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NONLINEAR FIELD ORIENTED CONTROL OF INDUCTION MOTORS USING THE BACKSTEPPING DESIGN

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Abstract

Using backstepping, which is a recursive nonlinear design method, a novel approach to control of induction motors is developed. The resulting scheme leads to a nonlinear controller for the torque and the amplitude of the field. A combination of nonlinear damping and observer backstepping with a simple flux observer is used in the design. Assuming known motor parameters the design achieves stability with guaranteed region of attraction. It is also shown how a conventional field oriented controller may be obtained by omitting parts of the nonlinear controller.

Keywords : Nonlinear control, backstepping design, induction motors.

NOMENCLATURE

a	complex spatial operator $e^{j2\pi/3}$
$i_{sA,B,C}$	stator phase currents A,B and C
$u_{sA,B,C}$	stator phase voltages A,B and C
$\bar{i}_s$	stator current complex space vector
$\bar{u}_s$	stator voltages complex space vector
R_s, R_r	resistances of a stator and rotor phase winding
L_s, L_r	self inductance of the stator and the rotor
L_m	magnetizing inductance
T_r	rotor time constant ($T_r = L_r / R_r$)
σ	leakage factor ($1 - L_m^2 / (L_s L_r)$)
R'_r	referred rotor resistance ($R'_r = (L_m / L_r)^2 R_r$)
L'_s	referred stator inductance ($L'_s = \sigma L_s$)
L'_m	referred magnetizing inductance ($L'_m = (1 - \sigma) L_s$)
p	time derivative operator ($p \equiv d/dt$)
Z_p	number of pole pair
ω_{mech}	angular speed of the rotor
ω_{mR}	angular speed of the rotor flux
i_{mR}	rotor magnetizing current
ρ	rotor flux angle
c_m	torque factor ($c_m = 1.5 Z_p L'_m$)

1 Introduction

The development of the design of high-performance controllers for drives using an induction motor as an actuator is shortly stated by Leonhard [6] as "30 Years Space Vectors, 20 Years Field Orientation and 10 Years Digital Signal Processing with Controlled AC-drives". The relevance of field oriented control is witnessed by a large numbers of investigations carried out both from a theoretical and a practical point of view [5]. The scheme works with a controller which approximately linearizes and decouples the relation between input and output variables by using the simplifying hypothesis that the actual motor flux is kept constant and equal to some desired value.

In the last 10 years with digital signal processing, significant advances have been made in the theory of nonlinear state feedback control [3], and particular feedback linearization and input-output decoupling techniques have been successfully applied for control of induction motor drives [7], [4], [1], [2], [8], [9] and [10].

In Krstic, Kanellakopoulos and Kokotovic [11], a view is opened to a largely unexplored landscape of nonlinear systems with uncertainties. The recursive design methodology developed is called backstepping. With this method the construction of nonlinear feedback control laws and associated Lyapunov functions is systematic and guarantee that the designed system will possess desired properties globally or in a specified region of the state space.

While feedback linearization methods require precise models and often cancel some useful nonlinearities, backstepping designs offer a choice of design tools for accommodation of certain nonlinearities and can avoid wasteful cancellations.

In this paper the method is used for design of a field

oriented controller for an induction motor assuming measured rotor speed and stator currents and voltages. The control objectives are tracking of

- the amplitude of the estimated flux
- the estimated electrical torque

A Lyapunov function for the designed system guarantee desired properties in the state space region where the motor is magnetized.

The key items in this paper are

- a model assuming the speed as a parameter
- rotor field orientation
- a simple flux estimator
- control of torque and rotor field amplitude based on nonlinear feedback
- stability analyzed by a Lyapunov function

The paper also shows how a conventional field oriented controller may be derived from the developed nonlinear controller by omitting some terms.

2 Induction motor model

Figure 1: Definitions of transformation angles and symbols

The motor model is described in a rotating reference frame $e^{j\rho}$ with $\omega = \frac{d\rho}{dt}$, having the d-axis in the direction of $e^{j\rho}$ and the q-axis orthogonal to the d-axis.

For the currents and voltages given in stator coordinates (i_{sA}, i_{sB}, i_{sC}) and (u_{sA}, u_{sB}, u_{sC}) the following equations based on the angle definition given in Fig. 1 give the transformation from stator coordinates to rotating (d, q) -coordinates:

$$\begin{aligned}\bar{i}_s &= i_{sd} + j i_{sq} = \frac{2}{3}(i_{sA} + a i_{sB} + a^2 i_{sC})e^{-j\rho} \\ \bar{u}_s &= u_{sd} + j u_{sq} = \frac{2}{3}(u_{sA} + a u_{sB} + a^2 u_{sC})e^{-j\rho}\end{aligned}$$

The motor model is then given by:

$$\frac{d}{dt} \begin{Bmatrix} L'_s i_{sd} \\ L'_s i_{sq} \\ L'_m i_{md} \\ L'_m i_{mq} \end{Bmatrix} = \begin{Bmatrix} u_{sd} \\ u_{sq} \\ 0 \\ 0 \end{Bmatrix} - \begin{Bmatrix} R_s i_{sd} - \omega L'_s i_{sq} + R'_r(i_{sd} - i_{md}) - Z_p \omega_{mech} L'_m i_{mq} \\ R_s i_{sq} + \omega L'_s i_{sd} + R'_r(i_{sq} - i_{mq}) + Z_p \omega_{mech} L'_m i_{md} \\ R'_r(i_{sd} - i_{md}) + (\omega - Z_p \omega_{mech}) L'_m i_{mq} \\ R'_r(i_{sq} - i_{mq}) - (\omega - Z_p \omega_{mech}) L'_m i_{md} \end{Bmatrix}$$

and the developed electrical torque is:

$$m_e = \frac{3}{2} Z_p L'_m (i_{md} i_{sq} - i_{mq} i_{sd}) \quad (1)$$

The mechanical equation is:

$$J \frac{d\omega_{mech}}{dt} + f_0 \omega_{mech} = m_e - m_L$$

Because the rotor magnetizing current $\bar{i}_m = i_{md} + j i_{mq}$ is not measured an estimator has to be constructed. A common method used is based on the current equations. Introducing $T_r = L'_m / R'_r$ and $\omega_r = Z_p \omega_{mech}$ in the equations gives

$$\begin{aligned}\frac{d}{dt} \hat{i}_{md} &= \frac{1}{T_r} (i_{sd} - \hat{i}_{md}) \\ \frac{d}{dt} \hat{i}_{mq} &= \frac{1}{T_r} (i_{sq} - \hat{i}_{mq}) - (\omega - \omega_r) \hat{i}_{md} = 0\end{aligned}$$

gives for $\hat{i}_{md} \neq 0$ and $\hat{i}_{mq} = 0$

$$\omega_{slip} = \omega - \omega_r = \frac{i_{sq}}{T_r \hat{i}_{md}}$$

The estimation error $\tilde{i}_m = i_m - \hat{i}_m$ has the dynamics

$$\frac{d}{dt} \begin{Bmatrix} \tilde{i}_{md} \\ \tilde{i}_{mq} \end{Bmatrix} = \begin{Bmatrix} -\frac{1}{T_r} \tilde{i}_{md} + \omega_{slip} \tilde{i}_{mq} \\ -\frac{1}{T_r} \tilde{i}_{mq} - \omega_{slip} \tilde{i}_{md} \end{Bmatrix}$$

The convergence is based on the Lyapunov function

$$V_{obs} = \frac{1}{2} (\tilde{i}_{md}^2 + \tilde{i}_{mq}^2)$$

whose derivative along the solutions is

$$\dot{V}_{obs} = \tilde{i}_{md} \left(-\frac{1}{T_r} \tilde{i}_{md} + \omega_{slip} \tilde{i}_{mq} \right) + \tilde{i}_{mq} \left(-\frac{1}{T_r} \tilde{i}_{mq} - \omega_{slip} \tilde{i}_{md} \right)$$

$$\dot{V}_{obs} = -\frac{1}{T_r} (\tilde{i}_{md}^2 + \tilde{i}_{mq}^2) \leq 0$$

3 Backstepping

The control objectives are tracking of

- the amplitude of the estimated magnetizing current $\hat{i}_{md}$
- the estimated electrical torque $\frac{3}{2} Z_p L'_m \hat{i}_{md} i_{sq}$

Step 1.

We first consider the tracking objective of the magnetizing current. A tracking error $z_1 = \hat{i}_{md} - i_{md,ref}$ is defined and the derivative becomes

$$\dot{z}_1 = \frac{1}{T_r}(i_{sd} - \hat{i}_{md}) - \frac{di_{md,ref}}{dt}$$

To initiate backstepping, we choose i_{sd} as our first virtual control. If the stabilizing function is chosen as

$$i_{sd,ref} = \hat{i}_{md} - c_1 T_r z_1 + T_r \frac{di_{md,ref}}{dt}$$

we get

$$\dot{z}_1 = -c_1 z_1 + \frac{1}{T_r}(i_{sd} - i_{sd,ref})$$

Due to the fact that i_{sd} is not a control input an error variable $z_2 = i_{sd} - i_{sd,ref}$ is defined and we have

$$\dot{z}_1 = -c_1 z_1 + \frac{1}{T_r} z_2$$

Step 2.

The derivative of the error variable $z_2 = i_{sd} - i_{sd,ref}$ becomes

$$\begin{aligned} \dot{z}_2 = & -\frac{1}{T_r}(i_{sd} - \hat{i}_{md}) \\ & + c_1(i_{sd} - \hat{i}_{md} - T_r \frac{di_{md,ref}}{dt}) - T_r \frac{d^2 i_{md,ref}}{dt^2} \\ & \frac{1}{L'_s} u_{sd} - \frac{1}{L'_s} (R_s i_{sd} - \omega L'_s i_{sq} + R'_r(i_{sd} - \hat{i}_{md})) \\ & + \frac{R'_r}{L'_s} \tilde{i}_{md} + \omega_r \frac{L'_m}{L'_s} \tilde{i}_{mq} \end{aligned}$$

Viewing $\tilde{i}_{md}$ and $\tilde{i}_{mq}$ as unknown disturbances we apply nonlinear damping [11] to design the control function

$$\begin{aligned} \frac{1}{L'_s} u_{sd} = & \frac{1}{L'_s} (R_s i_{sd} - \omega L'_s i_{sq} + R'_r(i_{sd} - \hat{i}_{md})) \\ & (\frac{1}{T_r} - c_1)(i_{sd} - \hat{i}_{md}) \\ & + c_1 T_r \frac{di_{md,ref}}{dt} + T_r \frac{d^2 i_{md,ref}}{dt^2} \\ & - c_2 z_2 - \frac{1}{T_r} z_1 - d_2 \phi^2 z_2 + \phi_1 \tilde{i}_{md} + \phi_2 \tilde{i}_{mq} \end{aligned}$$

Defining $\phi_1 = \frac{R'_r}{L'_s}$, $\phi_2 = \omega_r \frac{L'_m}{L'_s}$ and $\phi^2 = \phi_1^2 + \phi_2^2$ insertion of the control function in the dynamics for the error variable z_2 gives

$$\dot{z}_2 = -c_2 z_2 - \frac{1}{T_r} z_1 - d_2 \phi^2 z_2 + \phi_1 \tilde{i}_{md} + \phi_2 \tilde{i}_{mq}$$

Step 3.

We now turn our attention to the torque tracking objective. A tracking error is for $\hat{i}_{md} \neq 0$ defined as $z_3 = i_{sq} - m_{e,ref}/(\frac{3}{2} Z_p L'_m \hat{i}_{md})$ and the derivative is

$$\begin{aligned} \dot{z}_3 = & \frac{1}{L'_s} u_{sq} - \frac{1}{L'_s} (R_s i_{sq} + \omega L'_s i_{sd} + R'_r i_{sq} + \omega_r L'_m \hat{i}_{md}) \\ & - \frac{R'_r}{L'_s} \tilde{i}_{mq} + \omega_r \frac{L'_m}{L'_s} \tilde{i}_{md} \\ & + \frac{3}{2} Z_p L'_m \hat{i}_{md}^2 \frac{1}{T_r} (i_{sd} - \hat{i}_{md}) \\ & - \frac{1}{\frac{3}{2} Z_p L'_m \hat{i}_{md}} \frac{dm_{e,ref}}{dt} \end{aligned}$$

Figure 2: Nonlinear Field Oriented Control

Viewing $\tilde{i}_{md}$ and $\tilde{i}_{mq}$ as unknown disturbances we apply nonlinear damping [11] to design the control function

$$\begin{aligned} \frac{1}{L'_s} u_{sq} = & \frac{1}{L'_s} (R_s i_{sq} + \omega L'_s i_{sd} + R'_r i_{sq} + \omega_r L'_m \hat{i}_{md}) \\ & - \frac{3}{2} Z_p L'_m \hat{i}_{md}^2 \frac{1}{T_r} (i_{sd} - \hat{i}_{md}) \\ & + \frac{1}{\frac{3}{2} Z_p L'_m \hat{i}_{md}} \frac{dm_{e,ref}}{dt} \\ & - c_3 z_3 - d_3 \phi^2 z_3 - \phi_1 \tilde{i}_{mq} + \phi_2 \tilde{i}_{md} \end{aligned}$$

Insertion of the control function in the dynamics for the error variable z_3 then gives

$$\dot{z}_3 = -c_3 z_3 - d_3 \phi^2 z_3 - \phi_1 \tilde{i}_{mq} + \phi_2 \tilde{i}_{md}$$

The combined controller is shown in figure 2 where we have

$$\begin{aligned} u_{sd,ff} &= R_s i_{sd} - \omega L'_s i_{sq} + R'_r(i_{sd} - \hat{i}_{md}) \\ u_{sq,ff} &= R_s i_{sq} + \omega L'_s i_{sd} + R'_r i_{sq} + \omega_r L'_m \hat{i}_{md} \\ u_{sd,nl} &= L'_s \{ (\frac{1}{T_r} - c_1)(i_{sd} - \hat{i}_{md}) - \frac{1}{T_r}(\hat{i}_{md} - \hat{i}_{md,ref}) \} \\ u_{sq,nl} &= -\frac{m_{e,ref}}{\frac{3}{2} Z_p L'_m \hat{i}_{md}^2} \frac{L'_s}{T_r} (i_{sd} - \hat{i}_{md}) \end{aligned}$$

Figure 3 shows a conventionally field oriented controller.

Figure 3: Conventional Field Oriented Control

Often the current controllers are of the PI type and the feedforward terms $u_{sd,ff}$ and $u_{sq,ff}$ are omitted. If we compare figure 2 with 3 it is seen that the conventional

is a special case of the nonlinear controller, with omitted feedforward terms from the references and nonlinear feedforwards $u_{sd,nl}$ and $u_{sq,nl}$.

4 Stability Analysis

Combining the above transformations give the system dynamics

$$\frac{d}{dt} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ \tilde{i}_{md} \\ \tilde{i}_{mq} \end{Bmatrix} = \begin{Bmatrix} -c_1 z_1 + \frac{1}{T_r} z_2 \\ -c_2 z_2 - \frac{1}{T_r} z_1 - d_2 \phi^2 z_2 + \phi_1 \tilde{i}_{md} + \phi_2 \tilde{i}_{mq} \\ -c_3 z_3 - d_3 \phi^2 z_3 - \phi_1 \tilde{i}_{mq} + \phi_2 \tilde{i}_{md} \\ -\frac{1}{T_r} \tilde{i}_{md} + \omega_{slip} \tilde{i}_{mq} \\ -\frac{1}{T_r} \tilde{i}_{mq} - \omega_{slip} \tilde{i}_{md} \end{Bmatrix} \quad (2)$$

This system has an equilibrium at $z_1 = z_2 = z_3 = \tilde{i}_{md} = \tilde{i}_{mq} = 0$

Furthermore the derivative of the function

$$V = \frac{1}{2} \{ z_1^2 + z_2^2 + z_3^2 + T_r \left(\frac{1}{d_2} + \frac{1}{d_3} \right) (\tilde{i}_{md}^2 + \tilde{i}_{mq}^2) \}$$

along the solution of (2) is nonpositive

$$\begin{aligned} \dot{V} = & -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2 - \frac{3}{4} \left(\frac{1}{d_2} + \frac{1}{d_3} \right) (\tilde{i}_{md}^2 + \tilde{i}_{mq}^2) \\ & - d_2 \left(\phi_1 z_2 - \frac{1}{2d_2} \tilde{i}_{md} \right)^2 - d_2 \left(\phi_2 z_2 - \frac{1}{2d_2} \tilde{i}_{mq} \right)^2 \\ & - d_3 \left(\phi_1 z_3 - \frac{1}{2d_3} \tilde{i}_{mq} \right)^2 - d_3 \left(\phi_2 z_3 + \frac{1}{2d_3} \tilde{i}_{md} \right)^2 \end{aligned}$$

It is seen too that $\dot{V} \leq -W \leq 0$ with

$$W = c_1 z_1^2 + c_2 z_2^2 + c_3 z_3^2 + \frac{3}{4} \left(\frac{1}{d_2} + \frac{1}{d_3} \right) (\tilde{i}_{md}^2 + \tilde{i}_{mq}^2)$$

Assuming boundedness of the reference values and a stable control for $m_{e,ref}$ based on ω_{mech} then we have for $\hat{i}_{md} > 0$ a solution giving $\lim_{t \rightarrow \infty} W \rightarrow 0$. Hence, the torque and field magnitude tracking objectives are achieved for any initial condition $\hat{i}_{md} > 0$.

5 Simulations

In this section sensitivity against change in rotor resistance and magnetizing inductance will be analyzed based on a simulation study. The nonlinear control method will be compared with the traditional rotor flux based Field Oriented Control method with d-q current control loops. Both methods will use the following flux estimator:

$$\begin{aligned} \hat{i}_{mR}(t) &= \frac{1}{1+pT_r} i_{sd}(t) \\ \hat{\rho}(t) &= Z_p \theta_{mech} + \int_0^t \frac{i_{sq}(\tau)}{T_r \hat{i}_{mR}(\tau)} d\tau \end{aligned}$$

which is defined for $i_{mR} \neq 0$.

The rotor resistance and the magnetizing inductance change considerably due to variation in temperature and

magnetic saturation. For our test motor, a GRUNDFOS 1.1 kW induction motor, these variation can be calculated. The magnetizing inductance is minimal when the motor is unloaded. In that case the motor is very close to saturation. The magnetizing inductance is maximum when the motor torque is maximum. The minimum, and maximum, values for the rotor resistance is obtain for cold and hot motor respectively. The analysis of the sensitivity against variation in R_r and L_m is based on a simulation study.

Figure 4: Field Oriented Control System with decoupling of torque and field amplitude

In the control systems the speed controller and field weakening function will be omitted as shown in figure 4

The load is simulated as

$$J \frac{d\omega_{mech}}{dt} = m_e - f_0 \omega_{mech}$$

and the reference signals $i_{mR,ref}$, $m_{e,ref}$ are chosen as:

$$\begin{aligned} i_{mR,ref} &= \begin{cases} 0 & \text{for } t < 0 \\ 0.8 & \text{for } 0 \leq t < 1 \\ 0.4 & \text{for } 1 \leq t \end{cases} \\ m_{e,ref} &= \begin{cases} 0 & \text{for } t < 0.5 \\ 0.4 & \text{for } 0.5 \leq t \end{cases} \end{aligned}$$

The parameters used for simulation are $J = 0.00077$, $Z_p = 1$, $\alpha_1 = 0.04$, $T_2 = 0.0005$.

	R_s	R_r	L_m	$L_s - L_m$	$L_r - L_m$
Nominal	9.20	6.61	0.5353	0.01228	0.01865
Cold motor	9.20	4.79	0.5353	0.01228	0.01865
196% load	9.20	6.61	0.6601	0.01228	0.01865

Figures 5 and 6 show the sensitivity to changes in rotor resistance and magnetizing inductance. The subfigures in the first and second column show the simulation results using the Nonlinear Backstepping method and Rotor Field Oriented Control method respectively. The dashed lines

Figure 5: Sensitivity to change in rotor resistance for Nonlinear Control (a and b) and Rotor Field Oriented Control (c and d). Dashed line (nominel motor) and solid line (cold motor)

Figure 6: Sensitivity to change in magnetizing inductance for Nonlinear Control (a and b) and Rotor Field Oriented Control (c and d). Dashed line (nominel motor) and solid line (196% load)

correspond to equal parameters in the motor and the model (hot motor, 100% load). The solid lines correspond to the case where motor parameters change due to temperature (cold motor) and load variations (200% load).

As the figures show the two methods demonstrate nearly the same dynamic behavior. Both methods are sensitive to changes in L_m and R_r . In the case of a change in L_m it would have been expected that the Rotor Field Oriented Control method would have been less sensitive due to the internal current loops.

6 Conclusion

A new method based on nonlinear control theory has been compared to the traditional Rotor Field Oriented Control method. Compared to other strategies evolving from the nonlinear control theory reported in the literature this method do not need an adaptation of the motor load. Elimination of this need for adaptation implies that servo performance is present even at momentary load torque changes. Both methods are sensitive to motor parameter variations. In the case of a change in L_m it would have been expected that the Rotor Field Oriented Control method would have been less sensitive due to the internal current loops, but the simulation studies show that the two methods have nearly the same sensitivity. The nonlinear method has the advantage that it opens for a systematic way of compensating for nonlinearities like magnetic saturation effects.

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