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Odgaard, Peter Fogh; Wickerhauser, M.V.

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Discrimination between different kind of surface defects on Compact Discs

P.F. Odgaard

Department of Control Engineering
Frederik Bajersvej 7C
Aalborg University
DK 9220 Aalborg
odgaard@control.auc.dk

M.V. Wickerhauser

Department of Mathematics
One Brookings Drive
Washington University, St. Louis
St. Louis MO 63130
victor@math.wustl.edu

Abstract— Compact Disc players have problems playing discs with surface defects such as scratches and finger prints. The problem is that handling normal disturbances such as mechanical shocks etc, require a high bandwidth of the controllers which keep the Optical Pick-Up focused and radial tracked on the information track on the disc. In order for the controllers to handle the surface defects it is required that they are non-sensitive to the frequency contents of the defect, since a defect is a disturbance on the measurements. A simple solution to this problem is to decrease the controller bandwidth during the defect. However, due to the variation of defects a more adaptive control strategy would be preferable. In this paper the defects are categorised into three groups. A discriminator is designed, based on the local most discriminating basis vectors of the Karhunen-Loève and Haar bases as well as the mean of groups basis. In these bases the discrimination rule is simple. The defect in question is a member of the group it is closest too. The Karhunen-Loève basis gives a correct classification rate of more than 85.7% with 3 basis vectors and the Haar basis of more than 94.6% with 5 basis vectors.

I. INTRODUCTION

Compact Disc players (CD players) have been on the market more than two decades, and most people have no problems with their players, except if they try to play a CD with surface defects like scratches, finger prints etc. Those defects cause the player to jump to another area of the disc, meaning jumps in the music, or might even stop playing. The Optical Pick-up Unit (OPU) which is use to retrieve the information from the disc, is kept focused and tracked at the information track by two control loops, since there is no physical contact. The OPU feeds the controllers with indirect measurements of the physical distances in the focus and radial tracking directions, e_f and e_r , see Fig. 1. During the occurrence of a defect these signals are degenerated, and if not handled in some way the controllers can force the OPU out of focus and radial tracking. The problem in handling disturbances it that they require a high controller bandwidth which is in conflict with the fact that handling a defect in principle requires a low bandwidth, see (Andersen *et al.*, 2001) and (Vidal *et al.*, 2001).

The OPU generates, in addition to e_f and e_r , two residuals which can be used to detect surface defects as scratches, see Fig. 1. Simple threshold method used on the residuals are widely used methods for surface defect detection, see (Philips, 1994), (Andersen *et al.*, 2001) and (Vidal *et al.*,

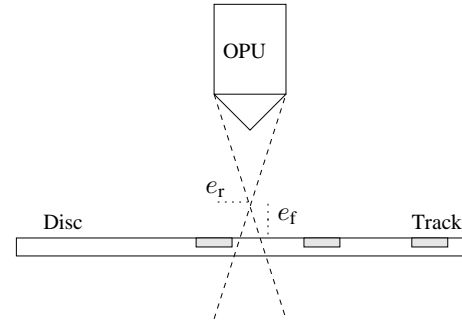


Fig. 1. The focus error e_f is the distance from the focus point of the laser beam to the reflection layer of the disc, the radial error is the distance from the centre of the laser beam to the centre of the track. The OPU emits the laser beam towards the disc surface and computes indirect measurements of e_f and e_r based on the received reflected light. In addition the OPU generates two residuals which can be used to detect surface defects as scratches.

2001).

In (Odgaard *et al.*, 2003b) and (Odgaard *et al.*, 2003a) some more discriminating residuals are described and computed. These are computed based on the OPU outputs and models of the OPU and the defects. Handling the defects can be done in a number of ways, most of them are dependent on a detection of the occurrence of the defect. This detection based on some residuals generated by the OPU is described in (Philips, 1994), (Andersen *et al.*, 2001) and (Vidal *et al.*, 2001).

In (Odgaard and Wickerhauser, 2003) the methods for detection and time localisation of the defects are improved. This time localisation is based on the fact that the given defect does not vary much from defect encounter number m to number $m + 1$. This is due to the fact that the distance between the track is $1.6 \mu\text{m}$, this distance is very small compared to the defects.

The set of all surface defects is a large set and the design of one controller handling them all would be a conservative controller. Instead it would better to discriminate the detected and time located defects into a number of groups, and use controllers adapted to the given defect group. Finger prints and small scratches can be merged into one group since finger prints seen from the control loops appear as a collection of small scratches. Larger scratches have a longer time duration and other frequency contents. This means that the optimal

handling of these two groups is not the same. The last group is a group of disturbances like defects, they are caused by other disc defects such as eccentricity, non constant reflection rate of the disc etc. Their frequency contents is in a lower frequency range than the other defects.

As a consequence of, the repetitive character of defects, it is possible to use more time for the feature extraction of the defect, in this case the discrimination of the defect into the three groups. This also means that the entire defect is available for the discrimination algorithm, and not only a small part of it.

Each defects are inside a window of n samples. In order to simplify the discrimination mentioned above the defects are transformed into some approximating bases: Karhunen-Loève (Mallat, 1999) , Haar (Mallat, 1999), FFT. The dimensions of these approximating subspaces are decreased by finding the four most local discriminating basis vectors, (LDB), see (Coifman and Saito, 1994) and (Saito, 1994), where the Fisher discriminator is used as cost functions, see (Johnson and Wichern, 2002) and (Flury, 1997). In this paper the 1-10 most discriminating basis vectors are used for the discrimination. These are in addition compared with a set consisting of the set of the means of the vectors in each group, this set of vectors does not span \mathcal{R}^n , but is a good comparison for the other discriminating bases.

In this paper defect groups are first defined based on experimental data, this data set is divided into a test and a training set. The various bases are shortly described, as well as the algorithm for finding the most local discriminating basis vectors. This is followed by a description of the decision rule based on the transform into these LDB vectors. In the end the different discriminating bases are compared based on the test data set.

II. DEFECT GROUPS

From the focus and radial residuals, α_f and α_r , defects are extracted based on the algorithm described in (Odgaard and Wickerhauser, 2003). Each detected defect is extracted into a column vector with the length of $256=2^8$ samples. This is a length is chosen since all defects in the dataset are shorter than 256 samples. The defects are extracted with symmetric geometric centre intended to be in the middle of the defect vector. Each vector can contain several defects (especially finger prints). In addition the centralisation is not totally successfully due to implementation. It was chosen that a given defect is only contained in one defect vector. I.e. the centre is not always in the middle of the vectors.

This extraction gives two matrices with defects. The defects in α_f are in A_f and the defects in α_r are in A_r . Where each column in the matrices are a defect vectors. All defects have by visual inspection been classified into three groups: G_1 Small defects, G_2 Disturbance like defects, G_3 Large defects,. These groups are described in Introduction, see Section I.

From each of the groups a training and a test set were formed by randomly taking 80% of the set to be the training set and keep the remaining part as test sets.

III. DISCRIMINATING ALGORITHM

The defects are in a block of 256 samples in time. In order to discriminate between the different kind of defects, the use of \mathcal{R}^n , $n = 256$ is a large set. It would be a better idea to use some approximating subspaces to reduce the needed order, and next find the most local discriminating basis vectors in this given basis, and eg. use the m most discriminating ones for the discrimination between the groups, where m is determined by test. The number of the discriminating basis vectors should be low, this means that the search for the optimal value of m is chosen to be in the interval: $[1, 10]$.

The local discriminating subspace is the most discriminating set of x basis vectors of a given basis, the groups which shall be discriminated between and a cost function, see (Saito, 1994) and (Coifman and Saito, 1994). In this paper the Fisher discriminator, see (Johnson and Wichern, 2002) and (Flury, 1997), is used as the cost function for finding the most discriminating basis vectors.

A. Fisher Discriminator

The Fisher discriminator gives the discriminating power of a number of groups in a given basis, see (Johnson and Wichern, 2002) and (Flury, 1997). Given an orthonormal basis : $\{x_1, \dots, x_n\}$, and $S = \{s_m : m = 1, \dots, M\}$ signals in G_1 , and $T = \{t_k : k = 1, \dots, K\}$ signals in G_2 , the discriminating power of the basis vector \mathbf{x}_i between groups 1 and 2, is defined as:

$$FD(G_1, G_2 | \mathbf{x}_i) = \frac{|E(\langle S, \mathbf{x}_i \rangle) - E(\langle T, \mathbf{x}_i \rangle)|^2}{Var(\langle S, \mathbf{x}_i \rangle) + Var(\langle T, \mathbf{x}_i \rangle)}, \quad (1)$$

and for the basis as a whole:

$$FD(G_1, G_2 | \mathbf{x}) = \sum_{i=1 \dots n} FD(G_1, G_2 | \mathbf{x}_i). \quad (2)$$

A good discriminating basis would have high discriminating power in a few basis vectors and almost nothing in the remaining majority of vectors, and a poor discriminating basis has the same discriminating power for all basis vectors.

In this work the basis is used to discriminate among three groups, this means that the discriminating powers among all the groups for each basis vector is computed:

$$\begin{aligned} FD(\mathbf{G} | \mathbf{x}_i) &= FD(G_1, G_2 | \mathbf{x}_i) \\ &+ FD(G_1, G_3 | \mathbf{x}_i) \\ &+ FD(G_2, G_3 | \mathbf{x}_i). \end{aligned} \quad (3)$$

When all these discriminating powers were computed, the m most discriminating basis vectors were found by choosing the m basis vectors with the highest discriminating powers.

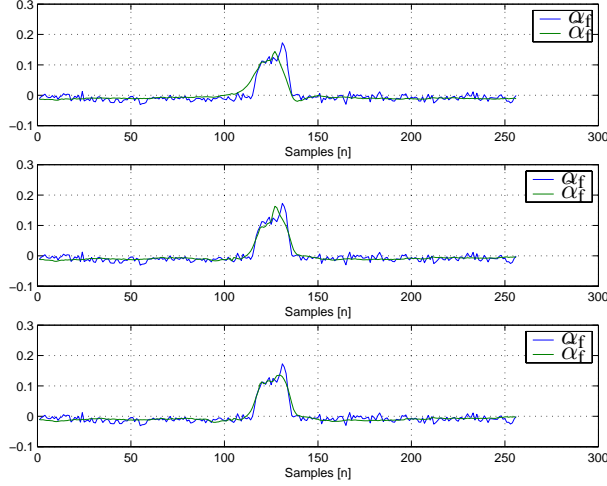


Fig. 2. Illustration of the Karhunen-Loève approximation of α_f which contain a typical scratch. The first approximation is based on the one most approximating coefficient, the second approximation is based on the five most most approximating coefficients, and the third approximation on the seven most approximating coefficient.

B. Choice of Discrimination basis

Four different bases are tried in this work: The Karhunen-Loève basis, the Haar basis, the frequency basis and the set of mean of groups.

1) *Karhunen-Loève basis*: This basis is chosen since it is the best approximating linear basis for a given training set. It is used to reduce the dimension of the subspace, for which the best discriminating basis vectors are found. These basis vectors are found in the following way, see (Mallat, 1999) and (Wickerhauser, 1994a).

- 1) Given the data set in A_f and A_r . Compute a data sets with zero mean, by subtracting the mean of each defect vector. This gives the data sets: \bar{A}_f and \bar{A}_r .
- 2) Then find the eigenvalues and eigenvectors of $\bar{A}_f \cdot \bar{A}_f^T$ and $\bar{A}_r \cdot \bar{A}_r^T$, these are the autocorrelation of the zero mean data sets. The eigenvectors are the Karhunen-Loève basis, and the eigenvalues are the variance of the given coordinates.

These eigenvectors/ Karhunen-Loève basis vector of the data set, are eigendefects. The approximating property of this basis is illustrated in Figs. 2 where a time serie of α_f containing a defect, is approximated with one, five and seven Karhunen-Loève basis vectors. From this it is clear that just few Karhunen-Loève basis vectors give a good approximation of the original signal.

2) *Haar basis*: Wavelet bases in general and the Haar basis specific are much more simple (faster in computations) than the Karhunen-Loève basis, but on the other hand not as good approximating basis. The following generalised Haar basis is chosen as a basis, since it is a simple basis. It is a generalised Haar basis since some other properties are needed than those given by the normal Haar basis. No

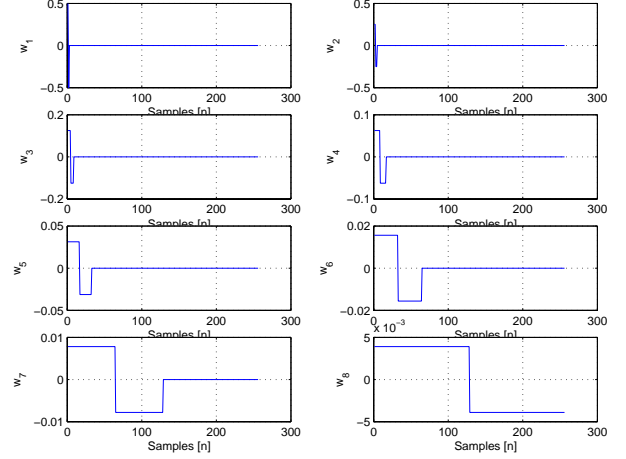


Fig. 3. The 8 generalised Haar basis vectors.

reconstruction is needed based on this basis, this means that the basis does not need to be orthogonal, due to the Fisher discriminator, it needs to be almost orthogonal, but it is still a normalised basis. It also have to time invariant, but due to the non requirement of reconstruction this can be handled easier than normal time invariant Haar transforms, see (Wickerhauser, 1994b) and (Mallat, 1999). The basis vectors, w_i , has the length of $2^8 = 256$. For the vector number n it is formed as follows. The first n elements take the value 1, and the next n elements take the value -1, and the remaining ones take the value zero. The vector is next normalised by multiplication of the factor 2^{-n} . The first 8 vectors are defined in this way. The last vector is the maximum value of the signal which shall be transformed. The basis vectors w_1, \dots, w_8 , are illustrated in Fig. 3. These basis vectors are all orthogonal.

Since the defects cannot be assured to be centred in the data set, this transform has to be time invariant. Since this transform is only used for analysis. It can be handled simple by computing the coordinates, c_j , by:

$$c_j = \max(|s * w_j|). \quad (4)$$

s denotes the signal, and w_j the j 'th basis vector. The basis' orthogonal property is lost in this time invariance handling. However, it is close to be orthogonal. The orthogonality is only lost if the maximum of the convolution relates to a time shift in the basis vector which makes the basis non orthogonal.

3) *Mean of group set*: If one wants to discriminate between two known signals, the best way is to convolute it with the signal itself, and the convolution giving the highest result is the convolution with the signal itself. But if the signal is not perfectly known or one which is a discrimination of groups containing more than one signal, this method is not so good anymore. However, these arguments indicate the usability of a discriminating basis consisting of the mean of the groups.

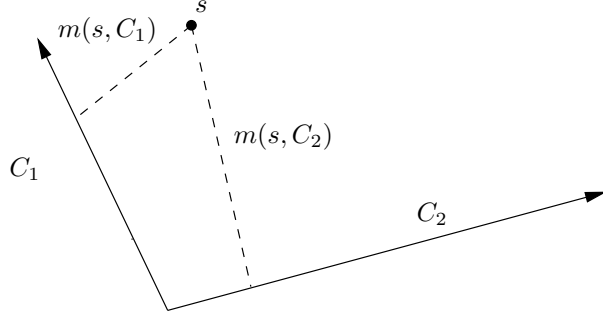


Fig. 4. Illustration of the decision rule of the discriminator. s is the defect in question. C_1 and C_2 are the centre of two groups. $m(s, C_1)$ and $m(s, C_2)$ are the measures of the distances between the defect in question and the respective group centres. Since s is closest to C_1 , this means that $m(s, C_1) < m(s, C_2)$. The decision is that the defect in this example is a member of group 1, since it is closest to centre of group 1.

This set is not orthogonal and neither does it span \mathcal{R}^n , but it is a good basis for the other bases to compare with, since it normally has good discriminating properties.

4) *FFT basis*: It is clear that the time axis is not a good discriminating basis of these defects, this leads to the question: Is the frequency basis a good discriminating basis for the defects? To test the frequency basis's discriminating power, a FFT basis was also used, with 128 elements linear distributed from 0 Hz to 17.5 kHz.

5) *Finding the discriminating basis vectors*: The Fisher discriminating power function is next used to find the four most discriminating vectors in each basis. These four most discriminating basis vectors are following used for discrimination.

C. The algorithm

For each group the centre of the group in the given discriminating basis is computed, based on the training set. This gives three vectors: C_1, C_2, C_3 .

The discriminating algorithm is: Find the group which has a centre with the smallest distance to the sample, given a metric. A successful measure has been found to be the geometric distance between the sample s and the vectors defined as the coordinates of the centre:

$$m(s, C) = \|s - \langle C, s \rangle \cdot C\|_2. \quad (5)$$

The decision rule and measure are illustrated in Fig. 4. In this illustration the defect in question, s , is closest to the centre of group 1, and the defect is as consequence a member of group 1. To illustrate the algorithm's success, an array is defined, where the rows indicate which group the defect is contained by, and the column which groups they are detected as.

D. Results

The discriminator's results are computed for the 1 to 10 most discriminating basis vectors. For 1 to 2 basis vectors the results are not impressive. For 3 to 6 basis vectors, the

Karhunen-Loève basis α_f				Karhunen-Loève basis α_r			
3 Basis vectors				3 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	85.7%	25.0%	0.0%	G_1	83.8%	0.0%	0.0%
G_2	9.1%	50.0%	0.0%	G_2	6.8%	0.0%	16.7%
G_3	5.2%	25.0%	100%	G_3	9.5%	100%	83.3%
4 Basis vectors				4 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	87.0%	25.0%	0.0%	G_1	85.1%	0.0%	0.0%
G_2	10.4%	50.0%	0.0%	G_2	8.1%	0.0%	0.0%
G_3	2.6%	25.0%	100%	G_3	6.8%	100%	100%
5 Basis vectors				5 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	87.0%	25.0%	0.0%	G_1	83.8%	0.0%	0.0%
G_2	10.4%	50.0%	0.0%	G_2	8.4%	0.0%	16.7%
G_3	2.6%	25.0%	100%	G_3	9.5%	100%	83.3%
6 Basis vectors				6 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	89.6%	50.0%	0.0%	G_1	83.8%	0.0%	0.0%
G_2	9.1%	25.0%	0.0%	G_2	2.7%	100%	0.0%
G_3	1.3%	25.0%	100%	G_3	13.5%	0.0%	100%

TABLE I

THE RESULTS OF THE DISCRIMINATOR BASED ON THE 3 TO 6 MOST DISCRIMINATING KARHUNEN-LOÈVE BASIS VECTORS ARE SHOWN IN THIS TABLE. THE LEFT HALF PART OF THE TABLE IS BASED ON THE RESIDUAL α_f , AND THE OTHER HALF PART ON THE RESIDUAL α_r . THESE PARTS ARE AGAIN SEPARATED INTO ARRAYS WITH THE RESULTS OF EACH NUMBER OF BASIS VECTORS. THE HORIZONTAL G_1, G_2, G_3 ARE THE GROUP IN WHICH THE TEST DATA ARE CONTAINED AND THE VERTICAL ONES ARE THE GROUPS THEY ARE CLASSIFIED AS BEING IN. THIS CAN BE ILLUSTRATED BY AN EXAMPLE. TAKE α_f WITH 3 BASIS VECTORS. 85.7% G_1 DEFECTS ARE CLASSIFIED AS BEING G_1 DEFECTS, 9.1% WAS CLASSIFIED AS BEING G_2 DEFECT AND THE REMAINING 5.2% WAS CLASSIFIED AS BEING G_3 DEFECTS.

improvement of the results is clear. The frequency basis achieves non acceptable results, the results of the discriminator based on the Karhunen-Loève basis, the Haar basis and the mean of group set are illustrated in Tables III-D, III-D and III-D. Before choosing the best discriminator from these results in Tables III-D-III-D, it is necessary to define some requirements to the discriminator. The most important issue is to classify G_3 defects as G_3 defects, since the controllers are maybe forced into severe problems if these defects are not classified correct. The second most important thing is to have as high correct classification of G_1 defects. G_2 defects are non common and are as consequence not as important to classify correct. Another important thing is to limit the required computations. This means that a low number of basis vectors is better than a high number of basis vectors. It also means that the Haar basis has a disadvantage in the way the basis transformation is done. It is made time invariant by finding the max of the auto correlation of the basis vectors and the residuals, where the other two basis transformations

Haar basis α_f				Haar basis α_r			
3 Basis vectors				3 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	49.4%	50.0%	0.0%	G_1	75.5%	0.0%	0.0%
G_2	19.5%	25.0%	0.0%	G_2	4.1%	100%	66.7%
G_3	31.2%	25.0%	100%	G_3	20.3%	0.0%	33.3%
4 Basis vectors				4 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	35.1%	25.0%	0.0%	G_1	79.7%	0.0%	0.0%
G_2	42.9%	50.0%	0.0%	G_2	16.2%	0.0%	16.7%
G_3	22.1%	25.0%	100%	G_3	4.1%	100%	83.3%
5 Basis vectors				5 Basis vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	54.5%	25.0%	0.0%	G_1	94.6%	0.0%	0.0%
G_2	31.1%	50.0%	0.0%	G_2	1.4%	100%	0.0%
G_3	14.3%	25.0%	100%	G_3	4.1%	0.0%	100%
Basis order 6				Basis order 6			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	84.4%	25.0%	0.0%	G_1	77.0%	0.0%	0.0%
G_2	10.4%	50.0%	0.0%	G_2	17.6%	100%	0.0%
G_3	5.2%	25.0%	100%	G_3	5.4%	0.0%	100%

TABLE II

THE RESULTS OF THE DISCRIMINATOR BASED ON 3 TO 6 MOST DISCRIMINATING HAAR BASIS VECTORS ARE SHOWN IN THIS TABLE. THE LEFT HALF PART OF THE TABLE IS BASED ON THE RESIDUAL α_f , AND THE OTHER HALF PART ON THE RESIDUAL α_r . THESE PARTS ARE AGAIN SEPARATED INTO ARRAYS WITH THE RESULTS OF EACH NUMBER OF BASIS VECTORS. THE HORIZONTAL G_1, G_2, G_3 ARE THE GROUP IN WHICH THE TEST DATA ARE CONTAINED AND THE VERTICAL ONES ARE THE GROUPS THEY ARE CLASSIFIED AS BEING IN. THIS IS THE SAME PRINCIPLE AS IN TABLE III-D.

are done by convoluting the basis vectors with the residuals. This means that a Haar has to perform significantly better than the other bases to be chosen as the best one. Inspection of these three tables with the results shows that the best performance is achieved by using the Haar transform with 5 basis vectors on α_r , where G_1 defects were classified correct with 94.6% success, and the two other groups were correctly classified with 100% success. The mean of group set achieves the best performance for the α_r residual, with 82.4% success for G_1 defects and 100% success for the two others. The Karhunen-Loève basis performs best at the α_f residual. The Karhunen-Loève basis does not improve its performance much from order 3 to 6. It is interesting to compare the result of order 3 and 4 discriminator based on the Karhunen-Loève basis, with mean of group set based discriminations. The Karhunen-Loève based discriminator achieves 50% success for G_2 defects for both 3 and 4 basis vectors, and 100% for G_3 defects for both the 3 and 4 basis vectors. The G_1 success rate is 85.7% for the Karhunen-Loève 3 basis vectors and 87% for the Karhunen-Loève 4 basis vectors.

Even though the Haar basis based discriminator performs the best, it is presumably not preferable due its high demands of computations, the time invariant property. Comparing the

Mean set α_f				Mean set α_r			
3 vectors				3 vectors			
	G_1	G_2	G_3		G_1	G_2	G_3
G_1	79.2%	0.0%	50.0%	G_1	82.4%	0.0%	0.0%
G_2	13.0%	75.0%	25.0%	G_2	17.6%	100%	0.0%
G_3	19.5%	0.0%	25.0%	G_3	0.0%	0.0%	100%

TABLE III

THE RESULTS OF THE DISCRIMINATOR BASED ON THE MEAN OF GROUP SET OF THE ORDER 3, THIS SET HAVE ONLY 3 VECTORS, IS SHOWN IN THIS TABLE. THE LEFT HALF PART OF THE TABLE IS BASED ON THE RESIDUAL α_f , AND THE OTHER HALF PART ON THE RESIDUAL α_r . THE HORIZONTAL G_1, G_2, G_3 ARE THE GROUP WHICH THE TEST DATA ARE CONTAINED IN AND THE VERTICAL ONES ARE THE GROUPS THEY ARE CLASSIFIED AS BEING IN. THIS IS THE SAME PRINCIPLE AS IN TABLE III-D.

Karhunen-Loève basis and the mean of group basis, they have the same good performance regarding discriminating G_3 defects. The Karhunen-Loève basis is 3 or 5 % point better success rate of G_1 defects, but it does not perform as well at G_2 . However, as written before, the G_2 is rare and is as a consequence not as important to discriminate well. This means that if the number of computations are not a large problem, the projection of α_r on the 5 Haar basis vectors is the best discriminator. If the number of computation is a problem with best discriminator is the projection of α_f on the 3 most discriminating Karhunen-Loève basis vectors.

IV. CONCLUSION

Based on the test data from real world challenging CDs three defect groups are defined. These groups are used to design a discriminator, which is designed to discriminate between these groups. This discriminator is found based on the local discriminating basis of some approximating bases: Karhunen-Loève, Haar etc. After the basis transformation, the discriminator finds the group which the given defect is closest to in the given basis. The Karhunen-Loève basis based detection has rates higher than 85.7% for the important short defects, G_1 , and large defects, G_3 . The much more computationally demanding Haar basis based discriminator has success rates higher than 94.6% for all the three defect groups. In addition to these high success rates the Haar basis based discriminator is time invariant, which is an important property if it can not ensured that the defects are symetrically placed in the middle of the data block.

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