosion, wear and erosion are associated with significant uncertainty. Observations of the degree of damage  $D(t)$  can increase the reliability of predictions using Bayesian statistical techniques as illustrated in figure 2. Generally an inspection at time  $T_1$  and associated maintenance/repair will decrease the uncertainty and the expected mean damage level at time  $T_2$  will be smaller since most realizations with large damage level at time  $T_1$  can be expected to be maintained/repared.

The model in section 2.1 is now extended to include time-varying degradation and damage accumulation (e.g. wear, fatigue and corrosion). It is assumed that the wind turbine is not rebuild in case of failure.

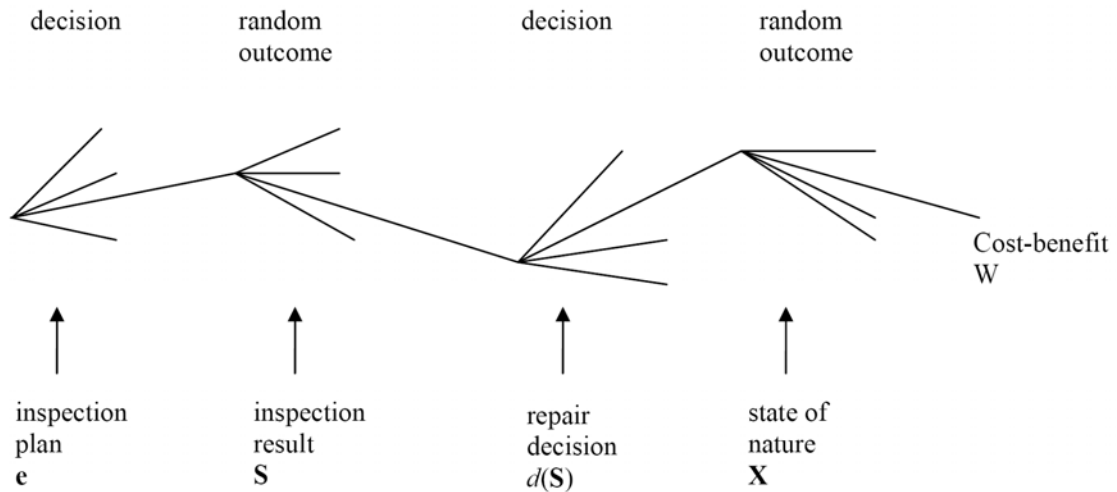


Figure 3. Inspection planning decision tree.

The performance of wind turbines is subject to a number of uncertainties. These include operational conditions, material characteristics and environmental exposure. The uncertainties are due to inherent physical randomness and uncertainties associated with the models used to assess the performance of the systems. If, furthermore, the statistical basis for the assessment of the uncertainties is limited then also statistical uncertainties may be important.

When inspection planning for wind turbines is considered, it is important to take all these uncertainties into consideration, as they will strongly influence the future performance of the systems. It is also important to realize that the degree of control of the engineering systems achieved by the inspections is strongly influenced by the reliability of the inspections, i.e. their ability to detect and size degradation. The reliability of inspections themselves may be subject to significant uncertainty and this must be taken into account in the planning of inspections.

The decision problem of identifying the cost optimal inspection plan may be solved within the framework of pre-posterior analysis from the classical decision theory see e.g. Raiffa and Schlaifer [6] and Benjamin and Cornell [7]. Here a short summary is given, see e.g. Sørensen et al. [8], Madsen & Sørensen [9], Faber et al. [2] and Sørensen et al. [3]. The inspection decision problem may be represented as shown in figure 3 which is extracted from the general decision tree in figure 1.

In the general case the parameters defining a inspection plan are the possible repair / maintenance actions which are modeled by the decision rule  $d$ , the number of inspections  $N$  in the service life  $T_L$ , the time intervals between inspections and possible repair/maintenance  $\mathbf{t} = (t_1, t_2, \dots, t_N)$  and the inspection qualities  $\mathbf{q} = (q_1, q_2, \dots, q_N)$ .

These inspection parameters are written as  $\mathbf{e} = (N, \mathbf{t}, \mathbf{q})$ . The outcome of inspections (typically a damage level, e.g. a crack size, the extent of corrosion or wear) is modeled by a random variable  $S$  since it is unknown at the time of decision making. A decision rule  $d(S)$  is then applied to the outcome of the inspection to decide whether or not repair / maintenance should be performed. The different uncertain parameters (stochastic variables) modeling the state of nature such as load variables and material characteristics are collected in  $\mathbf{X} = (X_1, X_2, \dots, X_n)$ .

If the total expected costs are divided into fabrication, inspection, repair, maintenance, strengthening and failure costs and a constraint related to a maximum annual (or accumulated) failure probability  $\Delta P_F^{\max}$  is added then the optimization problem can be written

$$\begin{aligned} \max_{\mathbf{z}, \mathbf{e}, d} \quad & W(\mathbf{z}, \mathbf{e}, d) = B(\mathbf{z}, \mathbf{e}, d) - C_I(\mathbf{z}, \mathbf{e}, d) - C_{IN}(\mathbf{z}, \mathbf{e}, d) - C_{REP}(\mathbf{z}, \mathbf{e}, d) - C_F(\mathbf{z}, \mathbf{e}, d) \\ \text{s.t.} \quad & z_i^l \leq z_i \leq z_i^u, \quad i = 1, \dots, N \\ & \Delta P_t(\mathbf{z}, \mathbf{e}, d) \leq \Delta P_F^{\max}, \quad t = 1, 2, \dots, T_L \end{aligned} \quad (3)$$

$W(\mathbf{z}, \mathbf{e}, d)$  is the total expected benefits minus costs in the service life time  $T_L$ ,  $B$  is the expected benefits,  $C_I$  is the initial costs,  $C_{IN}$  is the expected inspection costs,  $C_{REP}$  is the expected costs of repair and  $C_F$  is the expected failure costs. The annual probability of failure in year  $t$  is  $\Delta P_{F,t}$ . The  $N$  inspections are assumed performed at times  $0 \leq T_1 \leq T_2 \leq \dots \leq T_N \leq T_L$ .

The total capitalized benefits are written

$$B(\mathbf{z}, \mathbf{e}, d) = \sum_{i=1}^N B_i (1 - P_F(T_i)) \frac{1}{(1+r)^{T_i}} \quad (4)$$

where the  $i$ th term represents the capitalized benefits in year  $i$  given that failure has not occurred earlier,  $B_i$  is the benefits in year  $i$ ,  $P_F(T_i)$  is the probability of failure in the time interval  $[0, T_i]$  and  $r$  is the real rate of interest.

The total capitalized expected inspection costs are

$$C_{IN}(\mathbf{e}, d) = \sum_{i=1}^N C_{IN,i}(\mathbf{q})(1 - P_F(T_i)) \frac{1}{(1+r)^{T_i}} \quad (5)$$

where the  $i$ th term represents the capitalized inspection costs at the  $i$ th inspection when failure has not occurred earlier,  $C_{IN,i}(q_i)$  is the inspection cost of the  $i$ th inspection.

The total capitalized expected repair costs are

$$C_{REP}(\mathbf{e}, d) = \sum_{i=1}^N C_{R,i} P_{R_i} \frac{1}{(1+r)^{T_i}} \quad (6)$$

where  $C_{R,i}$  is the cost of a repair at the  $i$ th inspection and  $P_{R_i}$  is the probability that a repair is performed after the  $i$ th inspection when failure has not occurred earlier.

The total capitalized expected costs due to failure are estimated from

$$C_F(\mathbf{e}, d) = \sum_{t=1}^{T_L} C_F(t) \Delta P_{F,t} P_{COL|FAT} \frac{1}{(1+r)^t} \quad (7)$$

where  $C_F(t)$  is the cost of failure at the time  $t$ .  $P_{COL|FAT}$  is the conditional probability of collapse of the wind turbine given fatigue failure of the considered component and models the importance / consequence of fatigue failure. The probabilities of failure at year  $t$  and the probability of repair can be determined as described in e.g. Madsen et al. (1990). It is noted that if the inspection / maintenance option is removed then the optimization problem (3) is simplified to (2).

The above model is in principle related to a single wind turbine and a single component. For wind turbine placed in a wind farm with many critical components the same basic formulation can be used, but the initial costs, inspection, repair and failure costs should be formulated as a basic cost plus marginal costs for each extra wind turbine in the park.

### 3. Maintenance planning

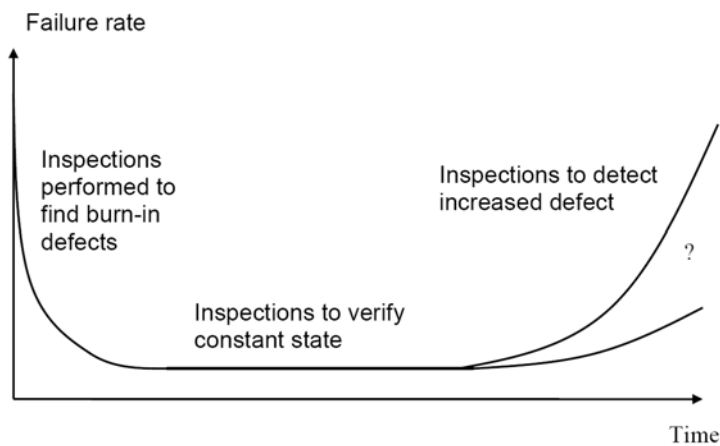


Figure 4. Bath-tub model for lifetime failure rate.

For many components subject to degradation / damage accumulation the model in figure 4 can be used to illustrate the development of the failure rate during the lifetime. Initially a high failure

rate can be expected due to fabrication / burn-in defects. Next, a period with a 'normal' constant failure / defect rate will take place. Corrective maintenance is performed in this period. At the end of the lifetime of the component the failure / defect rate can be expected to increase. If the failure rate increases strongly (time to failure is „known“) then preventive maintenance should be performed. If the failure rate is moderately increasing deterioration / damage can be observed before failure, and condition control / condition & risk based maintenance should be performed and planned using the principles described above for risk-based inspection & maintenance/repair planning.

The risk-based methods described in section 2 can thus be used to optimal planning of decisions on

- future inspections / condition monitoring (time and type), and
- maintenance / repair actions based on (unknown) observations from future inspections / monitoring

taking into account uncertainty and costs.

The next section shows in more detail how this can be used for typical components in wind turbines.

#### 4. Example – mechanical component in wind turbine: gearbox

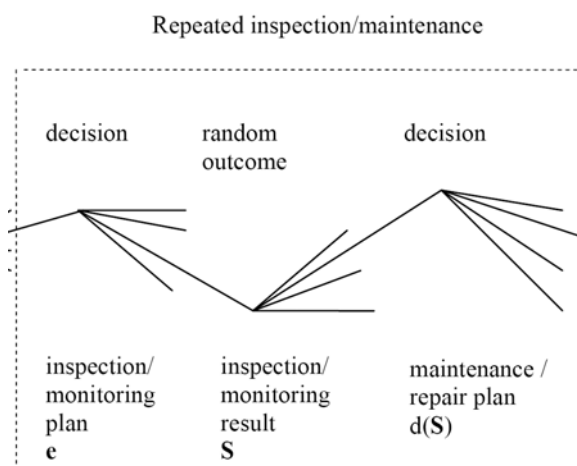


Figure 5. Repeated inspection/maintenance.

In this example the theoretical models outlined in section 2 and 3 are illustrated considering gearboxes in wind turbines. Only the process of planning repeated inspection and maintenance / repair is considered, see figure 5. Examples of inspection methods and inspection results for gearboxes are:

- Visual inspection: though inspection covers: indication of extent of wear
- Oil analysis: with time intervals a sample is taken indicating extent of wear
- Magnet: with time intervals a representative sample is taken indicating extent of wear material
- Investigation of oil filters: with time intervals a representative sample is taken indicating extent of wear material
- Particle counting online: continuously representative samples are taken indicating extent of wear material
- Condition monitoring: continuously the vibration response is monitored and used to indicate mechanical changes



All these assessment methods give indirect information on the damage/deterioration state of the gearbox, since the damage/deterioration state is not measured directly. The indicators will have different reliabilities with respect to information about the real damage state and they will have different costs. These aspects can be modelled using the risk-based inspection & maintenance planning approach described in section 2. In order to use the pre-posterior Bayesian decision models optimal, risk-based inspection & maintenance planning it is further necessary to formulate decision rules for maintenance / repair actions given inspection/monitoring results, i.e. it is beforehand decided which future maintenance / repair to perform when future inspection results become available.

The models to be established are:

- A deterministic model for damage / deterioration accumulation – as function of time:  $D(t)$
- A stochastic model for uncertain parameters in the damage accumulation model such that a probabilistic model for the damage accumulation can be obtained, i.e. the probability of certain damage levels can be calculated
- A stochastic model for the uncertainty / reliability of each inspection type
- A decision model  $d(S)$  for repair / maintenance (action) given future result of inspection / condition monitoring, ...
- A model for costs related to inspections, maintenance, repair and possible failure (including loss of income)

It is noted that information from continuous quality control and monitoring systems can be used to establish the stochastic models.

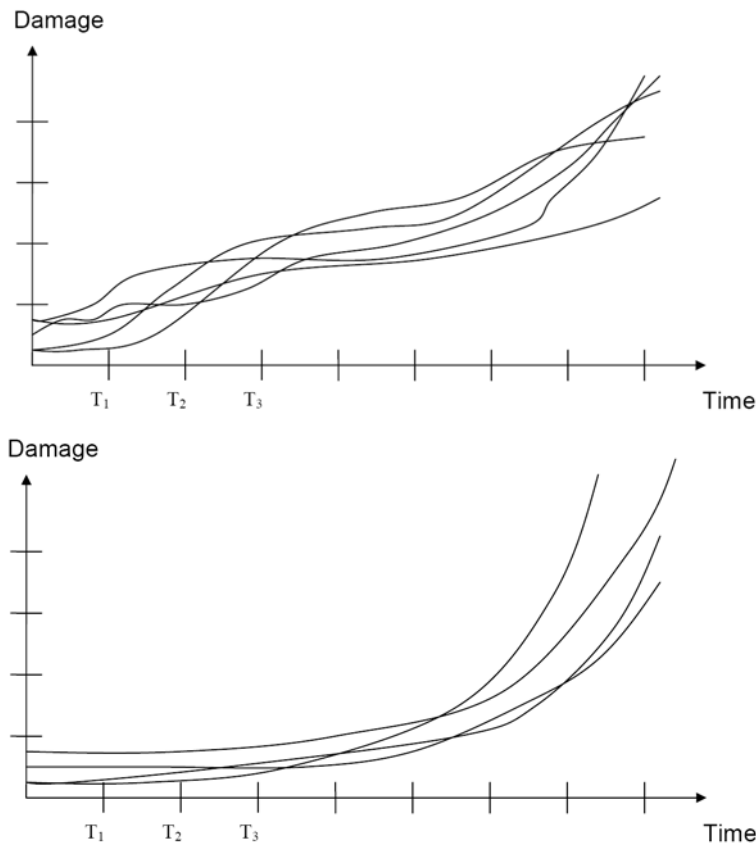


Figure 6. Examples of damage accumulation.

Figure 6 shows examples of realisations of damage accumulation as function of time. If time and damage are discretized in damage states  $T_1, T_2, T_3, \dots$  and  $D_1, D_2, D_3, \dots$  then the probabilistic information needed is:

- Conditional probabilities related to damage accumulation process:  $P(D(T) = d_i | D(t) = d_j)$ , i.e. the probability that the damage at time  $T$  is  $d_i$  given that the damage at time  $t$  is  $d_j$ .
- Conditional probabilities related to inspection method:  $P(D(T) = d_i | I(T) = I_{obs})$  where  $I(T)$  is the uncertain (unknown) inspection result at time  $T$  giving indirect information about the damage state  $D(T)$  at time  $T$ .  $I_{obs}$  is the actual inspection result.

Using these probabilities the probability of failure at time  $T$  given an inspection result  $I_{obs}$  at time  $T_1$  can be estimated by:

$$P_F(T | I_{obs}) = \int P(D(T) \geq D_F | D(T_1) = x) P(D(T_1) = x | I(T_1) = I_{obs}) dx \quad (8)$$

where  $D_F$  is the damage level corresponding to failure where e.g. a complete exchange is necessary.

It is noted that the model for the reliability of the inspection methods modelled though  $P(D(T) = d_i | I(T) = I_{obs})$  can be formulated in different ways dependent on the type of information available using Bayes rule.

An example decision model  $d(S)$  could be:

- If  $D_M \leq D(T) < D_R$  then maintenance
- If  $D_R \leq D(T) < D_F$  then repair

where  $D_M$  and  $D_R$  are damage levels corresponding to maintenance and repair.

The probability of repair at time  $T_1$  given an inspection result  $I_{obs}$  at time  $T_1$  is estimated by:

$$P_R(T_1 | I_{obs}) = \int P(D_R \leq D(T_1) < D_F | D(T_1) = x) P(D(T_1) = x | I(T_1) = I_{obs}) dx \quad (9)$$

The probabilities in (8) and (9) are used in equations (5) - (7) to estimate the total expected costs and next in equation (3) to obtain the optimal inspection times.

The model can easily be extended with:

- Information from:
  - More inspection methods
  - Monitoring systems
  - Inspection / monitoring from other (correlated) components
- Decisions on
  - Which of several inspection methods to use (incl. simultaneous use of several inspection methods)
  - Which of several maintenance methods to use
- Overall planning for:
  - Many components
  - Wind farms
- Detailed planning for offshore operations (e.g. in case of failure) taking into account
  - Uncertainties in weather forecasts
  - Cost: materiel, loss of income, ...

## 5. Conclusions

A risk-based life-cycle approach for optimal planning of operation and maintenance is described. The approach is theoretically based on pre-posterior Bayesian decision theory and can be used when deterioration mechanisms such as fatigue, corrosion, wear and erosion are present and can be observed by inspection and/or monitoring before failure of the component considered. The risk based approach can rationally take into account the uncertainty related to the deterioration and the future costs related to inspection/monitoring, maintenance, repair and failure (loss of income).

Observations of the degree of deterioration damage can increase the reliability of predictions, especially in connection with condition-based maintenance using Bayesian updating. The approach can be used for gearboxes, generators, cracks, corrosion, etc. Further, it is described how probabilistic indicators can be used to quantify indirect information about the damage state for critical components. The approach is illustrated for application to gear-boxes in situations where deterioration can be observed before failure.

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