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## Safety and inspection planning of older installations

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**Abstract:** A basic assumption often made in risk- and reliability-based inspection planning is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information (from inspections and repairs) becomes available. The Bayesian approach and a no-crack detection assumption imply that the inspection time intervals usually become longer and longer with time. For ageing platforms several small cracks should be expected to be observed according to the bath-tub curve development often assumed implying an increased risk for crack initiation (and coalescence of small cracks) and increased crack growth. This should imply shorter inspection time intervals for ageing structures. Different approaches for updating inspection plans for older installations are proposed. The most promising method consists of increasing the rate of crack initiations at the end of the expected lifetime corresponding to a bath-tub hazard rate effect. The approach illustrated is for welded steel details in platforms. Systems effects are considered, including the use of dependence between inspection and failure events in different components for inspection planning.

**Keywords:** reliability and risk-based inspection planning, ageing offshore installations, Bayesian approach, system reliability, fatigue

### 1 INTRODUCTION

Reliability and risk-based inspection (RBI) planning for offshore structures has been an area of high practical interest over recent decades. The first developments were in inspection planning for welded connections subject to fatigue crack growth in fixed steel offshore platforms. This application area for RBI is now the most developed. Formerly, practical applications of RBI required significant expertise in the areas of structural reliability theory and fatigue and fracture mechanics [1]. This made practical implementation in industry difficult. Recently, generic and simplified approaches for RBI have been formulated, making it possible to base inspection planning on a few key parameters commonly applied in the deterministic design of structures, e.g. the fatigue design factor (FDF) and the reserve strength ratio (RSR) [2, 3].

The basic assumption made in risk/reliabilitybased inspection planning is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information (from inspections) becomes available. Further, the RBI approach for inspection planning presented in references [2] and [3] is based on the assumption that at all future inspections no cracks are detected. If a crack is detected, then a new inspection plan should be developed. The Bayesian approach and the no-crack detection assumption imply that the inspection time intervals usually become longer and longer with time.

By its nature, damage will accumulate in structures exposed to conditions of stress cycles. The structures will ultimately reach a state in which they are judged to be no longer fit for service. By then, unless repaired or re-rated, the structure may be said to have reached the end of its life. As damage accumulates, failure becomes increasingly probable, and if not withdrawn from service, failure of some kind will eventually occur. There is a dearth of guidelines for inspection planning of ageing offshore installations in both scientific papers and regulations and standards to cope with this problem. Within the aviation industry this subject has been given more attention as a result

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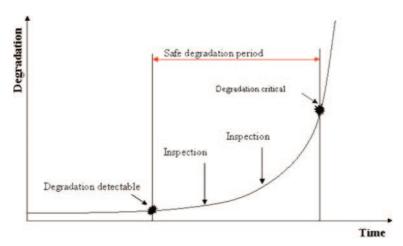


Fig.1 Theoretical damage tolerance inspection regime to detect cracks before they become critical. Adapted from reference [4]

of, among other things, the fuselage failure of the Aloha Boeing 737-200 aircraft on 28 April 1988 [4]. The resulting regulation for ageing aircraft has led to decreased inspection intervals. The guiding principles for inspection planning for ageing aircraft are based on the damage tolerance philosophy, or safety by inspection. According to reference [4] 'The damage tolerance philosophy is based on the principle that while cracks due to fatigue and corrosion will develop in the aircraft structure, the process can be understood and controlled. A key element is the development of a comprehensive programme of inspections to detect cracks before they can affect flight safety. That is, damage tolerant structures are designed to sustain cracks without catastrophic failure until the damage is detected in scheduled inspections and the damaged part is repaired or replaced.' To ensure that this occurs, there should be at least two opportunities to detect the crack prior to it reaching its critical length [4], as illustrated in Fig. 1.

A similar approach could be taken for ageing offshore installations that also reflects accessibility for inspection and consequence classes, represented by FDFs. This would require N=FDF opportunities to detect the crack prior to it reaching its critical length in areas that have passed their fatigue life. This approach has in practice been required by the Petroleum Safety Authority Norway.

This normally leads to an inspection interval decreasing with age, and is more in line with expectations of the need to inspect ageing structures. In contrast, the Bayesian approach and the no-crack detection imply that the inspection time intervals usually become longer and longer.

Further, inspection planning based on the RBI approach implies that single fatigue critical components are considered, one at a time, but with acceptable reliability levels assessed based on the

consequences for the whole structure in case of the fatigue failure of single components.

Examples and information on RBI and maintenance planning can be found in a number of papers, e.g. references [1–3] and [5–14]. Important aspects are systems considerations, design using robustness considerations by accidental collapse limit states, and use of monitoring by the 'leak before break' principle to identify damage.

The above methods for RBI have been developed in principle for new offshore structures, where possible ageing effects are unimportant. Figure 2 shows the installation years for fixed offshore structures in the Norwegian part of the North Sea. It is seen that a considerable number of platforms are more than 30 years old, and have passed or are close to their design lifetime.

For ageing installations an increasing number of small defects/cracks are expected to be observed according to the bath-tub curve development often assumed. This assumption has not yet been supported by observation of offshore installations, but experience from other industries such as aviation indicates that widespread fatigue damage can be a significant problem for ageing structures, especially in combination with corrosion; e.g. see reference [15]. This study is thus based on an assumption of potential increased crack initiation being present for ageing structures, and the intention of the study is to identify models resulting in more frequent inspections for ageing structures. Widespread fatigue implies an increased risk for defect/crack initiation (and the coalescence of small defects/cracks) and increased growth - thus illustrating a bath-tub effect, see Fig. 3, where, in particular, the part related to the last stage of its lifetime is uncertain.

In this paper it is assumed that installations in life extension should have the same safety levels as

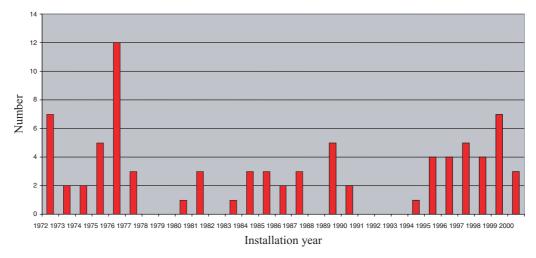


Fig. 2 Installation years for Norwegian fixed platforms

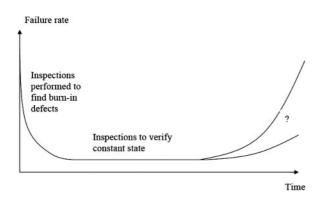


Fig. 3 Bath-tub model for failure rate for ageing installations

installations in design life, thus giving the same safety for both people and the environment. A sufficient safety level in life extension can be obtained by reducing uncertainty about the installation through knowledge from operation, dedicated maintenance with respect to ageing, modifications of structure, change from 'safe life thinking' to 'damage tolerant thinking', or by an appropriate risk-based maintenance approach. This paper considers different ways of formulating a risk-based maintenance approach for inspection planning of ageing installations.

Due to common loading, common model uncertainties, and correlation between inspection qualities, it can be expected that information obtained from inspection of one component can be used, not only to update the inspection plan for that component, but also for other nearby components. Such system effects can also lead to increased probability of simultaneous failure in nearby correlated components. This aspect is illustrated by an example in this paper. Further, it is noted that initiation of small defects/cracks which may coalescence to larger defects/cracks can occur at more than one position, i.e. a systems effect along, for example, a welding can be of importance depending on the length of

the weld and the dependence between the defects/fatigue cracks.

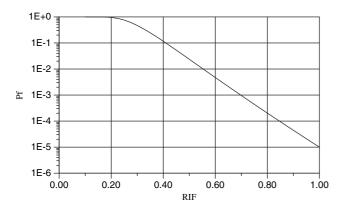
This paper is a summary of a project performed by John Dalsgaard Sørensen for the Petroleum Safety Authority in Norway within their 'Aging Installations' project.

#### 2 RISK-BASED INSPECTION PLANNING

In RBI planning the inspection plans are determined such that the annual probability of failure is less than a maximum acceptable annual probability of failure,  $\Delta P_F^{\rm max}$ , which is dependent on the consequences of fatigue failure being the total collapse of the structure. Further, the inspection plan should be determined such that the lifetime total expected costs of inspection, repair, strengthening, and eventual failure are minimized.

For fatigue failures the requirements of safety are typically given in terms of a required FDF. As an example, reference [16] specifies FDF=10 where there are 'Substantial consequences of fatigue failure' and 'No access or in the splash zone'. From the FDFs it is possible to establish the corresponding annual probabilities of failure for a specific year. For the joints to be considered in an inspection plan, the acceptance criteria for the annual probability of fatigue failure may be assessed through the RSR (reserve stress ratio)-given failure of each of the individual joints, to be considered together with the annual probability of joint fatigue failure.

If the RSR-given joint fatigue failure is known (e.g. obtained from an USFOS analysis), it is possible to establish the corresponding annual collapse failure probability-given fatigue failure,  $P_{\rm COL|FAT}$ , if information is available on the applied characteristic values for the capacities, live loads, and wave height;



**Fig. 4** Example relationship between RIFs and annual collapse probability of failure – *P*(COL|FAT)

ratios of the environmental load to the total load; coefficient of variation of the capacity and the load.

In order to assess the acceptable annual probability of fatigue failure for a particular joint in a platform, the reliability of the considered platform must be calculated conditional on fatigue failure of the considered joint. The importance of a fatigue failure is measured by the residual influence factor (RIF) defined as

$$RIF = \frac{RSR^{damaged}}{RSR^{intact}} \tag{1}$$

where RSR<sup>intact</sup> is the RSR value for the intact structure and RSR<sup>damaged</sup> is the RSR value for the structure damaged by fatigue failure of a joint. The principal relation between RIF and annual collapse probability is illustrated in Fig. 4.

The implicit code requirement of the safety of the structure in regard to total collapse may be assessed through the annual probability of joint fatigue failure (in the last year in service)  $P_{\rm FAT_{\it j}}$  for a joint for which the consequences of failure are 'substantial' (i.e. design fatigue factor 10). This probability can be regarded as an acceptance criteria, i.e.  $P_{\rm AC}$ . A typical maximal allowed annual probability of collapse failure is in the order of  $P_{\rm AC}=10^{-5}$ .

On this basis it is possible to establish joint- and member-specific acceptance criteria in regard to fatigue failure. For each joint j, the conditional probabilities of structural collapse given failure of the considered joint  $P_{\mathrm{COL}|\mathrm{FAT}_j}$  are determined and the individual joint acceptance criteria for the annual probability of joint fatigue failure are found by a first crude approximation

$$\Delta P_{\text{FAT},j}^{\text{max}} = \frac{P_{\text{AC}}}{P_{\text{COL}|\text{FAT}_i}} \tag{2}$$

The inspection plans must then satisfy

$$P_{\mathrm{FAT}_{j}} {\leqslant} \Delta P_{\mathrm{FAT},j}^{\mathrm{max}} \tag{3}$$

for all years during the operational life of the structure.

Ersdal [11] considered the life extension of existing offshore jacket structures including fatigue degradation and inspection effects in a life extension. A predictive Bayesian approach is used. Different inspection and repair methods are considered, indicating that degradation of the structure due to fatigue crack growth can be controlled by inspection, and repair leads to a significant extended life. Investigations show that systems effects related to life extension and possible combined hazard of wave-in-deck loading are found to be very important.

In many situations there will be a number of fatigue crack critical details (components) in an offshore steel platform. Assessment of the acceptable annual fatigue probability of failure for a particular component can be dependent on the number of fatigue critical components. The acceptable annual probability of fatigue failure of a component is obtained by considering the importance of the component through the conditional probability of its failure. Given  $P_{\rm AC}$ , RIF, and the number of fatigue critical components, the maximum acceptable component annual probability of fatigue failure  $\Delta P_{\rm FAT}^{\rm max}$  can be calculated using a simple upper bound on the probability of failure; see reference [3]. As an example  $\Delta P_{\rm FAT}^{\rm max}$  is shown in Fig. 5 for  $P_{\rm AC}=10^{-5}$  and N=1, 2, 5, and 10 critical components.

In generic inspection planning, inspection planning is made by interpolation in a predetermined database with plans covering the potential application domain; see e.g. reference [2]. Given:

- (a) the type of fatigue-sensitive detail and thereby code-based SN-curve;
- (b) the fatigue strength measured by FDF;
- (c) the importance of the considered detail for the ultimate capacity of the structure, measured by e.g. RIF and RSR;
- (d) the member geometry (thickness);
- (e) the inspection, repair, and failure costs;

the optimal inspection plan, i.e. the inspection times and inspection qualities, can be determined. This inspection plan is generic in the sense that it is representative of the given characteristics of the considered details, i.e. the SN-curve, FDF, RSR, and the inspection, repair, and failure costs.

This inspection planning procedure requires information on costs of failure, inspections, and repairs. Often, these are not available, and the inspection planning is based on the requirement that the annual probability of failure in all years has to satisfy the reliability constraint implied by  $\Delta P_{\rm F}^{\rm max}$ . Further, in RBI planning, the assumption that no cracks are found at the inspections is usually made. If a crack is found, it is often assumed to be perfectly repaired and a new inspection plan has to be made based on that observation. The action after detection of a crack

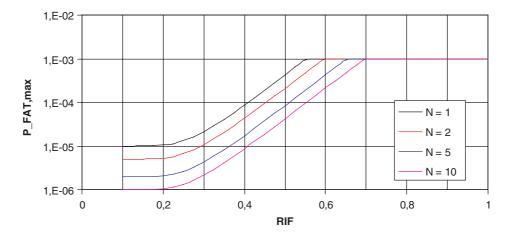


Fig. 5 Maximum acceptable annual probability of fatigue failure  $\Delta P_{\rm FAT}^{\rm max}$  for each fatigue critical detail as a function of RIF.  $P_{\rm AC} = 10^{-5}$ 

can on the other hand be that small cracks are grinded and large cracks are repaired by welding, for example.

The reliability of inspections can be modelled in many different ways. Often probability of detection (POD) curves are used to model the reliability of the inspections.

In order to model the influence of inspections and estimate the probability of failure, a probabilistic fracture mechanical (FM) model is needed. This model is often calibrated such that it gives the same reliability level as a code-based probabilistic SN-approach using Miner's rule of linear accumulation of damage.

If a bilinear SN-curve is applied, the SN-relation can be written

$$N = K_1 (\Delta s)^{-m_1} \qquad \text{for } N \leqslant N_{\mathcal{C}}$$
 (4)

$$N = K_2(\Delta s)^{-m_2} \qquad \text{for } N > N_C \tag{5}$$

where  $\Delta s$  is the stress range; N is the number of cycles to failure;  $K_1$ ,  $m_1$  are the material parameters for  $N \leq N_{\rm C}$ ;  $K_2$ ,  $m_2$  are the material parameters for  $N > N_{\rm C}$ ;  $N_{\rm C}$  is the number of cycles where the slope of the SN-curve changes from  $m_1$  to  $m_2$ ; and  $\Delta s_{\rm C}$  is the stress range corresponding to  $N_{\rm C}$ .

The probability of failure is calculated using the limit state equation

$$g = \Delta - \sum_{s_i \ge \Delta s_C} \frac{n_i T_L}{K_1(X_S s_i)^{-m_1}} - \sum_{s_i < \Delta s_C} \frac{n_i T_L}{K_2(X_S s_i)^{-m_2}}$$
(6)

 $\Delta$  is the model uncertainty related to Palmgren–Miner's rule for linear damage accumulation,  $T_{\rm L}$  is the service life,  $s_i$  is the stress range in group i, and  $X_{\rm S}$  is a stochastic variable modelling the model uncertainty related to waves and SCF (wave load response).  $X_{\rm S}$  is assumed lognormal distributed with

**Table 1** Example of stochastic model for the SN-approach

Variable	Distribution*	Expected value	Standard deviation
Δ	LN	1	0
$Z_{ m SCF}$	LN	1	$COV_{SCF}$
$Z_{ m wave}$	LN	1	$COV_{wave}$
$m_1$	D	3	
$\log K_1$	N	12.048	0.218
$m_2$	D	4	
$\log K_2$	N	13.980	0.291

\*D: deterministic, N: normal, LN: lognormal.

mean value = 1 and  $\text{COV} = \sqrt{\text{COV}_{\text{wave}}^2 + \text{COV}_{\text{SCF}}^2}$ . The coefficient of variation  $\text{COV}_{\text{wave}}$  models the uncertainty on the wave load, foundation stiffness, and stress ranges.  $\text{COV}_{\text{SCF}}$  models the uncertainty in the stress concentration factors (SCFs) and local joint flexibilities (LFJs).  $\log K_i$  is modelled by a normal distributed stochastic variable according to a specific SN-curve.

Using the illustrative stochastic model in Table 1, based on Faber *et al.* [3] and equation (6), the probability of failure in the service life and the annual probability of failure can be obtained. It is noted that the stochastic model for  $\log K_1$  and  $\log K_2$  is based on fatigue tests with variable amplitude load.

A fracture mechanical modelling of the crack growth is applied assuming that the crack can be modelled by a two-dimensional semi-elliptical crack. It is assumed that the fatigue life may be represented by a fatigue initiation life and a fatigue propagation life

$$N = N_{\rm I} + N_{\rm P} \tag{7}$$

where N is the number of stress cycles to failure,  $N_{\rm I}$  is the number of stress cycles to crack propagation, and

 $N_{\rm P}$  is the number of stress cycles from initiation to crack through.

The number of stress cycles from initiation to crack through is determined on the basis of a two-dimensional crack growth model. The crack growth can be described by the following two coupled differential equations

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_{\mathrm{A}} (\Delta K_{\mathrm{A}})^{m} \qquad a(N_{0}) = a_{0}$$

$$\frac{\mathrm{d}c}{\mathrm{d}N} = C_{\mathrm{C}} (\Delta K_{\mathrm{C}})^{m} \qquad c(N_{\mathrm{I}}) = c_{0}$$
(8)

where  $C_A$ ,  $C_C$ , and m are material parameters, and  $a_0$  and  $c_0$  describe the initial crack depth a and crack length c respectively, after  $N_I$  cycles. The stress intensity ranges are  $\Delta K_A$  and  $\Delta K_C$ . The crack initiation time  $N_I$  is modelled as Weibull distributed with expected value  $\mu_{10}$  and coefficient of variation equal to 0.35 [17]. The limit state function is written

$$g(\mathbf{x}) = N - nt \tag{9}$$

where t is time in the interval from 0 to the service life  $T_{\rm L}$ .

In order to model the effect of different weld qualities, two different values of the crack depth at initiation  $a_0$  can be used: 0.1 mm and 0.4 mm corresponding approximately to high and low material control. The critical crack depth  $a_c$  is often taken as the thickness of the tubular member. An example of a probabilistic modelling used in a fracture mechanical reliability analysis is shown in Table 2.

The parameters  $\mu_{\ln C_C}$  and  $\mu_0$  are fitted such that the difference between the probability distribution functions for the fatigue life determined using the SN-approach and the fracture mechanical approach is minimized as illustrated in the examples below.

A steel jacket structure with service life  $T_{\rm L}=40$  years and located in the North Sea is considered. The characteristics for some fatigue sensitive details are shown in Table 3, where  $T_{\rm F}$  is the fatigue lifetime for deterministic design. The resulting inspection intervals are shown in Table 4 for a maximum acceptable annual probability of failure,  $\Delta P_{\rm F}^{\rm max}=10^{-5}$ . It is seen that the time to first inspection increases with the FDF, FDF =  $T_{\rm F}/T_{\rm L}$ , and that after the first inspection, the inspection time intervals generally increase with time; but for low FDFs they decrease in the first part of the design lifetime.

Note that a basic assumption in the reliability-based inspection planning approach used in this paper is that a Bayesian approach can be used. This implies that probabilities of failure can be updated in a consistent way when new information becomes available. The Bayesian approach is also consistent with rational risk analysis and decision-making based on the framework of pre-posterior analysis from

**Table 2** Example of uncertainty modelling used in the fracture mechanical reliability analysis

Variable	Distribution*	Expected value	Standard deviation
$N_{ m I}$	W	$\mu_{\mathrm{I0}}$	$0.35 \mu_0$
$a_0$ $\ln C_{\rm C}$ $m$	D N D	0.4 mm $\mu_{\ln C_{\mathbb{C}}}$ m-value corresponding to the low	0.77
		cycle part of the bilinear SN-curve	
$Z_{ m SCF}$	LN	1	
$Z_{ m wave}$	LN	1	
$a_{\rm c}$	D	T (thickness)	
Y	LN	1	0.1

 $\ln C_{\rm C}$  and  $N_{\rm I}$  are correlated with correlation coefficient  $\rho_{\ln(C_{\rm C}),N_{\rm I}}=-0.5$ 

**Table 3** Example cases

Case	$COV_{wave}$	$COV_{SCF}$	T (mm)	T <sub>F</sub> (year)
1	0.1	0.15	20	100
2	0.1	0.15	20	120
3	0.1	0.15	20	140
4	0.1	0.15	20	160
5	0.1	0.15	20	180
6	0.1	0.15	20	200

**Table 4** Example inspection time intervals in years

Inspection number	1	2	3	4	5
Case 1 - FDF = 2.5 Case 2 - FDF = 3.0 Case 3 - FDF = 3.5 Case 4 - FDF = 4.0 Case 5 - FDF = 4.5 Case 6 - FDF = 5.0	13 16 19 22 25 28	6 7 9 10 12	5 7 9	7 9	9

classical Bayesian decision theory [18, 19] and implemented as described in reference [9]. This basic assumption is also very important in order to understand why longer inspection time intervals are obtained when no-finds at the inspections are assumed.

The uncertainties in Table 2 can be divided into aleatory and epistemic. Aleatory uncertainty is inherent variation associated with the physical system or the environment – it can be characterized as irreducible uncertainty or random uncertainty. Epistemic uncertainty is uncertainty due to lack of knowledge of the system or the environment – it can be characterized as subjective or reducible uncertainty. The epistemic uncertainties such as  $Z_{\rm SCF}$ ,  $Z_{\rm wave}$ , and Y are time-invariant and therefore the event inspection and 'no-find' can be considered as a proof-loading effect, implying that each inspection reduces these uncertainties, giving rise to a decrease

<sup>\*</sup>D: deterministic, N: normal, LN: lognormal, W: Weibull.

in the total uncertainty after each inspection – and thus longer inspection intervals in the normal design lifetime. In general, epistemic uncertainties are dominant in many applications for offshore installations.

# 3 INSPECTION PLANNING FOR OLDER INSTALLATIONS

This section describes various investigations in reliability-based inspection planning with the aim of discussing and investigating how decreased inspection time intervals could be obtained when time approaches and goes beyond the design lifetime. A shorter interval between inspections is intuitively expected when structure is used beyond the design lifetime, but as seen above, traditional reliability-based inspection techniques, when applied after the design life, normally result in increasing inspection time intervals.

The following assumptions/observations are included in the considerations for a modified method for reliability-based inspection planning for older installations:

- For ageing platforms, several small cracks are assumed to be observed – implying an increased risk of crack initiation (and coalescence of small cracks) and growth – thus modelling a bath-tub effect.
- 2. The repair of cracks can imply weakening of the material, implying subsequent crack initiation and growth.
- 3. Observed cracks can be divided into cracks due to fabrication defects and fatigue-growing cracks:
  - (a) fabrication cracks should be detected by fabrication control and/or an initial inspections, and are therefore not considered in the following;
  - (b) growing fatigue cracks which should be detected by inspections typically 10 per cent (of welds) are inspected and, from these, 5 per cent have cracks (defects).

The following models are considered for modifying inspection intervals for older installations:

- (a) increase of expected value of initial crack size with time owing to the coalescence of smaller cracks;
- (b) non-perfect repairs by detection of cracks, the repair is not perfect, and a new crack is initiated;
- (c) human errors in inspections (beyond uncertainty included in POD-curves);
- (d) increased rate of crack initiation adjustment of the crack initiation time such that the initiation of cracks increases with time (bath-tub effect). The increase of crack initiation can be in excess of the crack initiation expected at the design stage (and obtained by reliability-based calibration to SN-curves) owing to the ageing effects (e.g. by coalescence of small defects/cracks).

The above effects also apply in the case of extended lifetime. Representative examples are used to evaluate the different models.

The basic assumption in the RBI approach described in section 2 is that in a critical detail a defect/crack initiates at some time and is modelled by a stochastic variable. However, as mentioned above, it is assumed that for ageing installations the damage initiation rates follow a bath-tub form; see Fig. 3. Initial damages are mainly due to fabrication/construction defects, and at the end of the expected lifetime the damage rate is assumed to increase owing to widespread fatigue damage effects. In Fig. 6 a simple combined model is illustrated where the 'bath-tub' effect is combined with the 'usual' defect/crack initiation model. The model could be modified in different ways, but the main idea is to introduce more cracks at the end of the lifetime.

In model (d) it is assumed that more defects/cracks are initiated when time is approaching the design lifetime (owing to weakening by age effects) than assumed in the initial calibration of the fracture mechanics model. This model corresponds to the model in Fig. 6.

In the examples below, the extra cracks are assumed to be initiated following a simple linear

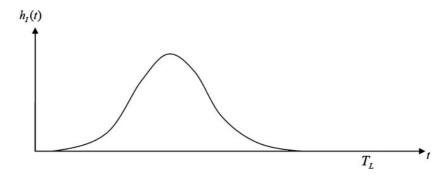


Fig. 6 A combined model for damage initiation with ageing defects

or constant model in the time interval  $[T_0, T_{\rm E}]$ ; see Figs 7 and 8. The extra cracks are assumed to be initiated in the fatigue critical area considered. Further, it is assumed that the expected number of extra cracks is  $(1-\delta)/\delta$  such that  $\alpha_{\rm I}=2(1-\delta)/(T_{\rm E}-T_0)$  with linear increase in the initiation rate and  $\alpha_{\rm I}=(1-\delta)/(T_{\rm E}-T_0)$  with constant increase in the initiation rate; see Figs 7 and 8.

Monte Carlo simulations are used to estimate the reliability as a function of time by the SN-approach and by the FM approach for the models proposed above. In order to reduce the computational effort, a one-dimensional fracture mechanics model is used. The stochastic models used are shown in Tables 5 and 6.

The parameters in the fracture mechanical model are calibrated to

$$\mu_{I0}$$
 = 5 years and  $\mu_{\ln C_c}$  = -25.5

The reliability index (based on accumulated probability of failure) is shown in Fig. 9. It is seen that a satisfactory agreement between the SN and the FM approach is obtained.

RBI planning with no modifications results in the inspection times for  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$  shown in Table 7. It is seen that inspection time intervals increase with time – most of the fastest growing cracks are detected and repaired in the first inspections, and thus only a few critical cracks are left when time approaches the design lifetime.

The three models (a), (b), and (c) described above do not result in decreased inspection time intervals. The main reason is believed to be the statistical effect of the inspection, namely that fast-growing cracks are detected by the first inspections – or, if not, then by

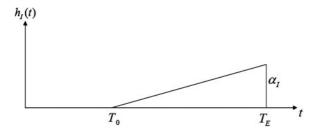


Fig. 7 Initiation rate of extra defects/cracks – linear model

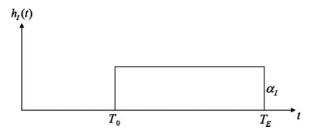


Fig. 8 Initiation rate of extra defects/cracks – constant model

one of the following inspections – leading to increasing inspection time intervals.

In model (d) extra cracks are assumed to initiate in the time interval  $[T_0, T_E]$ ; see Figs 7 and 8.

The inspection time intervals (in years) are with  $\Delta P_F^{\text{max}} = 10^{-4}$  determined in four situations:

- 1.  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$ ; see Tables 8a (linear) and 9a (constant).
- 2.  $\delta = 0.25$  and  $[T_0, T_E] = [40-60]$ ; see Tables 8b (linear) and 9b (constant).
- 3.  $\delta = 0.50$  and  $[T_0, T_E] = [25-60]$ ; see Tables 8c (linear) and 9c (constant).
- 4.  $\delta = 0.50$  and  $[T_0, T_E] = [40-60]$ ; see Tables 8d (linear) and 9d (constant).

Firstly, it is noted that owing to a limited number of simulations  $(2\times10^6)$  the inspection times have a 'standard error' of 1–2 years – therefore, some deviations in the inspection times for year 25 are observed. From the figures, it is seen that when many extra cracks  $(\delta=0.25)$  are initiated, the inspection time intervals decrease, especially if the extra cracks start early (after 25 years). As expected the constant model results in the largest decrease at the beginning of the time interval [40–60]. Note that a reason for the difference in ratio between the first and second

 Table 5
 Stochastic model for SN-approach in examples

Variable	Distribution	Expected value	Standard deviation
Δ	LN	1	0.2
$Z_{\rm SCF}$	LN	1	$COV_{SCF} = 0.10$
$Z_{ m wave}$	LN	1	$COV_{wave} = 0.30$
$T_{ m L}$	D	25 years	
$T_{ m F}$	D	75 years	
$m_1$	D	3	
$\log K_1$	N	12.048	0.218
$m_2$	D	4	
$\log K_2$	N	13.980	0.291

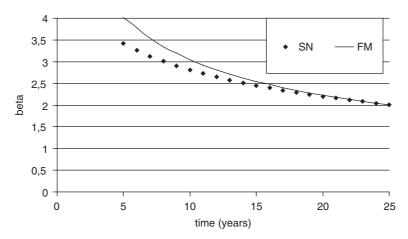
 $\log K_1$  and  $\log K_2$  are assumed fully correlated

**Table 6** Uncertainty modelling used in the fracture mechanical reliability analysis

Variable	Distribution*	Expected value	Standard deviation
$N_{\rm I}$ $a_0$	W D	$\mu_{10}$ (fitted) 0.4 mm	$0.35~\mu_0$
$\ln C_{\rm C}$	N D	$\mu_{\ln C_{\mathbb{C}}}$ (fitted)	0.77
$Z_{\mathrm{SCF}}$	LN	1	0.10
$Z_{ m wave}$ $a_{ m c}$	LN D	I T (thickness)	0.30
Y T	LN D	1 50 mm	0.1
$T_{ m L}$	D	25–60 years	

 $\ln C_{\rm C}$  and  $N_{\rm I}$  are correlated with correlation coefficient  $\rho_{\ln(C_{\rm C}),N_{\rm I}}\!=\!-0.5$ 

<sup>\*</sup>D: deterministic, N: normal, LN: lognormal, W: Weibull.



**Fig. 9** Reliability index (accumulated) as a function of time for the SN-approach and calibrated FM-approach

**Table 7** Inspection times and inspection time intervals in years. 'No modification.'  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

Inspection time Inspection interval	6	7	9	11	14	18	22	29	39	53
Inspection interval	O	1	2	2	3	4	4	1	10	14

Table 8a	Inspection times and inspection time intervals in years. 'Linear model' with $\delta = 0.25$ and
	$[T_0, T_{\rm E}] = [25-60] \cdot \Delta P_{\rm F}^{\rm max} = 10^{-4}$

Inspection time	6	8	9	11	13	17	22	30	38	43	48	53	58
Inspection interval	6	2	1	2	2	4	5	8	8	5	5	5	5

**Table 8b** Inspection times and inspection time intervals in years. 'Linear model' with  $\delta = 0.25$  and  $[T_0, T_{\rm E}] = [40-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

Inspection time	6	8	9	11	13	17	22	30	40	51	55	59	
Inspection interval	6	2	1	2	2	4	5	8	10	11	4	4	

**Table 8c** Inspection times and inspection time intervals in years. 'Linear model' with  $\delta = 0.50$  and  $[T_0, T_{\rm E}] = [25-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

Inspection time         7         8         10         12         15         18         23         30         40         47         55           Inspection interval         7         1         2         2         3         3         5         7         10         7         8													
Inspection interval ( 1 2 2 3 3 5 ( 10 ( 8	msbection time	7	8	10	12	15	18	73	30	40			
	Inspection interval	7	1	2	2	3	3	5	7	10	7	8	

**Table 8d** Inspection times and inspection time intervals in years. 'Linear model' with  $\delta = 0.50$  and  $[T_0, T_{\rm E}] = [40-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

	- 0, 2-		- I							
Inspection time	6	8	10	12	15	18	24	31	41	55
Inspection interval	6	2	2	2	3	3	6	7	10	14

**Table 9a** Inspection times and inspection time intervals in years. 'Constant model' with  $\delta = 0.25$  and  $[T_0, T_{\rm E}] = [25-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

Inspection time	6	7	9	11	13	16	20	27	33	38	42	46	52	58
Inspection interval	6	1	2	2	2	3	4	7	6	5	4	4	6	6

**Table 9b** Inspection times and inspection time intervals in years. 'Constant model' with  $\delta = 0.25$  and  $[T_0, T_E] = [40-60]$ .  $\Delta P_E^{\rm max} = 10^{-4}$ 

Inspection time	6	7	9	11	13	16	20	27	35	45	49	54	58
Inspection interval	6	1	2	2	2	3	4	7	8	10	4	5	4

**Table 9c** Inspection times and inspection time intervals in years. 'Constant model' with  $\delta = 0.50$  and  $[T_0, T_{\rm E}] = [25-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$ 

Inspection time	6	8	10	12	15	19	24	31	38	45	53
Inspection interval	6	2	2	2	3	4	5	7	7	7	8

**Table 9d** Inspection times and inspection time intervals in years. 'Constant model' with  $\delta = 0.50$  and  $[T_0, T_E] = [40-60]$ .  $\Delta P_E^{\text{max}} = 10^{-4}$ 

Inspection time	6	8	10	12	15	19	24	31	42	51	57	
Inspection interval	6	2	2	2	3	4	5	7	11	9	6	

**Table 10** Inspection times and inspection time intervals in years. 'No modification.'  $\Delta P_{\rm F}^{\rm max}=10^{-3}$ 

Inspection time	14	22	37
Inspection interval	14	8	15

**Table 11** Inspection times and inspection time intervals in years. 'Linear model' with  $\delta = 0.25$  and  $[T_0, T_{\rm F}] = [25-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$ 

Inspection time	14	21	31	45	56	
Inspection interval	14	7	10	14	11	

**Table 12** Inspection times and inspection time intervals in years. 'Constant model' with  $\delta = 0.25$  and  $[T_0, T_{\rm E}] = [25-60]$ .  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$ 

Inspection time	14	21	31	43	52	
Inspection interval	14	7	10	12	9	

inspection intervals in Tables 4 and 8/9 is the different maximum annual probability of failure (Table 4:  $\Delta P_{\rm F}^{\rm max}=10^{-5}$ , and Tables 8/9:  $\Delta P_{\rm F}^{\rm max}=10^{-4}$ ).

The inspection time intervals (in years) are with  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$  determined in three situations:

- 1. no extra cracks; see Table 10;
- 2.  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$ ; see Table 11 (linear);
- 3.  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$ ; see Table 12 (constant).

Results similar as for  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$  are observed. The inspection time intervals decrease when extra cracks are initiated, especially with the 'constant model'.

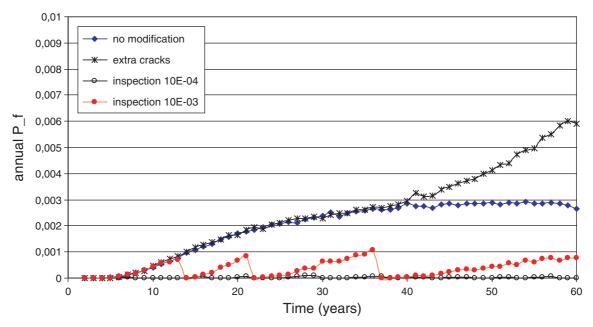
Figure 10 shows the annual probability of failure as a function of time without extra crack initiation (i.e. unmodified model and denoted 'no modification'),

with extra crack initiation (linear [10; 25] and  $\alpha_{\rm I} = 3 \times 2/15$ ), and with inspections when  $\Delta P_{\rm F}^{\rm max} = 10^{-4}$  and  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$ . The annual probability of failure is seen to increase significantly when extra initiation of cracks is included. However, using inspections, it is seen that it is possible to obtain a maximum annual probability of failure below  $\Delta P_{\rm F}^{\rm max}$ .

# 4 SYSTEMS EFFECTS FOR OLDER INSTALLATIONS

For many installations there will be a (large) number of critical details (components), implying the following important aspects.

- Assessment of the acceptable annual fatigue probability of failure for a particular component can depend on a number of fatigue critical components. The acceptable annual probability of failure of a component is obtained considering the importance of the component through the conditional probability of failure given failure of the component.
- 2. Owing to common loading, common model uncertainties, and correlation between inspection qualities, it can be expected that information obtained from inspection of one component can be used to update the inspection plan not only for that component, but also for other nearby components. Further, the common history and loading also imply an increased risk of several correlated components failing at almost the same time.
- 3. In some cases the development of a defect/crack in one component causes a stiffness reduction and an increased damping, implying that loads could be redistributed, thereby increasing the stress ranges in some of the other critical details.



**Fig. 10** Annual probability of failure as a function of time. 'Linear model' with  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$  and with inspections when  $\Delta P_E^{\text{max}} = 10^{-4}$  and  $\Delta P_E^{\text{max}} = 10^{-3}$ 

Table 13 illustrates the stochastic variables typically used in a fracture mechanical model for fatigue analysis, based partly on Table 6.

Considering as an example two critical components, the limit state equations can be written

$$g_1(t) = a_{c,1} - a_1(\mathbf{X}_{\text{Load},1}, \mathbf{X}_{\text{Strength},1}, t)$$
 (10)

$$g_2(t) = a_{c,2} - a_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, t)$$
(11)

where  $a_j(\mathbf{X}_{\text{Load},j}, \mathbf{X}_{\text{Strength},j}, t)$  is the crack depth at time t for component j;  $a_{\text{c},j}$  is the critical crack depth for component j;  $\mathbf{X}_{\text{Load},j}$  are the load variables ( $\mathbf{Z}_{\text{SCF}}$ ,  $\mathbf{Z}_{\text{wave}}$ , a, and b) for component j; and  $\mathbf{X}_{\text{Strength},j}$  are the strength variables ( $N_{\text{I}}$ ,  $a_0$ ,  $\ln C_{\text{C}}$ , and Y) for component j.

The events corresponding to detection of a crack at time T can similarly be written

$$h_1(T) = c_{d,1} - c_1(\mathbf{X}_{Load,1}, \mathbf{X}_{Strength,1}, T) \leq 0$$
 (12)

$$h_2(T) = c_{d,2} - c_2(\mathbf{X}_{Load,2}, \mathbf{X}_{Strength,2}, T) \le 0$$
 (13)

where  $c_j(\mathbf{X}_{\text{Load},j}, \mathbf{X}_{\text{Strength},j}, c_{\text{d},j}, T)$  is the crack length at time T for component j, and  $c_{\text{d},j}$  is the smallest detectable crack length for component j. Note that the crack depth  $a_j(t)$  and crack length  $c_j(t)$  are related through the coupled differential equations in (8).

The stochastic variables in different components will typically be dependent. The load-related variables can be assumed to be fully dependent as the loading is common to most components. However, in special cases different types of component, and components placed a long distance between each

**Table 13** Stochastic variables for fracture mechanical analysis

	Variable	Description
Strength variables	$N_{ m I}$ $a_0$ $\ln C_{ m C}$	Number of stress cycles to initiation of crack Initial crack length Crack growth parameter
Load variables	$Z_{\mathrm{SCF}}$ $Z_{\mathrm{wave}}$ $a, b$	Geometry function Uncertainty stress range calculation Uncertainty wave load Weibull parameter in long-term stress range distribution
Inspection quality	$c_{ m d}$	POD curve

other, can be less dependent. The strength variables  $N_{\rm I}$ ,  $a_{\rm 0}$ , and  $\ln C_{\rm C}$  will typically be independent as the material properties vary from component to component. However, some dependence can be expected for components fabricated with the same production techniques and from the same basic materials.

Updated probabilities of failure of components 1 and 2 given no detection of cracks in detail 1 and 2 are

$$P_{\text{F.1}|1} = P[g_1(t) \le 0 | h_1(T) > 0] \tag{14}$$

$$P_{F,2|2} = P[g_2(t) \le 0 | h_2(T) > 0]$$
(15)

$$P_{F,2|1} = P[g_2(t) \le 0 | h_1(T) > 0]$$
(16)

$$P_{F,1|2} = P[g_1(t) \leqslant 0 | h_2(T) > 0]$$
(17)

Equations (14) and (15) represent situations where a component is updated with inspection of the same component. Equations (16) and (17) represent situations where a component is updated with inspection of another component. The above formulae can easily be extended to cases where more components are inspected.

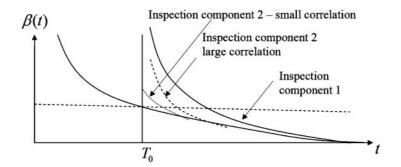
Figure 11 illustrates the effect of inspection planning on a component if this or another nearby component is inspected. The largest effect on reliability updating and thus inspection planning is obtained upon inspecting the same component or inspecting another component with a large correlation with the considered component.

As an example, two components are considered of the same stochastic model as in Table 6. It is assumed that component 1 is inspected, and if a crack is detected, then both components 1 and 2 are repaired. Further, it is assumed that each of the stochastic variables  $Z_{\rm SCF}$ ,  $Z_{\rm wave}$ , and Y are fully correlated in the two

elements, e.g.  $Z_{\rm SCF}$  in component 1 is fully correlated with  $Z_{\rm SCF}$  in component 2, and  $Z_{\rm SCF}$  and  $Z_{\rm wave}$  are independent.  $N_{\rm I}$  and  $\ln C_{\rm C}$  are assumed independent in the two components. No extra cracks are initiated and  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$ . Inspections should be performed in years 14, 23, and 35. Figure 12 shows the accumulated probability of failure for the two components. It is seen that the probability of failure for component 2 decreases, compared with no inspection, but is much higher than for the inspected component 1.

Next, it is assumed that extra cracks are initiated with the 'linear model' with  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$ .

Inspections should be performed in years 12, 17, 28, 38, 48, and 54. Figure 13 shows the accumulated probability of failure for the two components. It is seen that the probability of failure for component 2 decreases slightly compared with no inspection, but is much higher than for the inspected component 1.



**Fig. 11** Reliability index as a function of time for component 1 and updated reliability if inspection of component 1 at time  $T_0$ , or of component 2 at time  $T_0$  with large and small positive correlation with component 1

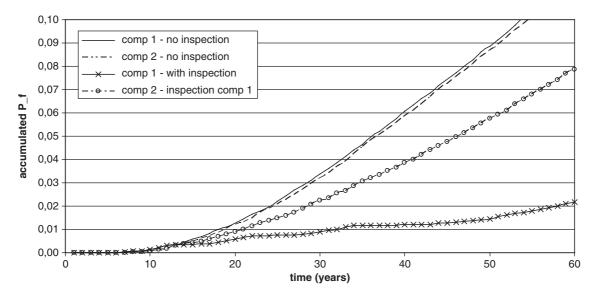


Fig. 12 Accumulated probability of failure as a function of time, without extra crack initiation and  $\Delta P_{\rm F}^{\rm max} = 10^{-3}$ 

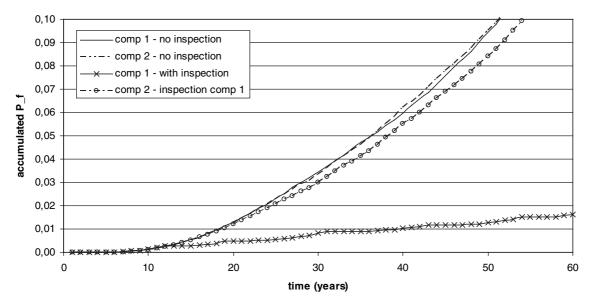
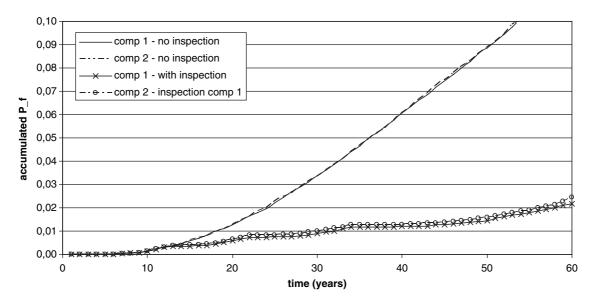


Fig. 13 Accumulated probability of failure as a function of time, with extra crack initiation – linear model with  $\delta = 0.25$  and  $[T_0, T_E] = [25-60]$  and  $\Delta P_F^{max} = 10^{-3}$ 



**Fig. 14** Accumulated probability of failure as a function of time, without extra crack initiation and  $\Delta P_{\rm F}^{\rm max}=10^{-3}$ 

If it is assumed that the stochastic variables modelling  $\ln C_{\rm C}$  in component 1 and 2 are fully correlated, and if no extra cracks are initiated, inspections should be performed in years 14, 23, and 35. Figure 14 shows the accumulated probability of failure for the two components. It is seen that now the probability of failure for component 2 is almost the same as for component 1.

These results indicate that a relatively high degree of correlation between the uncertain parameters in different components is needed in order to obtain substantial information that can be used in inspection planning.

### **5 SUMMARY**

The basic principles in reliability and RBI planning are described. The basic assumption made in risk/reliability-based inspection planning is that a Bayesian approach can be used. The Bayesian approach and the no-crack detection assumption imply that the inspection time intervals usually become longer and longer. Further, inspection planning based on the RBI approach implies that single components are considered, one at a time, but with the acceptable reliability level assessed based on the consequence for

the whole structure in case of fatigue failure of a single component.

The following two aspects are considered with the aim to develop/extend the RBI approach for older installations; namely, that for ageing structures several small defects/cracks are often observed – implying an increased risk for defect/crack initiation (and coalescence of small defects/cracks) and increased defect/crack growth. This should imply shorter inspection time intervals for ageing structures.

Different approaches for updating inspection plans for older installations are proposed in order to achieve decreased inspection intervals as the structures are ageing. The most promising method consists in increasing the rate of defects/crack initiation at the end of the expected lifetime – corresponding to a bath-tub hazard rate effect. The approach is illustrated for welded steel details in platforms, and implies that inspection time intervals decrease at the end of the platform lifetime. Note that data are needed to verify the increased crack initiation model. These data can be direct observations of cracks in older installations or indirect information from inspection programmes.

The approaches described are especially developed for the inspection planning of fatigue cracks, but can also be used for various other deterioration processes where inspection is relevant, including corrosion, chloride ingress in concrete with possible corrosion of reinforcement and wear.

Different system aspects are considered, including the assessment of the acceptable annual probability of failure for one component dependent on the number of critical components. Common loading, model uncertainties, etc. imply that information obtained from the inspection of one component can be used to update the inspection plan not only for that component, but also for other nearby components. Further, the common history and loading also imply an increased risk that several correlated components can fail at almost the same time. An example indicates that a high degree of correlation between the uncertain parameters in different components is needed in order to obtain substantial information that can be used in inspection planning.

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