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Blanke, M.; Knudsen, Morten Haack

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Mogens Blanke and Morten Knudsen

Department of Control Engineering, Aalborg University, DK-9220 Aalborg, Denmark
E-mail:{blanke,mk}@control.auc.dk

Abstract: Simulation of maneuvering and design of motion controls for marine systems require non-linear mathematical models, which often have more than one-hundred parameters. Model identification is hence an extremely difficult task. This paper discusses experiment design for marine systems identification and proposes a sensitivity approach to solve the practical experiment design problem. The applicability of the sensitivity approach is demonstrated on a large non-linear model of surge, sway, roll and yaw of a ship. The use of the method is illustrated for a container-ship where both model and full-scale tests were conducted.

Keywords: marine systems, modeling, system identification, non-linear ship models, experiment design.

1 INTRODUCTION

Parameters in non-linear mathematical models of ship dynamics are used for simulation of ship motions, and for the design of closed loop control systems. Model structure and values of parameters can be estimated in model basin tests, but continuous validation of experimental techniques and computational methods still require full-scale sea trials. They are employed to improve model to full scale predictions and investigate dynamic phenomena with new forms of hulls of surface ships, subsea vehicles and floating offshore structures. Despite two decades of experience with system identification, it is still a fairly difficult task to design experiments for system identification using the hydrodynamic non-linear differential equations. Reasons are that the non-linear models have extremely high complexity and the possibilities for excitation is fairly limited in full scale.

Methods for parameter identification in continuous-time systems have been studied earlier and some results are indeed available for non-linear systems. In the marine systems case, two problems need to be solved. One is the complexity of mathematical models and over-parameterization in the equations of motion, which renders some parameters heavily correlated in certain experiments. The second is the limited excitation possibilities in full scale. The consequence is that we need to design experiments carefully to be able to determine a model.

In this paper, we introduce a sensitivity approach that treats the combined problem of experiment design and parameter identification in a theoretically well motivated way. The classical result of parameter uncertainty, usually expressed in Fishers information criterion, are discussed. It is found that the largest source of parameter error is due to under-modeling, which is not considered with commonly used methods. The approach presented in the paper is shown to be a practical way to design experiments and get assessment of real parameter errors. Results from full-scale tests with a container vessel illustrate the concepts.

2 THE EXPERIMENT DESIGN PROBLEM

Popular tests for verification of properties of surface ships include the turning circle test Diudonné (1953) and the reversed spiral test Bech (1968). Using an easy experimental technique, both investigate the directional stability properties from a plot of the quasi-steady relation between turn rate and rudder angle - the steering characteristic. Other tests are zig-zag tests. A $\{\delta_{com}, \Delta\psi\}$ zig-zag test Comstock (ed.) apply a rudder angle δ_{com} until a heading change of $\Delta\psi$ has been obtained. The rudder command is then changed to -δ_{com} which value is held until -$\Delta\psi$ is reached, etc. Time-history plots are used to determine overshoot in heading and the time-constants of the response. These give indications of stability properties for the trained observer. The development of the classical tests can be seen as early attempts to design experiments to assess basic stability properties and crude dynamic measures of ship "responsiveness" in steering. With the advent of system identification theory, much more sophisticated analysis are at our disposal in widely available tool-boxes. The present paper shows how full-scale experiments can be designed to obtain identification of selected parts of the linear and non-linear parts of a mathematical model. The theory shows which parameter errors can be expected with a given level of measurement noise and disturbances from wind and sea.

An example of a mathematical model for a marine system is that of a container ship in four degrees of freedom (surge, roll, sway, yaw), was published in Blanke and Jensen (1997). The structure of these equations is used for experiment design.

2.1 Model structure

The motion of a marine vehicle in six degrees of freedom can be conveniently expressed in a reference system, the origin of which is chosen in the water plane and in the centre of symmetry of the hull. Motions in surge, sway, yaw and roll are considered. The coordinate systems used by the Danish Maritime Institute (DMI) remains horizontal with the xy plane in the water-plane and moves with the ship Chislett (1990). The Newtonian equations are then:

$$\begin{aligned}
&m(\dot{u} - vr - x_G\, r^2 + z_G\, pr\cos(\varphi)) = X\\
&m(\dot{v} + ur - z_G\,\dot{p}\cos(\varphi) + x_G\,\dot{r}) = Y\\
&I_{zz}\,\dot{r} + mx_G\,(ur + \dot{v}) = N\\
&I_{xx}\,\dot{p} - mz_G\,(ur + \dot{v})\cos(\varphi) = K - \rho g\nabla G_z(\varphi)
\end{aligned} \tag{1}$$

The SNAME standard notation and sign conventions have been used SNAME (1950). A model of a ship in four degrees of freedom was the object of an experimental research effort with the RPMM at DMI. Results were described in Blanke and Jensen (1997) where the vessel is a 230 m long container ship, with a displacement of 46.000 m^3.

3 IDENTIFICATION OF PHYSICAL PARAMETERS

The basic principle being direct identification of physical parameters is to adjust model parameters directly until reaching the minimum of the 2-norm of the deviation between the sampled system output and the simulated model output. This procedure appears very similar to the classical output error approach for estimating the discrete-time parameters Ljung (1987). The important difference is that the procedure for minimizing the performance function is operating directly on the physical, continuous-time parameters, Blanke and Knudsen (Oct 1998).

The parameters are estimated as follows. The simulation model may be in state-space form or a combination of linear dynamic transfer functions and static non-linearities. In both cases the model output can be expressed as in Eq. 2. The functional of the output vector for the ship (vector of all measured states) is predicted by the non-linear model using measured input excitation μ as input to the model,

$$\hat{y}(k) = f\left(\mu_N, \hat{\theta}\right) \tag{2}$$

where $\hat{\theta}$ is a vector containing estimates of physical parameters, μ_N is the input vector with N samples, and $f(\mu, \theta)$ is the non-linear equations of motion. The discrepancy between measurement and model prediction - when both are excited by the input signal μ - is the model error, ε,

$$\varepsilon(k) = y(k) - \hat{y}(k) \tag{3}$$

A performance function $V(\theta)$ to be minimized is then conveniently taken to be quadratic,

$$V\left(\hat{\theta}\right) = \frac{1}{2N}\sum_{k=1}^{N}\varepsilon^2\left(k, \hat{\theta}\right) \tag{4}$$

The parameter estimate $\hat{\theta}$ based on N input-output data points, μ_N and y_N, is the value $\hat{\theta}_N$ that minimizes $V\left(\mu_N, y_N, \hat{\theta}\right)$

$$\hat{\theta}_N = \arg\min_{\theta}\left(V\left(\mu_N, y_N, \hat{\theta}\right)\right) \tag{5}$$

The estimate $\hat{\theta}_N$ can be obtained through minimization of this criterion. The minimization can be done with several methods. Tiano and Blanke (1997) used a simulated annealing search method. Here, a Gauss-Newton algorithm is employed. This requires the Hessian H, which can be approximately determined from the model gradient $\Psi(k)$

$$\Psi(k) = \frac{\partial \hat{y}(k)}{\partial \hat{\theta}} \tag{6}$$

and

$$H = \frac{\partial^2 V\left(\hat{\theta}\right)}{\partial \hat{\theta}\ \partial \hat{\theta}^T} = \frac{1}{N}\sum_{k=1}^{N} \Psi(k)\ \ \Psi^T(k) \tag{7}$$

The gradient $\Psi(k)$ can be determined analytically in some cases, but is always available through numerical differentiation.

The normed root mean square output error

$$\varepsilon_{RMSn} = \frac{\varepsilon_{RMS}}{y_{RMS}} \tag{8}$$

is a more significant number for expressing the model fir than the not-scaled performance function $V(\theta)$. It is intuitively comprehensible that an accurate estimate of a parameter 'θ_i requires $V(\theta)$ to be sensitive to θ_i, and that - roughly - the most sensitive parameters will be estimated most accurately. Relative parameter sensitivities are more meaningful, and they can be obtained by introducing relative parameters and normed signals. The relative normed Hessian is then

$$H_{rn} = y_{RMS}^{-2} LHL \tag{9}$$

where L is a diagonal matrix $L = \mathrm{diag}(\theta)$. However, a good fit, i.e. small values of ε_{RMSn} and V, only indicates that the model structure is adequate for expressing the system behavior for a particular input signal. In Knudsen (1994) it is shown how characteristic sensitivity measures are very convenient for determining whether a good fit also implies accurate parameter estimates.

The minimum sensitivity of the parameter dependent part of the model error $\varepsilon_{p,RMSn}(\theta)$ with respect to one relative parameter θ_n- for arbitrary values of the remaining parameters - is found to be Knudsen (1994)

$$S_{i\,\min} = \sqrt{\left(H_{rn}^{-1}(\theta_N)_{ii}\right)^{-1}} \tag{10}$$

Also, the ratio R of the maximum and minimum sensitivities in any direction in the parameter space is essential

$$R = \frac{S_{\max}}{S_{\min}} \tag{11}$$

$$S_{\max} = \sqrt{\lambda_{\max}} \quad and \quad S_{\min} = \sqrt{\lambda_{\min}}$$

where λ denotes eigenvalues of H_{rn} and represent the sensitivity in the parameter space. These sensitivity measures are used for input design. In the sequel, we also need sensitivity measures for the individual parameters

$$S_i = \sqrt{H_{rn}(\theta_N)_{ii}} \tag{12}$$

and

$$R_i = \frac{S_i}{S_{i\,\min}} \tag{13}$$

Input design Within a class of input signals the input giving a minimum value of R is the one where determined. The input signals shall depend on few input signal parameter. One or more parameters shall control the frequency spectrum and, since the model is non-linear, one or more additional input signal parameters shall control the amplitude distribution.

It can be shown Knudsen (1996) that the estimated parameter error is inversely proportional to the sensitivity S_{imin}.

The total, relative estimated error for the i'th parameter can be determined as

$$\tilde{\theta}_{ri} = \frac{\varepsilon_{RMSn}^{x}}{S_{i\,\min}\sqrt{N}} + \frac{\varepsilon_{RMSn}^{m}}{S_{i\,\min}} \tag{14}$$

where ε_{RMSn}^{x} and ε_{RMSn}^{m} are the root mean square errors caused by noise and under-modeling, respectively. When input design has to be done prior to sea trials, we may employ the fully non-linear model and compare simulated responses of this with responses of the reduced complexity model.

Throughout the exercise of optimizing the input signal, the goal is to maximize the sensitivity $S_{i\,\min}$ for individual parameters since this minimizes both under-modeling and stochastic errors.

4 EXCITATION OF LINEAR AND NON-LINEAR PARTS

When designing experiments, the practical possibilities for excitation is the first thing to clarify. For the ship, rudder and propeller are the available actuators. From the structure of the equations, it is obvious, that the rudder is the prime actuator for motions in sway-roll-yaw.

Linear parameters of particular interest include the hydrodynamic parameters $Y_v^{'}$,$Y_r^{'}$ $N_v^{'}$, $N_r^{'}$, $K_v^{'}$, $K_p^{'}$. The stability lever Blanke (1982) in yaw is a combination of these linear terms and well determined parameters, such as the ship's mass

$$L_r^{'} = \frac{N_r^{'} - m^{'}x_G^{'}}{Y_r^{'} - m^{'}} \quad (>0); \quad L_v^{'} = \frac{N_v^{'}}{Y_v^{'}} \quad (>0) \tag{15}$$

The ship is directional stable if

$$L_r^{'} - L_v^{'} > 0 \tag{16}$$

Nonlinear terms of particular interest are $Y_{v|v|}^{'}$, $Y_{v|r|}^{'}, N_{v|v|}^{'}$, $N_{r|v|}^{'}$, $K_{v|v|}^{'}$. The sensitivity method makes it possible to investigate identifiability of individual parameters or to work with natural combinations such as the stability levers.

4.1 Test signal design

Test signals for determination linear and non-linear parameters in dynamic systems were treated in Godfrey (1993). For the marine case, the requirement of experiment conditions being practical, makes it is useful to reconsider properties of both the classical maneuvering tests and optimized dynamic test sequences.

4.1.1 Multi-frequency sine signal

A multi-sine test sequence is

$$\delta_s(t) = \delta_o \left(\sum_{i=1}^{7} \delta_i \; sin\left(\omega_i t + \varphi_{i,ran}\right) \right) \tag{17}$$

where ω_i is a selected test frequency, δ_i the amplitude of the ith component, and $\varphi_{i,ran}$ an initial, in principle random value of phase angle.

As an illustration of the optimization procedure, we select a limited number of linear parameters and design a test signal with only two sine components. The frequencies are fixed to 25 and 600s, for reasons of brevity, and the amplitude ratio of the two can be varied. The purpose is to identify parameters in linear steering and roll motions. The following parameters are selected for illustration of the concept: $N_v^{'}$, $N_r^{'}$, $K_v^{'}$, $K_p^{'}$. The sensitivities obtained for variation of the amplitude of the sine with the lower frequency are shown in Fig. 1.

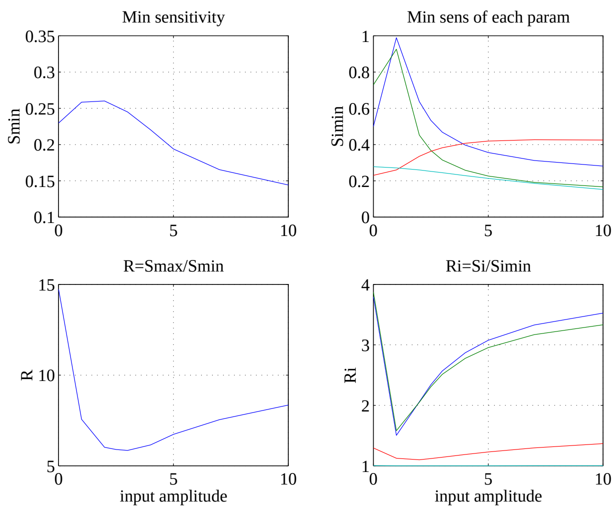

Fig. 1. Sensitivity measures plotted for varying input amplitude ratio between the two sines in the test signal. The minimum of the combined sensitivities is at $a = 3$. The sensitivity of two parameters is particularly high at $a = 1$, as seen in Simin, but this is not the overall optimum.

5 PRACTICAL TEST SEQUENCES

The optimality of a test sequence for non-linear systems identification is by no means unique. Signal shape, frequency content and amplitude are free parameters. Step changes in rudder are the most practical for actual testing and used as the basic waveform here.

5.1 Multi-frequency binary test

A multi-frequency binary test sequence Jensen (1959) is one where the amplitude is limited to $\pm\,\delta_o$,

$$\delta_m(t) = sign\left\{\delta_s(t)\right\}$$

The amplitudes and initial phase angles are selected to optimize the spectrum of $\delta_m(t)$ such that a large part of the excitation signal energy is present at the desired frequencies. The binary multi-frequency test is easy to apply in practice Blanke and Jensen (1997). The multi-frequency binary test sequence is optimized to give persistent excitation and low parameter variance in both steering and roll. Furthermore, the signal is constructed to avoid inter-modulation phenomena in the frequency domain from quadratic terms. This is achieved using the sampling interval T$_s$ of the dataloging device in the generation of the signal. The duration of the sequence is T_{per}. With natural roll period $\omega_o$we get

$$\omega_k = round(\frac{\alpha}{\beta}\frac{T_{per}\omega_0}{2\pi})\frac{2\pi}{T_{per}}, k = 1,2,..5 \tag{18}$$

$$\omega_6 = 3\frac{2\pi}{T_{per}}; \quad \omega_7 = \frac{2\pi}{T_{per}} \tag{19}$$

where $\beta = 80$, $\alpha = [19,33,51,80,117]$ give five of the seven excitation frequencies. The remaining two are chosen from the experiment period available. Amplitude and initial phases are easily optimized for the particular application, the ship in this case.

The time-history of the rudder angle from a sea-trial with a container ship is plotted in Fig. 2 together with turn rate and roll angle signals. Results from several experiments at sea showed this signal to be very efficient in identifying linear steering and roll dynamics.

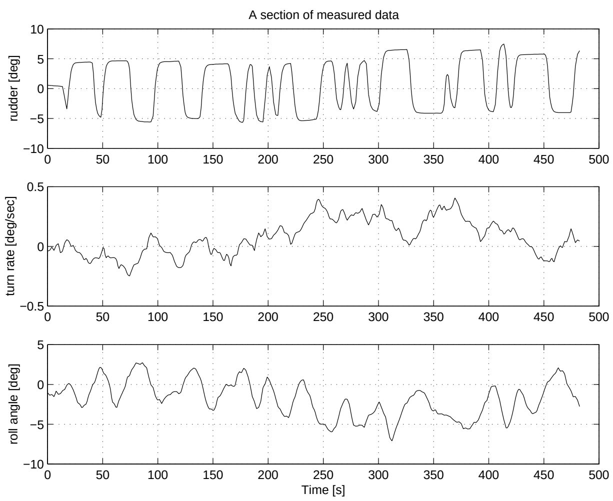

Fig. 2. Multifrequency binary test applied on a 46,000 m^3container vessel. The plot shows rudder angle, turn rate and roll angle. The entire test is 20 minutes long.

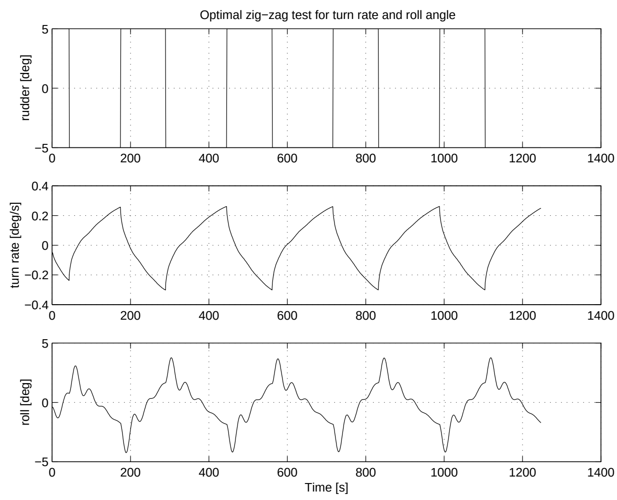

Fig. 3. Optimized test for identification of several linear and non-linear parameters. A 5-5 zig-zag test is the result.

δ_a	**2**	**5**	**10**	**20**	*Factor*
$S_{\min}$	.010	.015	.020	.027	
R	214	185	197	215	
N_v	.085	.071	.056	.041	$*1$
N_r	.079	.072	.062	.047	$*1$
N_δ	.136	.155	.102	.105	$*1$
Y_v	.098	.070	.058	.046	$*1$
Y_δ	.172	.154	.143	.152	$*1$
$Y_{v\vert v\vert}$	.011	.016	.022	.030	$*1$
$N_{v\vert v\vert}$	.025	.050	.068	.078	$*10$
$N_{r\vert r\vert}$	.034	.079	.107	.138	$*10$
$Y_{v\vert r\vert}$	.034	.057	.083	.115	$*500$
$N_{v\vert r\vert}$	.027	.062	.085	.104	$*50$

Table 1 Sensitivity of parameters with variation in amplitude of the zig-zag test

per.	**10**	**20**	**50**	**100**	**200**	
$S_{\min}$	.01	.03	.03	.02	.02	
R	552	167	158	284	395	
N_v	.04	.08	.06	.04	.03	$*1$
N_r	.07	.12	.08	.06	.05	$*1$
N_δ	.24	.21	.16	.09	.08	$*1$
Y_v	.05	.16	.13	.08	.07	$*1$
Y_δ	.19	.23	.21	.12	.09	$*1$
$Y_{v\vert v\vert}$	.01	.03	.04	.03	.02	$*1$
$N_{v\vert v\vert}$	.02	.10	.13	.08	.06	$*10$
$N_{r\vert r\vert}$	.06	.21	.27	.21	.16	$*10$
$Y_{v\vert r\vert}$	.06	.13	.14	.10	.08	$*500$
$N_{v\vert r\vert}$	.02	.15	.17	.12	.08	$*50$

Table 2 Sensitivity of parameters with variation of period for the modified spiral test

5.2 The zig-zag test

The zig-zag test is the everyday way to verify steering properties. It is a simple test used in acceptance trials. Several studies have treated the interpretation of this and similar tests, but the link to parameter accuracy has been weak. The sensitivity approach makes it easy to investigate the properties. In addition, optimization of amplitude and period of the test is straightforward. In our investigation, roll has been included as an output signal since roll properties are important for the steering properties. Fig. 3 shows the result of finding the optimal amplitude in a zig-zag test when the five most important linear terms are selected together with five non-linear coefficients. The resulting relative sensitivities are listed in Table 1. The Table shows, as expected, the linear terms to be best determined at low amplitudes of the test. The nonlinear ones improve in sensitivity the larger the amplitude. The three last nonlinear terms are quite insensitive in this form of excitation signal and $Y_{v|r|}$ can not be determined at all. The best experiment for an overall performance is the 5-5 zig zag test.

5.3 The spiral test

The classical spiral test of Diudonné or the inverse spiral test of Bech are likewise used in everyday practice. The two tests give a set of steady state values between rudder and turn rate. For a directional stable ship, rudder angle is the controlled variable, and the Diudonné test is a sequence of steps in rudder command: 0, 10, 20, -20 ...degrees. The result of optimizing the spiral test give the test signal shown in Figure 4. The optimization is given a fixed experiment length and a certain number of steps to execute.

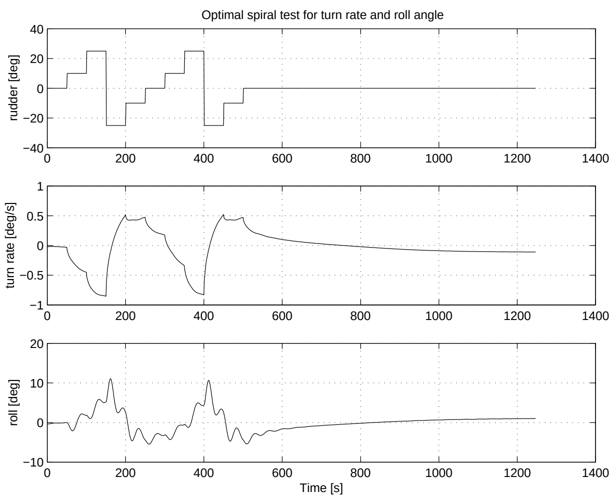

Fig. 4. Optimized spiral test to determine five linear and five non-linear hydrodynamic coefficients related with steering.

5.4 Evaluation of test signals

The ratio R, Equation (11), may be used for rating the three test sequences. The best is the optimal spiral test signal with $R = 159$, followed by the optimal zig-zag test signal with $R = 185$, and the multi-frequency signal with $R = 278$.

6 CONCLUSIONS

The paper has demonstrated the use of a sensitivity approach to design of input signals for use in experiments where the aim is a later identification of physical parameters in the manoeuvering equations. The sensitivity approach was shown to be efficient in determining which parameters could be identified with satisfactory accuracy using excitation from the rudder.

The method was applied to design optimized experiments for both linear and non-linear parameters. The classical manoeuvering tests, the zig-zag and the (inverse) spiral were found not to be optimal when carried out in the traditional ways. Instead, alternative test sequences were suggested that optimize identifiability of selected linear and non-linear steering parameters.

REFERENCES

Bech, M. I. (1968). The reversed spiral test as applied to large ships. *Shipping World and Shipbuilder* pp. 1753–1754.

Blanke, M. (1982). Ship Propulsion Losses Related to Automatic Steering and Prime Mover Control.. PhD thesis. Technical University of Denmark. Lyngby, Denmark.

Blanke, M. and A. G. Jensen (1997). Dynamic properties of vessels with low metacentric height. *Transactions of the Institution of Measurement and Control (UK).*

Blanke, M. and M. Knudsen (Oct 1998). A sensitivity approach to identification of ship dynamics from sea trial data. In: *IFAC Conference CAMS'98*. Fukuoka, Japan. pp. 261–269.

Chislett, M. S. (1990). The addition of heel-roll servo mechanism to the dmi horizontal planar motion mechanism. In: *Proc. MARSIM and ICSM.* Tokyo.

Comstock(ed.), J. P. (1967). *Principles of Naval Architecture.* Society of Naval Architecture and Marine Engineers. New York.

Diudonné, J. (1953). Collected french papers on the stability of route of ships at sea. Technical Report Technical Report DTMB-246. Naval Research and Development Center, Washington D.C. USA.

Godfrey, K. (1993). *Pertubation Signals for System Identification.* Prentice Hall Int.

Jensen, J. R. (1959). Notes on measurements of dynamic characteristics of linear systems. Technical report. Servoteknisk Forskningslaboratorium, Den Polytekniske Læreanstalt. København, Danmark.

Knudsen, M. (1994). A sensitivity approach for estimation of physical parameters. In: *10th IFAC Symposium on System Identification.* Vol. 2 p.231. Copenhagen, Denmark.

Knudsen, M. (1996). Determination of parameter estimation errors due to noise and under modelling. In: *IEEE Instrumentation and Measurement Technology Conference.* Brussels.

Ljung, L. (1987). *System Identification - Theory for the User.* Prentice Hall Int.

SNAME (1950). Nomenclature for treating the motion of a submerged body through a fluid. Technical Report Technical Research Bulletin No. 1-5. Society of Naval Architects and Marine Engineers. New York, USA.

Tiano, A. and M. Blanke (1997). Multivariable identification of ship steering and roll motions. *Proceedings Institute of Measurement and Control (UK).* Special Issue on Marine Systems Modelling and Control.