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SLIDING MODE ATTITUDE CONTROL FOR MAGNETIC ACTUATED SATELLITE

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Abstract: Magnetic torquing is attractive as a control principle on small satellites. The actuation principle is to use the interaction between the earth's magnetic field and magnetic field generated by a coil set in the satellite. This control principle is inherently nonlinear, and difficult to use because control torques can only be generated perpendicular to the local geomagnetic field vector. This has been a serious obstacle for using magnetorquer based control for three-axis attitude control. This paper deals with three-axis stabilization of a low earth orbit satellite. The problem of controlling the spacecraft attitude using only magnetic torquing is realized in the form of the sliding mode control. A three dimensional sliding manifold is proposed, and it is shown that the satellite motion on the sliding manifold is asymptotically stable.

Keywords: Attitude control, satellite control, sliding mode, time-varying systems, Lyapunov stability, quaternion feedback.

1 INTRODUCTION

This study was initiated as a part of the Danish Ørsted satellite project. The Ørsted satellite is a 60 kg auxiliary payload to be launched in March 1998 into 450 x 850 km orbit with 96 degrees inclination. The satellite is equipped with an 8 m long gravity gradient boom. The two primary science objectives of the mission are to measure the main geomagnetic field and study its interaction with the solar wind plasma. The satellite is actuated by three mutually perpendicular electromagnetic coils, magnetorquers. The information of the satellite attitude and angular velocity is available through an onboard attitude determination system, see Bak *et al.* (1996).

Magnetic torquing is attractive for small, cheap satellites, since the magnetic control systems are relatively lightweight, require low power and are inexpensive. The challenge was that three-axis control was not possible with an actuation prin-

ciple that leaves the system controllable in only two degrees of freedom because the control torque can only be generated perpendicular to the local magnetic field of the Earth.

The controller is developed for a satellite without appendages, since the concept was originally formulated for the Ørsted satellite during the boom pre-release phase. A characteristic feature of this configuration is that the principal moments of inertia are of the same order of magnitude.

The available literature on magnetorquing for three axis stabilization of satellites includes the work of (Cavallo *et al.*, 1993), where a configuration with two magnetic coils and a reaction wheel was addressed. A novel approach based on a rule-based fuzzy controller was proposed by (Steyn, 1994). Still another approach for three axis magnetic stabilization of a low earth near polar orbit satellite based on Lyapunov theory was presented by (Wiśniewski and Blanke, 1996), where a global

stabilizing magnetic controller was derived.

A number of internationally published papers dealing with magnetic attitude control can be extended to these addressing a linear control problem. (Arduini and Baiocco, 1997) proposed a control law in which the desired control torque was defined first and then the actual magnetic generated control torque was derived. Another concept cited in the literature was based on an idea of designing magnetic controller for the system with averaged parameters. This design strategy was used both for bias momentum satellites (Hablani, 1995) and three axis control (Martel *et al.*, 1988). The local stabilization of the satellite was achieved via implementation of the infinite-time-horizon linear quadratic regulator. An energy optimal solution with use of Riccati periodic equation was presented in (Wisniewski, 1997).

This study gives an application of the sliding control to the magnetic actuation of a spacecraft. The essence of the sliding controller design is outlined in Section 3. The design algorithm is split into two steps: the sliding manifold design and the sliding condition design. A three dimensional sliding manifold is proposed in Section 4. Furthermore, motion of the satellite on the sliding manifold is shown to be asymptotically stable. An ideal case of the sliding condition development is when the control torque is producible in x, y, and z directions independently. A solution to this control problem is given in Section 5. Section 6 considers a sliding condition for the magnetic generated control torque. The simulation study shows that the sliding control is stable for satellites, which the principal moments of inertia are of the same order of magnitude.

2 SATELLITE MODEL DESCRIPTION

2.1 Coordinate Systems

- Control CS, CCS is a right orthogonal coordinate system built on the principal axes of the satellite with the origin placed in the centre of mass. The x-axis is the axis of the maximum moment of inertia, and the z-axis is the minimum.
- Orbit CS, OCS is a right orthogonal coordinate system fixed in the centre of mass of the satellite. The z-axis points at zenith, the x-axis points in the orbital plane normal direction and its sense coincides with the sense of the orbital angular velocity vector.
- World CS, WCS is an inertial right orthogonal coordinate system with origin in the centre of mass of the satellite. The z-axis is parallel to the rotation axis of the Earth and

points towards the North Pole. The x-axis points at Vernal Equinox.

2.2 Nomenclature

${}^c\mathbf{v}, {}^o\mathbf{v}, {}^w\mathbf{v}$	Vector \mathbf{v} in CCS, OCS and WCS
$\mathbf{\Omega}_{cw}$	Angular velocity of CCS relative to WCS
$\mathbf{\Omega}_{co}$	Angular velocity of CCS relative to OCS
ω_o	Orbital rate.
\mathbf{I}	Principal inertia tensor, $\mathbf{I} = \text{diag}([I_x \ I_y \ I_z]^T)$, where $I_x \geq I_y \geq I_z$
$\mathbf{i}_o, \mathbf{j}_o, \mathbf{k}_o$	Unit vector along x-, y-, z-axis of OCS
\mathbf{N}_{ctrl}	Control torque
\mathbf{N}_{gg}	Gravity gradient torque
\mathbf{N}_{dis}	Disturbance torque
\mathbf{m}	Magnetic moment
\mathbf{B}	Magnetic field vector.
${}^c\mathbf{q}$	Quaternion representing CCS relative to
$= [\mathbf{q}^T \ q_4]^T$	OCS, where \mathbf{q} is vector part and q_4 is scalar part
$\mathbf{A}({}^c\mathbf{q})$	Attitude matrix based on ${}^c\mathbf{q}$
n_{coil}	Number of coil windings
A_{coil}	Coil area
i_{coil}	Current in coil
\mathbf{s}	Sliding variable
\mathbf{S}	Sliding manifold
\mathbf{E}	Identity matrix

2.3 Formulation of Equation of Motion

The mathematical model of a satellite is described by the dynamic equations and the kinematic equations of motion, see (Wertz, 1990). In this paper the attitude is parameterized by four components of a quaternion describing rotation of the Control CS in the Orbit CS.

The dynamic equations of motion of a satellite considered as rigid body is

$$\mathbf{I} \dot{{}^c\mathbf{\Omega}_{cw}}(t) = -{}^c\mathbf{\Omega}_{cw}(t) \times \mathbf{I} {}^c\mathbf{\Omega}_{cw}(t) + {}^c\mathbf{N}_{ctrl}(t) + {}^c\mathbf{N}_{gg}(t) + {}^c\mathbf{N}_{dis}(t). \quad (1)$$

Control torque is generated by an interaction of the geomagnetic field with the magnetorquer current $i(t)$ which gives rise to a magnetic moment $\mathbf{m}(t)$

$$\mathbf{m}(t) = n_{coil} i_{coil}(t) A_{coil}. \quad (2)$$

The control torque acting on the satellite is then

$${}^c\mathbf{N}_{ctrl}(t) = {}^c\mathbf{m}(t) \times {}^c\mathbf{B}(t). \quad (3)$$

Gravity gradient torque is given by

$${}^c\mathbf{N}_{gg}(t) = 3\omega_o^2 ({}^c\mathbf{k}_o \times \mathbf{I} {}^c\mathbf{k}_o). \quad (4)$$

The disturbance torque is mainly due to the aerodynamic drag, see (Wisniewski, 1995).

The kinematic equations are expressed by separate integrations of the vector and the scalar part of the attitude quaternion

$$\begin{aligned}\dot{\mathbf{q}} &= \frac{1}{2} {}^c\boldsymbol{\Omega}_{co} q_4 + \frac{1}{2} {}^c\boldsymbol{\Omega}_{co} \times \mathbf{q}, \\ \dot{q}_4 &= -\frac{1}{2} {}^c\boldsymbol{\Omega}_{co} \cdot \mathbf{q}.\end{aligned}\quad (5)$$

The relation between satellite angular velocity in World CS and angular velocity w.r.t. Orbit CS is obtained by

$${}^c\boldsymbol{\Omega}_{co} = {}^c\boldsymbol{\Omega}_{cw} - \omega_o {}^c\mathbf{i}_o. \quad (6)$$

3 SLIDING MODE CONTROLLER DESIGN

The satellite trajectory is expected to be in the vicinity of the reference for the most of the operational time, but there are certain transition or contingency phases, that the satellite motion can not be considered as rotation in the neighbourhood of a reference and the nonlinear terms in Eqs. 1 to 6 can become dominant. The problem is, thus, inherent nonlinear and nonlinear control methods are needed.

A sliding mode controller is implemented for the attitude corrections using magnetic torquing. Full attitude information in the form of the attitude quaternion, ${}^c\mathbf{q}$, and the satellite angular velocity with respect to the Orbit CS, ${}^c\boldsymbol{\Omega}_{co}$, are used as feedback signals. The objective of the attitude control is to turn the satellite such that the Control CS coincides with the Orbit CS, i.e. $\boldsymbol{\Omega}_{co}$ converges to $\mathbf{0}$, the vector part of the attitude quaternion, \mathbf{q} , converges to $\mathbf{0}$, and the scalar part, q_4 , approaches 1.

The design strategy of the sliding mode controller consists of two steps, see (Utkin, 1992), (Slotine and Li, 1991):

1. Sliding manifold design.
2. Sliding condition design.

Consider a manifold, a 3 dimensional hyperplane, in the state space of a 6th order system $[{}^c\boldsymbol{\Omega}_{co} \mathbf{q}]^T$. The sliding manifold is designed in such a way that the satellite trajectory, if on the hyperplane, converges to the reference. However, the satellite motion is not confined to the 3 dimensional hyperplane in general. Therefore a control law forcing the satellite motion toward the manifold is necessary for achieving stable satellite motion. The sliding condition keeps decreasing the distance from the state to the sliding manifold, such that every solution ${}^c\boldsymbol{\Omega}_{cw}$, \mathbf{q} originating outside the

sliding manifold tends to it. Now the manifold is an invariant set of the satellite motion and the trajectory of the system converges to the reference.

The result of the sliding condition design will be a desired control torque. When the desired control torque is implemented the satellite trajectory converges to the sliding manifold, but due to the satellite motion is stable on the sliding manifold, its trajectory converges to the reference. However the magnetic actuated satellite possesses one serious obstacle: the magnetic control torque is confined to lie perpendicular to the geomagnetic field vector and may not comply with the control torque which is desired to turn the satellite toward the sliding manifold. With the geomagnetic field varying along an orbit this implies, e.g., that over the earth's poles the rotation about the z-axis of the Orbit CS is totally uncontrollable.

4 SLIDING MANIFOLD DESIGN

It will be shown the satellite motion on a certain 3 dimensional hyperplane in the 6-dimensional state space of the vector part of the attitude quaternion, \mathbf{q} , and the satellite angular velocity, ${}^c\boldsymbol{\Omega}_{co}$, is stable.

First, let a sliding variable, \mathbf{s} , be defined as in Eq. 7

$$\mathbf{s} = \mathbf{I} {}^c\boldsymbol{\Omega}_{co} + \boldsymbol{\Lambda}_q \mathbf{q}, \quad (7)$$

where $\boldsymbol{\Lambda}_q$ is a positive definite matrix.

The sliding manifold is the subspace of the state space, where the sliding variable equals $\mathbf{0}$

$$\mathbf{S} = \{\mathbf{q}, {}^c\boldsymbol{\Omega}_{co} : \mathbf{s} = \mathbf{0}\}. \quad (8)$$

The definition of the sliding variable, \mathbf{s} , in Eq. 7 guarantees convergence of \mathbf{q} to zero and q_4 to 1 with an exponential rate. To prove this statement, consider a Lyapunov candidate function

$$v_q = \mathbf{q}^T \mathbf{q} + (1 - q_4)^2, \quad (9)$$

is equivalent to

$$v_q = 2(1 - q_4), \quad (10)$$

since $\mathbf{q}^T \mathbf{q} + q_4^2 = 1$.

The time derivative of the Lyapunov candidate function is calculated applying the kinematics in Eq. 5

$$\dot{v}_q = \mathbf{q}^T {}^c\boldsymbol{\Omega}_{co}, \quad (11)$$

thus

$$\dot{v}_q = -\mathbf{q}^T \mathbf{I}^{-1} \boldsymbol{\Lambda}_q \mathbf{q}. \quad (12)$$

The time derivative of the Lyapunov function is negative definite, since $\boldsymbol{\Lambda}_q$ is the positive definite matrix. According to Lyapunov's direct method,

the equilibrium ${}^c\mathbf{q} = [0 \ 0 \ 0 \ 1]^T$, ${}^c\mathbf{\Omega}_{co} = \mathbf{0}$ is asymptotically stable if the satellite is on the sliding manifold, \mathbf{s} .

The 3 dimensional hyperplane, Eq. 8, in 6 dimensional space $[{}^c\mathbf{\Omega}_{co}^T \ \mathbf{q}^T]^T$ is sufficient to describe the motion of the satellite (7th order differential equation) in the sliding mode. Notice that the equilibrium ${}^c\mathbf{q} = [0 \ 0 \ 0 \ -1]^T$, ${}^c\mathbf{\Omega}_{co} = \mathbf{0}$ is unstable even though ${}^c\mathbf{q} = [0 \ 0 \ 0 \ -1]^T$ and ${}^c\mathbf{q} = [0 \ 0 \ 0 \ 1]^T$ represent the same attitude (Control CS coincides with Orbit CS). Furthermore if the sliding variable is defined as

$$\mathbf{s} = \mathbf{I}\mathbf{\Omega}_{co} - \mathbf{\Lambda}_q\mathbf{q}, \quad (13)$$

it is possible to show using the Lyapunov candidate function

$$v_q = \mathbf{q}^T\mathbf{q} + (1 + q_4)^2, \quad (14)$$

that the equilibrium ${}^c\mathbf{q} = [0 \ 0 \ 0 \ -1]^T$, ${}^c\mathbf{\Omega}_{co} = \mathbf{0}$ is asymptotically stable and the equilibrium ${}^c\mathbf{q} = [0 \ 0 \ 0 \ 1]^T$, ${}^c\mathbf{\Omega}_{co} = \mathbf{0}$ is unstable.

5 SLIDING CONDITION DEVELOPMENT

The objective of the analysis is to derive the desired control torque turning the satellite trajectory toward the sliding manifold. The satellite motion is described in the space of the sliding variable, \mathbf{s} . A salient feature of this approach is that the reduced 3rd order system is considered. The representation of the satellite motion in the space of the sliding variable is calculated by differentiation of the sliding variable $\mathbf{s}(t)$ with respect to time.

$$\dot{\mathbf{s}} = \mathbf{I}^c\dot{\mathbf{\Omega}}_{cw} - \omega_o {}^c\dot{\mathbf{i}}_o + \mathbf{\Lambda}_q\dot{\mathbf{q}} \quad (15)$$

The derivatives of the satellite angular velocity and the attitude quaternion are calculated according to the equations of kinematics and dynamics, Eqs. 5 and 1

$$\begin{aligned} \dot{\mathbf{s}} &= -{}^c\mathbf{\Omega}_{cw} \times \mathbf{I}^c\mathbf{\Omega}_{cw} + 3\omega_o^2 {}^c\mathbf{k}_o \times \mathbf{I}^c\mathbf{k}_o \\ &\quad - \omega_o \mathbf{I}({}^c\dot{\mathbf{i}}_o \times {}^c\mathbf{\Omega}_{co}) + \frac{1}{2}\mathbf{\Lambda}_q({}^c\mathbf{\Omega}_{co}q_4 + {}^c\mathbf{\Omega}_{co} \times \mathbf{q}) \\ &\quad + {}^c\mathbf{N}_{ctrl}. \end{aligned} \quad (16)$$

Assume that the satellite trajectory is on the sliding manifold. The equivalent control is the control necessary to keep the satellite on the sliding manifold. In other words if the control torque is equal to the equivalent control then the time derivative of the sliding variable equals zero. If the satellite is not on the sliding manifold, the desired control torque equals the sum of the equivalent control and a part making the sliding variable converge to 0:

$$\mathbf{N}_{des} = \mathbf{N}_{eq} - \mathbf{\Lambda}_s\mathbf{s}, \quad (17)$$

the equivalent control torque, \mathbf{N}_{eq} is

$$\begin{aligned} \mathbf{N}_{eq} &= {}^c\mathbf{\Omega}_{cw} \times \mathbf{I}^c\mathbf{\Omega}_{cw} - 3\omega_o^2 ({}^c\mathbf{k}_o \times \mathbf{I}^c\mathbf{k}_o) \\ &\quad + \omega_o \mathbf{I}({}^c\dot{\mathbf{i}}_o \times {}^c\mathbf{\Omega}_{co}) - \frac{1}{2}\mathbf{\Lambda}_q({}^c\mathbf{\Omega}_{co}q_4 + {}^c\mathbf{\Omega}_{co} \times \mathbf{q}), \end{aligned} \quad (18)$$

and $\mathbf{\Lambda}_s$ is a positive definite matrix.

If the control torque were producible in each direction the desired control, \mathbf{N}_{des} could be substituted in Eq. 16 for the control torque, \mathbf{N}_{ctrl} . Thus the time derivative of the sliding variable, \mathbf{s} would be:

$$\dot{\mathbf{s}} = -\mathbf{\Lambda}_s\mathbf{s}. \quad (19)$$

The system described by differential equation in Eq. 19 is stable, hence the sliding condition is fulfilled. Unfortunately, the magnetic generated torque is perpendicular to the local geomagnetic field vector and can only partly comply with Eq. 17.

6 MODIFIED SLIDING CONDITION

The satellite appears uncontrollable if fixed at any instant of time due to the magnetic torque vector is constrained to always lie perpendicular to the local geomagnetic field vector. A modified sliding condition for magnetic stabilized satellites is discussed in this subsection.

The desired control torque is projected on a vector defined by the components of a sliding variable, i.e. the desired control torque is resolved into two components: perpendicular and parallel to the sliding variable vector. Magnetic generated torque is due to compensate only the component parallel to the sliding variable vector.

Consider a problem of the orthogonal projection of the desired control torque, $\mathbf{N}_{des}(t)$ onto the instant sliding variable vector, $\mathbf{s}(t)$ (see Fig. 1). The desired control torque, $\mathbf{N}_{des}(t)$ has two components: parallel, $\mathbf{N}_{des}^{prl}(t)$, and perpendicular, $\mathbf{N}_{des}^{prp}(t)$, to the vector $\mathbf{s}(t)$.

The control torque, \mathbf{N}_{ctrl} needs only compensate \mathbf{N}_{des}^{prl} , since \mathbf{N}_{des}^{prp} does not decrease the distance from the satellite trajectory to the sliding manifold. This control principle has an intuitive interpretation. The component \mathbf{N}_{des}^{prl} is responsible for diminishing of the sphere radius in Fig. 1, whereas \mathbf{N}_{des}^{prp} is responsible for movement on the sphere surface (sphere radius remains unchanged).

The same results are acquired as a result of theoretical analysis and are formalized in Theorem 1.

Theorem 1 *The control torque that compensates \mathbf{N}_{des}^{prl} makes the distance from the state*

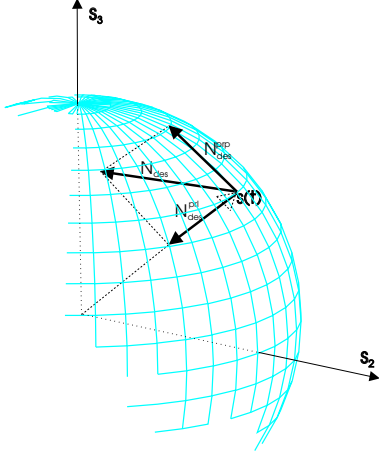


Fig. 1. Desired control torque resolved in s-space.

$[\mathbf{c}\Omega_{co}(t) \mathbf{q}(t)]^T$ to the sliding manifold in Eqs. 8 and 7 converge to zero, and the sliding condition is satisfied.

Proof of Theorem 1 *Barbalat's Lemma*, see (Slotine and Li, 1991) is used to prove that the manifold \mathbf{S} is an invariant set in the s-space. Construct a Lyapunov function:

$$v_s = \frac{1}{2} \mathbf{s}^T \mathbf{s}. \quad (20)$$

The motion in the s-space is described by the equation:

$$\dot{\mathbf{s}} = -\mathbf{N}_{eq} + \mathbf{N}_{ctrl}, \quad (21)$$

but the control torque compensates \mathbf{N}_{des}^{prl} , thus

$$\dot{\mathbf{s}} = -\mathbf{\Lambda}_s \mathbf{s} + \mathbf{N}^{prp}, \quad (22)$$

where \mathbf{N}^{prp} is a rest vector perpendicular to the vector $\mathbf{s}(t)$. Finally time derivative of the Lyapunov function is given by

$$\dot{v}_s = \mathbf{s}^T (-\mathbf{\Lambda}_s \mathbf{s} + \mathbf{N}^{prp}) = -\mathbf{s}^T \mathbf{\Lambda}_s \mathbf{s} \quad (23)$$

The time derivative of Lyapunov function is uniformly continuous and negative semidefinite, hence the conditions of Barbalat's lemma are fulfilled. ■

The control law, which has the objective to compensate \mathbf{N}_{des}^{prl} applying the magnetic actuation is only feasible when the geomagnetic field is not ideally parallel to the sliding variable, \mathbf{s} . Furthermore, if $\mathbf{c}\mathbf{B}$ and \mathbf{s} are near to parallel the amplitude of the control signal may be very large, since the large control torque \mathbf{N}_{ctrl} is desired to compensate even small \mathbf{N}_{des}^{prl} , see Fig. 2. In practice, the magnetic moment is confined, thus the ideal compensation of \mathbf{N}_{des}^{prl} is not possible. An approximate compensation is introduced

$$\mathbf{c}\mathbf{m} = \frac{\mathbf{c}\mathbf{B} \times \mathbf{N}_{des}^{prl}}{\|\mathbf{c}\mathbf{B}\|^2}, \quad (24)$$

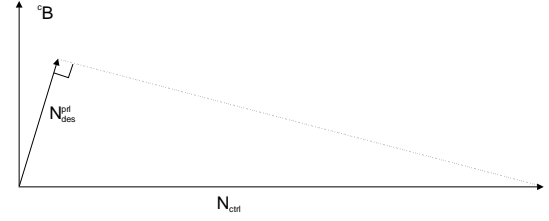


Fig. 2. Large \mathbf{N}_{ctrl} is necessary to compensate small \mathbf{N}_{des}^{prl} .

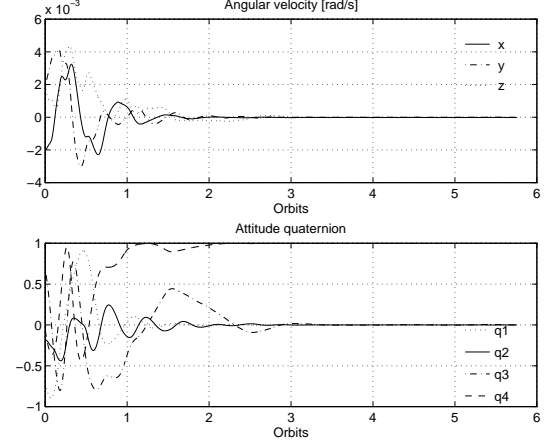


Fig. 3. The angular velocity, $\mathbf{c}\Omega_{co}$ and the attitude quaternion, $\mathbf{c}\mathbf{q}$. The attitude quaternion converges to the reference $[0 \ 0 \ 0 \ 1]^T$.

where

$$\mathbf{N}_{des}^{prl} = \frac{\mathbf{N}_{des} \cdot \mathbf{s}}{\|\mathbf{s}\|^2} \mathbf{s}. \quad (25)$$

Notice that the control law in Eq. 24 well compensates \mathbf{N}_{des}^{prl} , when $\mathbf{c}\mathbf{B}$ is perpendicular or nearly perpendicular to \mathbf{s} , and produces small control torque when $\mathbf{c}\mathbf{B}$ and \mathbf{s} are close to parallel.

The control law based on the approximate compensation of the desired control torque in Eq. 24 is observed to be locally stable for small values of the gain $\mathbf{\Lambda}_q$. Additional global stability property is gained when the principal moments of inertia are of the same order of magnitude (the satellite is in the boom stowed configuration). In this case the feedback generated according to Eq. 24 consists of the cross product of the angular velocity $\mathbf{c}\Omega_{co}$ with the local geomagnetic field vector, $\mathbf{c}\mathbf{B}$, plus small perturbation of the satellite attitude.

7 SIMULATION VALIDATION OF SLIDING MODE ATTITUDE CONTROL

The initial values corresponding to Euler angles are pitch 60 deg, roll 100 deg, and yaw -100 deg. Initial angular velocity is $\mathbf{c}\Omega_{co}$ is $[-0.002 \ 0.002 \ 0.002]^T$ rad/sec. The sliding mode attitude controller was evaluated by simulation for the Ørsted satellite with boom stowed configuration. A circular orbit with inclination of 96 de-

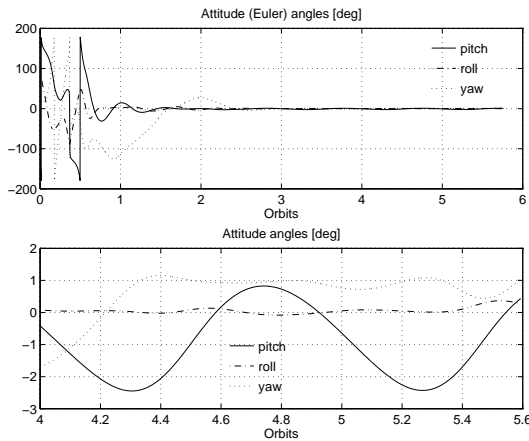


Fig. 4. Angular velocity and attitude quaternion of the Ørsted satellite on elliptical orbit. Motion is influenced by the aerodynamic drag.

grees has been simulated. The geomagnetic field has been computed using 8th order spherical harmonic model. Inertia tensor of the Ørsted satellite was equal $I = \text{diag}([3.4278 \ 2.9038 \ 1.2750])$. Various initial values of the angular velocity and the attitude were tested. The controller was evaluated for the initial values of the attitude both in the neighbourhood of the reference and for z-principal axis pointing up-side down relative to z-axis of the Orbit CS.

The control parameters were found empirically: $\Lambda_s = \text{diag}([0.003 \ 0.003 \ 0.003])$, $\Lambda_q = \text{diag}([0.002 \ 0.002 \ 0.002])$. Figure 3 shows simulation of the angular velocity and the attitude quaternion.

The satellite motion is influenced by the aerodynamic drag and eccentricity of the orbit. Simulation of the Ørsted satellite motion with disturbance torques acting on the satellite structure is depicted in Figure 4. Already after 2 orbits pitch, roll and yaw are within 10 deg. The sliding mode attitude controller keeps the steady state attitude error within ± 3 deg.

8 CONCLUSIONS

The sliding control law for three axis stabilization of a tumbling satellite was described and analysed in this paper. Both sliding manifold and continuous sliding condition were designed. The properties of the moments of inertia of a satellite body, when a gravity gradient boom is stowed, were used to provide the final control law. The controller was evaluated by simulations for the Ørsted satellite in realistic space environment. The controller shows to be applicable for satellites with pointing accuracy larger than 3 deg.

ACKNOWLEDGEMENT

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