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# ON-LINE MULTIPLE-MODEL BASED ADAPTIVE CONTROL RECONFIGURATION FOR A CLASS OF NONLINEAR CONTROL SYSTEMS

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Abstract: Based on the model-matching strategy, an adaptive control reconfiguration method for a class of nonlinear control systems is proposed by using the multiplemodel scheme. Instead of requiring the nominal and faulty nonlinear systems to match each other directly in some proper sense, three sets of LTI models are employed to approximate the faulty, reconfigured and nominal nonlinear systems respectively with respect to the on-line information of the operating system, and a set of compensating modules are proposed and designed so as to make the local LTI model approximating to the reconfigured nonlinear system match the corresponding LTI model approximating to the nominal nonlinear system in some optimal sense. The compensating modules are designed by the Pseudo-Inverse Method based on the local LTI models for the nominal and faulty nonlinear systems. Moreover, these modules should update corresponding to the updatings of local LTI models, which validations are determined by the model approximation errors and the optimal index of local design. The test on a nonlinear ship propulsion system shows the promising potential of this method for system reconfiguration.

Keywords: Multiple-model, adaptive control, control reconfiguration, nonlinear control systems

# 1. INTRODUCTION

Control Reconfiguration (CR) implies use and implementation of proper control techniques in order to recover the faulty system operation/functionality to its nominal level. In general, the CR can fall into two categories with respect to different design strategies:

• Requirement-oriented CR strategies: These strategies can be regarded as a kind of control design procedures, i.e., when some fault(s) happened inside the system, a new controller will be designed based on the faulty system information provided by FDI sub-systems so as to make the new closed-loop system (reconfigured system) still satisfy the requirements originally proposed for the nominal system. The example in this category is the Model Predictive Control based method (Huzmezan and Maciejowski (1999)).

• System-oriented CR strategies: Within these strategies, the CR is regarded as a kind of system property recovery, such as the dynamic or functionality recovery (Frei et al. (1999)). The CR design following this kind of strategies does not consider the concrete system requirements, alternatively, the whole design is based on the inherent information of nominal and faulty systems, such as the control mixer based methods by Gao and Antsaklis (1991) and the model-following method by Huang and Stengel (1990). The first kind of CR strategies depends on concrete system requirements, however, when these requirements are flexible, this kind of strategies will not be sufficient. With respect to the benefit of the essential dynamic/functionality recovery, the system-oriented strategies are causing more and more attention. From the system engineering point of view, the second kind of CR strategies fits the model following/matching scheme well if we regard the nominal system as the reference model. Once the (reconfigured) system output or state rate is required to follow that of the nominal system, these approaches are referred to as the explicit or implicit model-following methods as studied by Huang and Stengel (1990). When some system properties of the reconfigured system, such as the eigenstructure (Jiang (1994)), the closed loop system matrices (Gao and Antsaklis (1991)) or the I/O functionality (Yang and Blanke (2000)), are required to match those of the nominal system in some sense, these approaches are referred to as the model-matching methods (Yang (2000)). The biggest advantage of using the model-matching methods comparing with the model-following methods is that the reference model (nominal system) does not need to operate parallel with the practical system, therefore the reconfigured system has a simple structure and efficient computation.

However, no matter what kind of CR strategies, most current CR methods are restricted to linear system models, although in several applications, the CR design results derived based on linear system designs were applied into the nonlinear systems, these procedures are still very *ad hoc*, due to

- Lack of systematic analysis and guarantee for the validation of implementing the linear design results into nonlinear processes;
- Limited capability of one linear model to approximate a nonlinear system within its whole operating range. For example, the dynamical model of a airplane within the whole flight envelop should be represented by a family of LTI models owing to different flight heights and speeds.

Therefore, a lot of experiments and simulations were needed before those design results were applied in practice.

From the system-recoverable point of view, the CR for nonlinear systems can be regarded as a kind of nonlinear model-following or matching problems, however there are few work to deal with this kind of problems at present. The current non-linear design methods such as feedbacklinearization maybe can cope with the model-following CR problem, but it would be quite difficult for them to explore the model-matching CR methods,

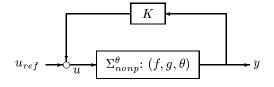


Fig. 1. Considered Nonlinear Control Systems

because til now it is not clear which kind of nonlinear system properties will play a critical rule in the nonlinear control reconfiguration. However, another challenging way for these problems is to use the multiple-model scheme, which has showed elegant potential in dealing with piecewise smooth nonlinear systems (Murray-Smith and Johansen (1997)). Besides that, within the multiple-model framework, many LTI-based design methods can still be used efficiently.

In this paper, an on-line multiple-model based adaptive control reconfiguration method is proposed from the system-oriented CR point of view. Instead of requiring the nominal and faulty nonlinear systems to match each other directly in some proper sense, a set of compensating modules are proposed and designed so as to make the local LTI model approximating to the reconfigured nonlinear system match the corresponding LTI model approximating to the nominal nonlinear system in some optimal sense. The compensating modules can be designed by the Pseudo-Inverse Method based on the local LTI models for the nominal and faulty nonlinear systems with respect to the model-matching strategy. Furthermore, these modules should update corresponding to the updatings of local LTI models. The test on a nonlinear ship propulsion system shows the promising potential of this method for system reconfiguration.

### 2. PROBLEM FORMULATION

Consider a class of nonlinear control systems with static output(state) feedback as shown in Fig. 1<sup>1</sup>. Let the nonlinear plant denote as  $\Sigma_{nonp}^{\theta}$ , which is assumed to have a general form:

$$\begin{cases} \dot{x}(t) = f(x(t), u(t), \theta(t)), \ x(t_0) = x_0 \\ y(t) = g(x(t), u(t), \theta(t)) \end{cases}$$
(1)

where  $x(t) \in X \subseteq R^n$ ,  $u(t) \in U \subseteq R^m$  and  $y(t) \in R^r$  are the state, input and output vectors with proper dimensions.  $\theta(t) \in R^p$  is called *fault parameter vector*, which is piecewise constant corresponding to abrupt faults and different entries  $\theta_i(t)$ ,  $i = 1, \dots, p$ , represent different fault situations in the plant. Therefore, case  $\theta(t) \equiv 0$ 

<sup>&</sup>lt;sup>1</sup> This configuration can also describe the cascaded control systems when we combine the cascaded controller and plant together into an augmented plant with unit feedback.

represents the nominal situation. Vector function  $f: R^n \times R^m \times R^p \mapsto R^n$  and  $g: R^n \times R^m \times R^p \times R^p \mapsto R^r$  are piecewise continuous. The static feedback as shown in Fig.1 can be described as  $u(t) = u_{ref}(t) - Ky(t)$  with assumption that the matrix  $K \in R^{m \times r}$  makes the nominal closed loop system stable, which is denoted as  $\Sigma_{non}^0$  with vector field  $\dot{x}_n$  and output  $y_n$ .

When some fault(s) happened inside the plant at instant  $t_f$ , assume the parameter  $\theta(t) = \theta_f \neq 0$ for  $t > t_f$  can be provided by FDI mechanism. Let the faulty plant and the corresponding faulty closed loop system denote as  $\sum_{nonp}^{\theta_f}$  and  $\sum_{non}^{\theta_f}$  respectively. Assume that the plant system can not change artificially during the operation, i.e., the CR design is only restricted to adjusting the feedback matrix K as well as pre/post-compensatingthe system input  $u_{ref}$  and output y signals, such as the possible reconfiguration structure as shown in Fig.2. Here we refer to these modules  $\mathcal{K}_f$ ,  $\mathcal{K}_u, \mathcal{K}_y$  and  $\mathcal{K}_u$  as compensating modules (Yang and Blanke (2000)). When we consider the matching characters of both the dynamic and output properties of the considered nonlinear control system, an optimal problem can be proposed as:

Synthesize modules  $\mathcal{K}_u$ ,  $\mathcal{K}_f$ ,  $\mathcal{K}_y$  and  $\mathcal{K}_d$ , denoted as  $\{\mathcal{K}_i\}$ , such that the reconfigured closed loop nonlinear system, denoted as:

$$\Sigma_{non}^{cr}: \begin{cases} \dot{x}_{cr}(t) = f_{cr}(x_{cr}(t), u_{ref}(t), \theta_f, \{\mathcal{K}_i\}), \\ y_{cr}(t) = g_{cr}(x_{cr}(t), u_{ref}(t), \theta_f, \{\mathcal{K}_i\}), \end{cases} (2)$$

satisfies  $\forall (x, u_{ref}) \in X \times U$ , there is

$$\min_{\{\mathcal{K}_i\}} J(x, u_{ref}, \theta_f, \{\mathcal{K}_i\}), \tag{3}$$

where

U.

 $J \doteq \alpha(x, u_{ref}) \|\dot{x}_n - \dot{x}_{cr}\|_2 + \beta(x, u_{ref}) \|y_n - y_{cr}\|_2$ Here  $\alpha(x, u_{ref})$  and  $\beta(x, u_{ref})$  are weighting functions for different operating points  $(x, u_{ref}) \in X \times$ 

It is obvious that this optimal problem is too conservative to get a solution. In this paper, in order to get the matrix-type compensating modules, a less conservative solution will be proposed based on the multiple-model scheme, i.e., Eq.(3) is almost satisfied at some countable state points which is obtained by solving some local optimal problem, meanwhile at any other point the variation of J is bounded by a scalar positive constant from a proper local optimal value.

#### 3. LINEAR MODEL APPROXIMATION

Consider the nonlinear system (1) within an open ball neighborhood of one operating point  $(x_0, u_0)$ ,

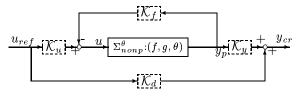


Fig. 2. Reconfiguration Modules

denoted as  $\mathcal{B}(x_0, u_0)$  with radius  $\delta_0$ . With respect to the nonlinear system theory (Isidori (1995)), once the vector functions f and g both are  $\mathcal{C}^1$ differentiable in x and u at point  $(x_0, u_0)$ , a  $\theta$ parameterized LTI system can be obtained as the local linear approximation of  $\Sigma^{\theta}_{nonp}$ , denoted the LTI model as  $\Sigma^{\theta}_{linp}$ . Then there is

**Lemma 1:** The closed loop LTI control system, denoted as  $\Sigma_{lin}^{\theta}$ , which is configured by the linear plant model  $\Sigma_{linp}^{\theta}$  with the same control structure as the nonlinear system  $\Sigma_{non}^{\theta}$ , is the local linear approximation of  $\Sigma_{non}^{\theta}$  within  $\mathcal{B}(x_0, u_{ref0})$ , if  $I_{r \times r} + D_0(\theta)K$  is full rank, and  $u_{ref0} = u_0 + Kg(x_0, u_0, \theta)$ , where  $D_0(\theta) = \frac{\partial g(x, u, \theta)}{\partial u}|_{(x_0, u_0)}$ .

**Lemma 2:** Once the vector functions f and g of (1) are  $\mathcal{C}^{\infty}$  differentiable in x and u at  $(x_0, u_0)$ , and all corresponding derivatives are  $l^2$ -induced norm bounded, there exist two positive reals related to  $\theta$  and  $\delta_0$ , denoted as  $\alpha_{cx}(\theta, \delta_0)$  and  $\alpha_{cy}(\theta, \delta_0)$ , satisfying  $\forall t \in (t_0, t_1), \forall u_{ref} \in \mathcal{B}_u \triangleq \{u_{ref} | (x, u_{ref}) \in \mathcal{B}(x_0, u_{ref0})\}$ , there is

$$\begin{aligned} \|\dot{x}_{n}(t) - \dot{x}_{clin}(t)\|_{2} &< \alpha_{cx}(\theta, \delta_{0}), \\ \|y_{n}(t) - y_{clin}(t)\|_{2} &< \alpha_{cy}(\theta, \delta_{0}), \end{aligned}$$
(4)

where  $\dot{x}_n(t)$  and  $y_n(t)$  ( $\dot{x}_{clin}(t)$  and  $y_{clin}(t)$ ) represent the tangent and output vectors of the closed loop nonlinear (linear) system at instant t, and ( $t_0, t_1$ ) denotes the time range when the system operates within  $\mathcal{B}(x_0, u_{ref0})$ .

**Theorem 1:** The nonlinear control system  $\Sigma_{non}^{\theta}$ within its whole operation range  $X \times U$  can be linearly approximated by a family of  $\theta$ parameterized LTI control systems, denoted as  $\{\Sigma_{lin}^{\theta^i}\}_{i=1}^N$ , once there exists a set of ordered points  $(x_i, u_{refi}) \in X \times U$  and a set of corresponding neighborhoods  $\mathcal{B}(x_i, u_{refi})$   $i = 1, \dots, N$ , where N can be a finite integer or  $+\infty$ , satisfying

$$\bigcup_{i=1}^{N} \mathcal{B}(x_i, u_{refi}) \supseteq X \times U, \text{ and}$$
$$\mathcal{B}(x_i, u_{refi}) \bigcap \mathcal{B}(x_{i+1}, u_{refi+1}) \neq \phi, \quad (5)$$

where  $\phi$  represents the empty set. Functions f and g belong to  $\mathcal{C}^1$  in x and u at any point  $(x_i, u_i)$ , and all the matrices  $I_{r \times r} + D_0^i(\theta) K$  are full rank, where  $D_0^i(\theta) = \frac{\partial g(x, u, \theta)}{\partial u}|_{(x_i, u_{refi})}$  and  $\Sigma_{lin}^{\theta^i}$  has the form

$$\begin{cases} \dot{x}^{i}_{clin} = A^{i}_{c}(\theta)x^{i}_{clin} + B^{i}_{c}(\theta)u_{ref} + \phi^{i}_{c}(\theta), \\ y^{i}_{clin} = C^{i}_{c}(\theta)x^{i}_{clin} + D^{i}_{c}(\theta)u_{ref} + \psi^{i}_{c}(\theta), \end{cases}$$
(6)

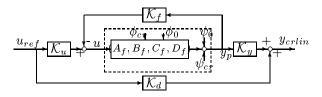


Fig. 3. Reconfiguration for Linear Control System

for  $x_{clin}^{i}(t_{0}^{i}) = x_{i}$  and  $(x_{clin}, u_{ref}) \in \mathcal{B}(x_{i}, u_{refi})$ . The parameters in (6) can be obtained by linearizing the nonlinear control system  $\Sigma_{non}^{\theta}$  at the point  $(x_{i}, u_{refi})$  or constructing from model  $\Sigma_{linn}^{\theta}$ .

#### 4. LOCAL CR DESIGN BY PSEUDO-INVERSE METHOD

Due to the simple computation and efficient implementation, the Pseudo-Inversed based methods have been popularly used in the CR design (Gao and Antsaklis (1991)). Here in order to the Pseudo-Inverse method for the local CR design, the faulty, nominal and reconfigured closed loop nonlinear systems  $\Sigma_{non}^{\theta_f}$ ,  $\Sigma_{non}^0$ ,  $\Sigma_{non}^{cr}$  and the LTI approximation models of them, denoted as  $\Sigma_{lin}^{\theta_f}$ ,  $\Sigma_{lin}^0$  and  $\Sigma_{lin}^{cr}$  respectively, need to be considered. Furthermore, we assume that a fault happened at  $t_f$  with  $t_f > t_0$  and the CR action begins at  $t_{cr}$  with  $t_{cr} > t_f$ . The corresponding state and input at  $t_{cr}$  is  $(x_0, u_{ref0}) \in X \times U$ . Denote a small open ball neighborhood of  $(x_0, u_{ref0})$  as  $\mathcal{B}(x_0, u_{ref0})$  with radius  $\delta_0$ .

#### 4.1 Design for Linear Control Systems

If we assume that no fault has happened till  $t_{cr}$ , the current operating plant should be  $\sum_{nonp}^{0}$ . We can get the LTI approximation of  $\sum_{nonp}^{0}$  within  $\mathcal{B}(x_0, u_{ref0})$ , and denote the LTI model as state space form  $(A_0, B_0, C_0, D_0, \phi_0, \psi_0)$  when matrix  $I + D_0 K$  is invertible. With respect to Lemma 1, the corresponding nominal closed loop linear system, denoted as  $\sum_{lin}^{0} ((x_{clin}, y_{clin}))$ , has the same form as (6), and we also denote  $\sum_{lin}^{0}$  as form  $(A_c, B_c, C_c, D_c, \hat{\phi}_c, \hat{\psi}_c)$  and  $x_{clin}(t_{cr}) = x_0$ .

However, the actually operating plant at  $t_{cr}$  is the faulty plant  $\Sigma_{nonp}^{\theta_f}$  instead of nominal plant  $\Sigma_{nonp}^{0}$  with respect to the fault assumption. Therefore, a local LTI approximation of  $\Sigma_{nonp}^{\theta_f}$  can also be obtained and denoted as  $(A_f, B_f, C_f, D_f, \phi_f, \psi_f)$  when  $I_{r \times r} + D_f K$  is invertible. Similarly, the corresponding closed loop linear system  $\Sigma_{lin}^{\theta_f}$  has the same form as (6).

We restrict the considered compensating modules to matrix-form, i.e.,  $K_f \in R^{m \times r}$ ,  $K_u \in R^{m \times m}$ ,  $K_y \in R^{r \times r}$  and  $K_d \in R^{r \times m 2}$ . Furthermore, in

order to deal with the  $\phi_c(\theta)$  and  $\psi_c(\theta)$  terms in (6), two local-constant vectors  $\phi_{cr} \in \mathbb{R}^n$  and  $\psi_{cr} \in \mathbb{R}^r$  are also introduced into this reconfigurable structure.

As a special case of CR design problem (3), when we choose  $\alpha(x, u_{ref}) \equiv 1$  and  $\beta(x, u_{ref}) \equiv 1$ , the CR design problem for LTI system  $\sum_{lin}^{\theta_f}$  within local region  $\mathcal{B}(x_0, u_{ref0})$  can be proposed as:

Find a set of compensating matrices, denoted as  $\mathcal{K}_{cr} \triangleq \{K_f, K_u, K_y. K_d, \phi_{cr}, \psi_{cr}\}$ , such that  $\forall (x, u_{ref}) \in \mathcal{B}(x_0, u_{ref0})$ , there is

$$\min_{\mathcal{K}_{cr}} (\|\dot{x}_{clin}(x, u_{ref}) - \dot{x}_{crlin}(x, u_{ref})\|_2, \\
+ \|y_{clin}(x, u_{ref}) - y_{crlin}(x, u_{ref})\|_2.$$
(7)

By employing the Pseudo-Inverse method, a Pseudo-solution can be obtained for (7) as:

$$\begin{cases}
K_{f}^{\star} = (I_{m \times m} - \Delta_{f} D_{f})^{+} \Delta_{f}, \\
K_{u}^{\star} = (B_{f} - B_{f} K_{f} (I_{r \times r} + D_{f} K_{f})^{+} D_{f})^{+} \\
(B_{0} - B_{0} K (I_{r \times r} + D_{0} K)^{-1} D_{0}), \\
K_{y}^{\star} = (I_{r \times r} + D_{0} K)^{-1} C_{0} ((I_{r \times r} + D_{f} K_{f})^{+} C_{f})^{+}, \\
K_{d}^{\star} = (I_{r \times r} + D_{0} K)^{+} D_{0} - K_{y} (I_{r \times r} + D_{f} K)^{+} D_{f}, \\
\phi_{cr}^{\star} = \phi_{0} - B_{0} K (I_{r \times r} + D_{0} K)^{+} \psi_{0} - \phi_{f} \\
+ B_{f} K_{f} (I_{r \times r} + D_{f} K_{f})^{+} \psi_{f}, \\
\psi_{cr}^{\star} = (I_{r \times r} + D_{0} K)^{+} \psi_{0} \\
- (I_{r \times r} + D_{f} K_{f})^{+} \psi_{f}.
\end{cases}$$
(8)

where  $M^+$  represents the Pseudo-Inverse of matrix M, and  $\Delta_f = B_f^+(A_f - A_0 + B_0K(I_{r \times r} + D_0K)^+C_0)C_f^+$ . This solution satisfies:

$$\begin{cases} a^{\star} \stackrel{c}{=} \min_{K_f} \|A_{cr} - A_c\|_2, \ b^{\star} \stackrel{c}{=} \min_{K_u} \|B_{cr} - B_c\|_2, \\ c^{\star} \stackrel{c}{=} \min_{K_y} \|C_{cr} - C_c\|_2, \ d^{\star} \stackrel{c}{=} \min_{K_d} \|D_{cr} - D_c\|_2, \ (9) \\ \|\hat{\phi}_{cr} - \phi_c\|_2 = 0, \qquad \|\hat{\psi}_{cr} - \psi_c\|_2 = 0. \end{cases}$$

Therefore, the solution (8) has the property:

$$\begin{cases} \mathcal{K}_{cr}^{\star} = \arg\min_{\mathcal{K}_{cr}} \|\dot{x}_{clin}(x, u_{ref}) - \dot{x}_{crlin}(x, u_{ref})\|_2, \\ \mathcal{K}_{cr}^{\star} = \arg\min_{\mathcal{K}_{cr}} \|y_{clin}(x, u_{ref}) - y_{crlin}(x, u_{ref})\|_2. \end{cases}$$

# 4.2 Design for Nonlinear Control Systems

The designed compensating matrices and vectors in Eq.(8) can be implemented directly into the faulty nonlinear control system  $\Sigma_{non}^{\theta_f}$ , thereby the reconfigured closed loop nonlinear system by using modules calculated in(8) has the form:

$$\Sigma_{non}^{cr}: \begin{cases} \dot{x}_{cr}(t) = f(x_{cr}(t), u(t), \theta_f) + \phi_{cr}, \\ y_p(t) = g(x_{cr}(t), u(t), \theta_f) + \psi_{cr}, \\ u(t) = K_u u_{ref}(t) - K_f y_p(t), \\ y_{cr}(t) = K_y y_p(t) + K_d u_{ref}(t), \end{cases}$$
(10)

with  $x_{cr}(t_{cr}) = x_0$  as shown in Fig.4.

<sup>&</sup>lt;sup>2</sup> Here we use the matrix notation  $K_i$  to represent the corresponding module  $\mathcal{K}_i$  for i = f, u, y, d.

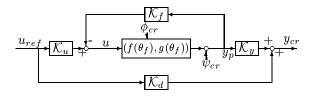


Fig. 4. Reconfiguration for Nonlinear Control System

Denote the reconfigured LTI control system by using modules (8) as  $\Sigma_{lin}^{cr}$ , which has a similar form as (6), then we have

**Theorem 2:** The (reconfigured) LTI system  $\Sigma_{lin}^{cr}$ is the linear approximation of the reconfigured nonlinear system  $\Sigma_{non}^{cr}$  within  $\mathcal{B}(x_0, u_{ref0})$ . Specially, once vector function f and g in (1) are  $\mathcal{C}^{\infty}$  in x and u, and all derivatives are  $l^2$ -induced norm bounded within  $\mathcal{B}(x_0, u_{ref0})$ , there exist two positive scalar constants related to  $\theta_f$ ,  $\delta_0$  and  $\mathcal{K}_{cr}$ , denoted as  $\alpha_{crx}(\theta, \delta_0, \mathcal{K}_{cr})$  and  $\alpha_{cry}(\theta, \delta_0, \mathcal{K}_{cr})$ , satisfying  $\forall t \in (t_{cr}, t_1')$ , there is

$$\begin{cases} \|\dot{x}_{cr}(t) - \dot{x}_{crlin}(t)\|_{2} < \alpha_{crx}(\theta, \delta_{0}, K_{cr}), \\ \|y_{cr}(t) - y_{crlin}(t)\|_{2} < \alpha_{cry}(\theta, \delta_{0}, K_{cr}), \end{cases}$$
(11)

where  $(t_{cr}, t'_1)$  represents the time interval that the nonlinear system  $\Sigma_{non}^{cr}$  operates within region  $\mathcal{B}(x_0, u_{ref0})$ .

**Proof**: From Lemma 1 and Lemma 2.

**Remark 1**: At the reconfiguring instant  $t_{cr}$ , the nonlinear systems  $\Sigma_{non}^{cr}$  and  $\Sigma_{non}^{0}$  have the relationship:

$$\begin{cases} \|\dot{x}_{cr}(t_{cr}) - \dot{x}_{n}(t_{cr})\|_{2} \leq a^{\star} \|x_{0}\|_{2} + b^{\star} \|u_{ref0}\|_{2} \\ \|y_{cr}(t_{cr}) - y_{n}(t_{cr})\|_{2} \leq c^{\star} \|x_{0}\|_{2} + d^{\star} \|u_{ref0}\|_{2}. \end{cases}$$
(12)

where  $a^*, b^*, c^*, d^*$  are determined in (9). Case  $a^* = b^* = c^* = d^* = 0$  means when the linear system  $\Sigma_{lin}^{\theta_f}$  can be completely recovered by modules in (8), then the nonlinear  $\Sigma_{non}^{\theta_f}$  can also be completely recovered at point  $(x_0, u_{ref0})$  by modules in (8).

#### 5. ON-LINE ADAPTIVE STRATEGY

From the system relationships, we have

$$\underbrace{\begin{array}{l} \|\dot{x}_{n}(t) - \dot{x}_{cr}(t)\|_{2} + \|y_{cn}(t) - y_{cr}(t)\|_{2} \leq \\ (\|\dot{x}_{n}(t) - \dot{x}_{clin}(t)\|_{2} + \|y_{cn}(t) - y_{cnlin}(t)\|_{2} + \\ (\|\dot{x}_{clin}(t) - \dot{x}_{crlin}(t)\|_{2} + \|y_{cnlin}(t) - y_{crlin}(t)\|_{2} \\ + \|\dot{x}_{cr}(t) - \dot{x}_{crlin}(t)\|_{2} + \|y_{cr}(t) - y_{crlin}(t)\|_{2} \\ + \|\dot{x}_{cr}(t) - \dot{x}_{crlin}(t)\|_{2} + \|y_{cr}(t) - y_{crlin}(t)\|_{2} \\ c_{r-term} \\ \end{array}}$$
(13)

It is obvious that the reconfiguration design can be decomposed into two cooperative parts: linear approximation design (cn-term and cr-term) and

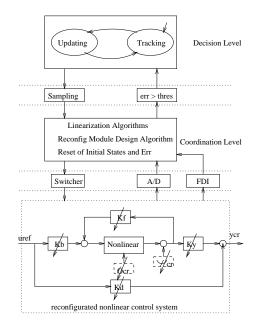


Fig. 5. Adaptive Reconfiguration Scheme

local CR design (*lcr-term*). In the following, an adaptive CR procedure is proposed after the FDI system provides the fault parameter  $\theta_f$ :

• Step 1: Sample one operating point  $(x_i, u_{refi})$ from the operating system  $\Sigma_{non}^{\theta_f}$  (probably through filters), then get the linear approximation  $\Sigma_{lin}^{\theta_{fi}}$  for  $\Sigma_{non}^{\theta_f}$  and  $\Sigma_{lin}^{0i}$  for the fictitious nominal system  $\Sigma_{non}^{0}$  both at  $(x_i, u_{refi})$ ;

• Step 2: Employ linear models  $\Sigma_{lin}^{\theta_f i}$  and  $\Sigma_{lin}^{0i}$  for the local compensating module design with respect to (8), and then implement the designed  $K_{cr}$  modules into the operating nonlinear system  $\Sigma_{non}^{\theta_f}$ . Thereby, the current operating system becomes the reconfigured nonlinear system  $\Sigma_{non}^{cr}$ . Meanwhile we need to get the linear approximation  $\Sigma_{lin}^{cri}$  for system  $\Sigma_{non}^{cr}$  in order to supervise the model updatings;

• Step 3: From the common initial point  $(x_i, u_{refi})$ , keep the fictitious linear systems  $\Sigma_{lin}^{0i}$  and  $\Sigma_{lin}^{cri}$ and nonlinear system  $\Sigma_{non}^{0}$  operating parallel (in software programs) to the practical operating system  $\Sigma_{non}^{cr}$ . Meanwhile, monitor the inequality:  $||Err(t)||_2 \leq Thres$ , where Err(t) represents proper combination of tracking error functions of linear models to corresponding nonlinear systems plus local CR design error, it could be the weighted right part of inequality (13). Thres is a given threshold, and it can be selected as an adaptive one with respect to the local CR design (8);

• Step 4: When  $||Err(t)||_2 \leq Thres$  is valid, keep the current nonlinear and linear systems operating, and this case is denoted as the *Tracking* state in Decision level in Fig.5; Otherwise, the fictitious linear systems as well as the compensating modules need to be updated according to step (1)-(2), and this procedure is abstracted as the *Updating* state in Decision level in Fig.5.

**Remark 2**: The model updatings and control mixer design should be finished quickly comparing with the system operation. Otherwise, the evaluation criterion (13) will have no sense.

### 6. BENCHMARK STUDY

The proposed method is tested in the ship propulsion benchmark proposed in Izadi-Zamanabadi and Blanke (1999). In order to reject the disturbances from the noises and external inputs, a filter is used for acquisition of the linearizing points. Here just the *cr-term* is used as Err(t) function, and *Thres* is selected as an adaptive one:  $Thres = w_{\theta} |\theta_{non}^{cr}| + w_n |n_{non}^{cr}| + w_u |U_{non}^{cr}|$ , where  $w_{\theta} = 0.1$ ,  $w_n = 0.1$  and  $w_u = 0.2$ .

(1) When  $k_r^f = -k_r$ , this fault caused the shaft speed overshot the maximum (12.5 rad/sec). When CR was implemented, the overshot was avoided as shown in Fig.6. There are total 46 times of LTI model updatings.

(2) when  $k_t^f = -k_t$ , this fault caused negative pitch angle and decreased the ship speed. When CR was implemented, the nonlinear system is completely recovered as shown in Fig.7. There are total 54 times of LTI model updatings.

# 7. CONCLUSIONS

A multiple-model based adaptive control reconfiguration method is proposed for nonlinear system consideration. The multiple-model concept has three manifolds: (1) A family of on-line LTI models is used to approximate the nonlinear system operation; (2) A set of linear approximations  $\{\Sigma_{lin}^{0i}\}$  and  $\{\Sigma_{lin}^{cri}\}$ ) are used simultaneously for the local model validations; and (3) The compensating module design is based on the on-line faulty and nominal linear approximation models  $\{\{\Sigma_{lin}^{0i}\}\}$ and  $\{\Sigma_{lin}^{\theta fi}\}$ ). This method fits into the hierarchical hybrid framework well. The test on a nonlinear ship propulsion system shows the promising potential of this method for system reconfiguration.

Discussions about the system stability and robustness by using this method will be the subjects of our future work.

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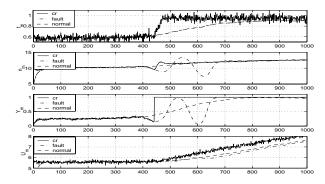


Fig. 6. Fault:  $k_r^f = -k_r$ , Time:×0.1 sec,  $T_f$ : 40sec,  $T_{cr}$ : 44sec

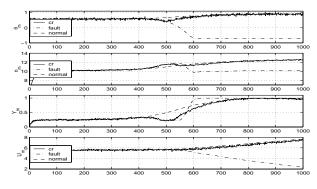


Fig. 7. Fault:  $k_t^f = -k_t$ , Time:×0.1 sec,  $T_f$  : 40sec,  $T_{cr}$  : 50sec

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